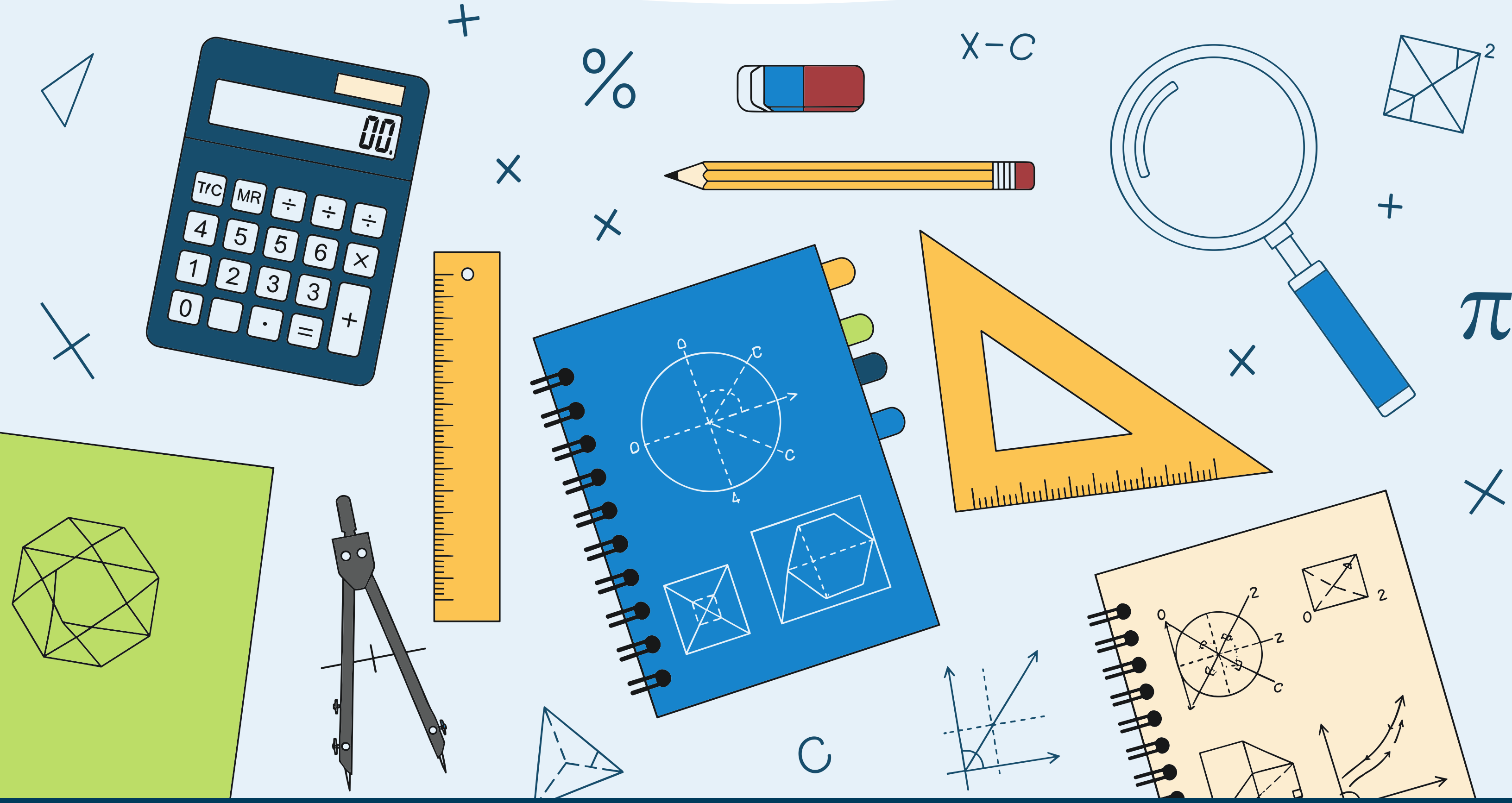
Illinois Comprehensive Numeracy Plan



ILLINOIS
STATE BOARD OF
EDUCATION

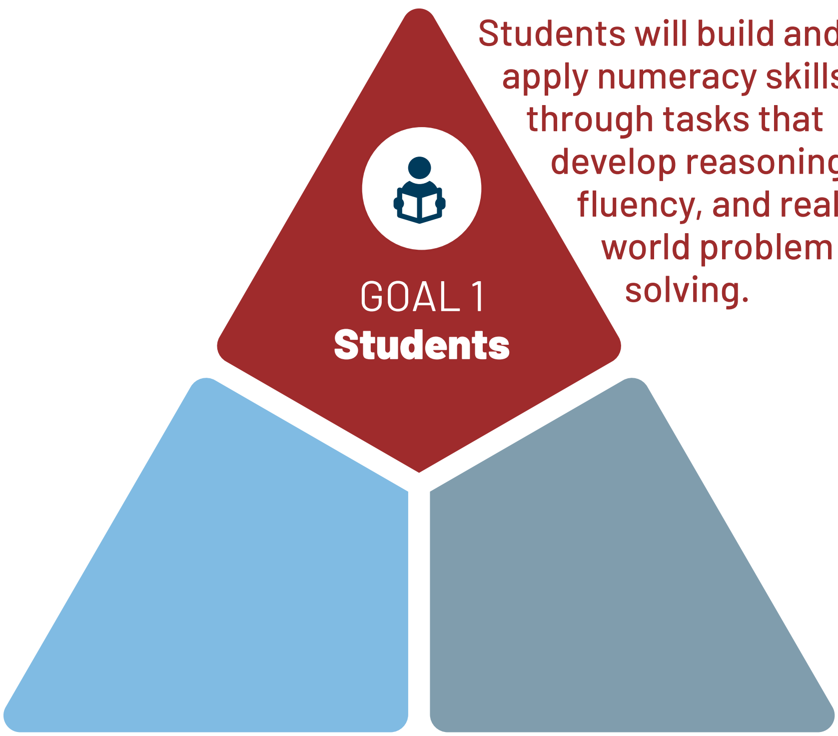
www.isbe.net/NumeracyPlan

June 2026

Section
01

FRAMEWORK FOR THE EVIDENCE-BASED DEVELOPMENT OF NUMERACY SKILLS

This section is dedicated to **Goal 1**:



Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

This section of the plan will cover the following main topics:

1. Evidence-Based Instruction
2. Cohesive Continuum of Learning
3. The Six Components of Numeracy
4. Instructional Considerations
5. Assessments

Section Overview

This section begins with defining evidence-based mathematics instruction, then discusses the importance of coherent and aligned curriculum to avoid fragmented instruction and create a cohesive continuum for learning. Next, the plan offers insight into the development of mathematical skills and carefully lays out the Six Components of Numeracy. This section elaborates on instructional considerations that benefit all learners and provides considerations for learners with specialized needs. Lastly, this portion of the plan offers information about assessment and ends with workbook pages tailored to each audience type and reflection questions regarding Goal 1 of the plan.

Evidence-Based Instruction

Evidence-based mathematics instruction draws on decades of research about how students learn. It is not a single program or method; it is a commitment to teaching in ways that a rigorous body of research consistently supports. As research advances and student demographics change, understandings of effective instruction evolve accordingly, making it essential to stay current with evidence-based best practices.

The Illinois Comprehensive Numeracy Plan follows the federal Every Student Succeeds Act (ESSA), endorsing instructional practices based on the Tiers of Evidence framework. This framework classifies practices into four evidence-based tiers that are determined by five factors:

Design of the study	Results of the study	Findings from related studies	Sample size and settings	Match *

*The extent to which the characteristics of the study align with the characteristics of the organization that wishes to implement the educational program. Practices with strong (Tier 1) and moderate (Tier 2) evidence should be prioritized. This approach ensures that teaching methods are not just theoretically sound but also have proven effectiveness in enhancing student learning.

UNDERSTANDING THE ESSA TIERS OF EVIDENCE

	TIER 1 **Strong Evidence**	TIER 2 **Moderate Evidence**	TIER 3 **Promising Evidence**	TIER 4 **Demonstrates a Rationale**
Study Design	Well-designed and implemented experimental study, meets WWC standards without reservations	Well-designed and implemented quasi-experimental study, meets WWC standards with reservations	Well-designed and implemented correlational study, statistically controls for selection bias[a]	Well-defined logic model based on rigorous research
Results of the Study	Statistically significant positive effect on a relevant outcome	Statistically significant positive effect on a relevant outcome	Statistically significant positive effect on a relevant outcome	An effort to study the effects of the intervention is planned or currently under way
Findings From Related Studies	No strong negative findings from experimental or quasi-experimental studies	No strong negative findings from experimental or quasi-experimental studies	No strong negative findings from experimental or quasi-experimental studies	N/A
Sample Size & Setting	At least 350 participants, conducted in more than one district or school	At least 350 participants, conducted in more than one district or school	N/A	N/A
Match	Similar population *and* setting to your setting	Similar population *or* setting to your setting	N/A	N/A

Figure 3: Understanding the ESSA Tiers of Evidence

a. Findings from experimental and quasi-experimental studies that either (a) meet the first three criteria for Tiers 1 and 2 but not the sample size, setting, or match requirements, or (b) do not meet What Works Clearinghouse (WWC) standards but statistically control for selection bias between the treatment and comparison groups are also eligible to meet Tier 3 Promising Evidence. *From Institute of Education Sciences. ESSA Tiers of Evidence. U.S. Department of Education, 2025.*

Evidence-Based Instruction IS...	Evidence-Based Instruction IS NOT...
✓ **A Collection of Research to Inform Instruction** Research about how children learn mathematics and what to do when a child encounters difficulty in learning mathematics. The research informs evidence-based instructional practices.	× **A Program, Intervention, or a Product for Purchase** The use of evidence-based instruction is an approach to teaching mathematics that is based on decades of research and evidence. It is not a specific program.
✓ **Ever-Evolving** New evidence and research are continuously being released. As populations, communities, and approaches evolve, so should practice. New research can impact the weight of evidence. The continuum of rigor and quality for research can help identify the weight of claims stemming from research.	× **Complete and Final** As with any research, it is never complete. Studies are constantly being conducted and new research continues to be released. Leaders, teachers, and families can work together to bring relevant evidence-based practices into the classroom.

Mathematics Instructional Approaches

High-quality mathematics instruction is intentional, data-informed, and grounded in a coherent integration of evidence-based instructional approaches. Research in mathematics education indicates that effective instruction relies on the thoughtful blending of multiple methodologies, including explicit instruction, unassisted discovery, guided inquiry, guided strategic development, and strategic coaching.[1] The selection and integration of these approaches must be driven by ongoing analysis of student data to ensure alignment with learners' strengths, needs, and instructional goals. Positive learning outcomes are associated with these approaches when interwoven implementation is intentional and aligned to student needs and instructional goals.[2] High-quality instruction reflects a balanced use of teacher-directed and student-centered practices to support procedural fluency, conceptual understanding, and mathematical reasoning.[3]

Cohesive Continuum of Learning

Numeracy skills develop over time as students build on prior knowledge and extend their understanding to new contexts.[4] A cohesive continuum of learning ensures that students experience mathematics as a connected progression of ideas rather than as disconnected topics. When learning is coherent, students are better able to make sense of mathematics, ultimately allowing them to make significant academic progress.[5]

Instructional coherence occurs when core instruction (including curriculum, assessments, and materials) and interventions are intentionally aligned to advance shared priorities, goals, and grade-level learning experiences for all students.[6] Students should encounter mathematical concepts multiple times across grade bands with increasing sophistication to foster students' confidence to apply mathematics flexibly and to transfer learning across domains and contexts.

To achieve this coherence, all mathematics instruction and assessments must be intentionally connected across grade levels and aligned to the Illinois Learning Standards for Mathematics. This includes both vertical alignment across grade levels and horizontal alignment within a grade level. This alignment ensures that students experience mathematics as a logical progression of ideas rather than isolated skills.[7] Vertical alignment also supports instructional coherence by allowing educators to understand how strategies and models are developed and used across grade levels, enabling them to make connections that help students build on prior learning and prepare for future concepts. Achieving such coherence requires educators to develop a deep understanding of the breadth and depth of the standards as well as a clear vision of learning within a grade, between grades, and within and between content strands. Additionally, standards should be understood as yearlong learning goals rather than isolated units of study. Instruction must be intentionally designed to ensure students are repeatedly exposed to and engage with all grade level standards.

The Illinois Learning Standards integrate conceptual understanding, procedural fluency, and application. This integration may occur within a single standard or develop across grade levels. For example, students' understanding of multiplication develops over time, beginning with early experiences such as sorting and evolves from grouping objects (concrete) to arrays and area models (pictorial), and eventually to symbolic algorithms (abstract) between second and sixth grade.[8] This concrete-pictorial-abstract (CPA) progression not only builds mathematical content knowledge but also shapes how students think and work mathematically.

These ways of thinking are articulated in the Standards for Mathematical Practice (SMPs), which describe the skills, dispositions, expertise, and understandings that all students should experience and develop. While the Illinois Learning Standards for Mathematics are grade-level specific in their identification of what students should know and be able to do, the eight Standards for Mathematical Practice are the same for each grade level in their description of the "habits of mind" that all students should develop as they progress through grade levels.[9] For example, the multiplication progression discussed above supports the development of flexible problem-solving skills and habits of mind, such as modeling (SMP 4), reasoning (SMP 3), and precision (SMP 6). These are the foundations for mathematical thinking and practice that are part of being a confident and proficient problem solver. These practices should permeate throughout the whole curriculum and as such are a unifying theme across the grades (i.e., the practices can be thought of as the "heart and soul" of what it means to do mathematics). The Standards for Mathematical Practice are not discrete grade-level skills but enduring practices that deepen over time, meaning students should apply mathematical practices during the learning of each lesson throughout the K-12 continuum.[10] Teachers are charged with establishing an environment through which students engage with mathematical practices and providing meaningful experiences for the students to develop these habits of mind for themselves.

This example of multiplication demonstrates why it is critical for educators to be deeply familiar with grade level standards and learning progressions. Without this understanding, students may be introduced to concepts or procedures that are developmentally inappropriate, such as standard algorithms, before foundational understanding is established. Instruction must be intentionally sequenced so that students first explore ideas through hands-on experiences, then transition to visual or diagrammatic representations, and finally to symbolic or abstract reasoning. Students should have ongoing opportunities to revisit concrete and pictorial models to reinforce understanding or address misconceptions.

Finally, the selection of high-quality core instructional materials and supplemental instructional materials is essential to supporting alignment and coherence in mathematics education. Tools such as EdReports, ISBE's Curriculum Evaluation Tool, and the CCNetwork's Guide to the Implementation of High-Quality Instructional Materials provide frameworks to assist educators and school leaders in evaluating curriculum quality.

Learning Standards

- Counting & Cardinality (K)
- Number & Operations in Base Ten (K-5)
- Ratios & Proportional Relationships (6-7)
- Number & Quantity (HS)
- Number & Operations-Fractions (3-5)
- The Number System (6-8)
- Algebra (HS)
- Operations & Algebraic Thinking (K-5)
- Expressions & Equations (6-8)
- Modeling (HS)
- Functions (8-12)
- Geometry (K-12)
- Measurement & Data (K-5)
- Statistics & Probability (6-12)

Additional Resources

- IELDS (K)
- KIDS (K)
- CTE (HS)
- Transitional Math (HS)
- Social Emotional Standards (K-12)
- WIDA (K-12)

<table>
<tr><th></th><th colspan="10">Grades</th></tr>
<tr><th></th><th>K</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>High School</th></tr>
<tr><td rowspan="7">Domains & Standards</td><td>Counting & Cardinality</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="6">Number & Operations in Base Ten</td><td colspan="2">Ratios & Proportional Relationships</td><td></td><td>Number & Quantity</td></tr>
<tr><td></td><td></td><td></td><td colspan="3">Number & Operations Fractions</td><td colspan="3">The Number System</td><td>Algebra</td></tr>
<tr><td colspan="6">Operations & Algebraic Thinking</td><td colspan="3">Expressions & Equations</td><td>Modeling</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td colspan="2">Functions</td></tr>
<tr><td colspan="10">Geometry</td></tr>
<tr><td colspan="6">Measurement & Data</td><td colspan="4">Statistics & Probability</td></tr>
<tr><td rowspan="4">Additional Resources</td><td>IELDS</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>CTE</td></tr>
<tr><td>KIDS</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>Transitional Math</td></tr>
<tr><td colspan="10">Social Emotional Standards</td></tr>
<tr><td colspan="10">WIDA</td></tr>
</table>

Figure 4: Learning Standards

The Six Components of Numeracy

Illinois has identified six components of numeracy detailed on the following pages.

Conceptual understanding
Comprehending mathematical concepts, operations, and relations

Procedural fluency
Skill in carrying out procedures flexibly, accurately, efficiently

Strategic Competence
Formulating, representing, and solving problems

Adaptive reasoning
Explaining and justifying mathematical thinking

Productive disposition
Seeing mathematics as sensible and useful

Discourse
Purposeful communication of mathematical ideas

The components, with the exception of discourse, were established by the National Research Council and offer a comprehensive overview of instructional practices. These practices align with the crucial components of numeracy. The instructional framework is not merely a theoretical construct; it is a practical tool for educators that is designed to enhance numeracy teaching and learning throughout Illinois.

The Six Components of Numeracy are not independent of one another; thus, they are not isolated skills. Each component works together to cultivate numeracy in students, which is a vital skill for success in both students' personal and professional lives. Effective mathematics instruction incorporates each component to build numeracy in students.

The following pages delve deeper into the Six Components of Numeracy. Each component is accompanied by "Instructional Practices and Strategies". The practices in these tables are specific to core, Tier 1 instruction. Not all practices are appropriate across all grade levels, as effectiveness varies by context. This list is not exhaustive and does not include evidence-based interventions designed for more targeted or intensive supports beyond core instruction. Professional judgement remains essential in determining how best to meet the diverse needs of students.

Conceptual understanding

Conceptual understanding is the ability for students to view mathematical concepts in a connected, meaningful way, recognizing their significance and organizing knowledge so that new concepts are linked to their existing understanding.[11] Benefits of conceptual understanding include increased retention rates of mathematical concepts, fewer computational errors, and students' ability to flexibly apply their knowledge, which ultimately develops their critical-thinking skills.[12] A strong conceptual understanding helps students develop a deep understanding of math concepts and their interconnectedness and allows them to approach math problems with a variety of strategies.

Why It Matters

Conceptual understanding is essential in mathematics because it enables students to make sense of mathematical ideas, relationships, and structures rather than relying on memorized procedures. When students understand the "why" behind the mathematics, they are better equipped to interpret and act on the quantitative information they encounter in everyday life such as comparing prices, managing a budget, evaluating interest rates, or making sense of data presented in the media. This depth of understanding allows students to translate real-world situations into mathematical representations and to interpret those results in ways that inform decisions. By engaging in authentic contexts, students learn to recognize patterns, evaluate claims, and determine whether results are reasonable, strengthening their ability to think critically in a data-rich world. These experiences position mathematics as a practical tool for problem solving and informed decision-making rather than a set of isolated skills. Ultimately, conceptual understanding prepares students to navigate personal, civic, and professional situations with confidence and sound reasoning.

- ☑ Supports informed decision-making in everyday contexts (e.g., budgeting, pricing, rates, and risk)
- ☑ Strengthens the ability to interpret and evaluate data encountered in media and daily life
- ☑ Builds durable understanding that transfers across contexts
- ☑ Reduces reliance on rote memorization and isolated procedures
- ☑ Connects multiple representations (e.g., graphs, equations, tables, models)
- ☑ Serves as the foundation for procedural fluency and continued learning

Conceptual understanding

Instructional Practices and Strategies[13]

Aligned with Evidence

- ✓ Use of concrete-pictorial-abstract (CPA) progression
- ✓ Emphasis on multiple representations (models, visuals, symbols)
- ✓ Tasks that require explanation of "why it works"
- ✓ Use of number talks and math routines
- ✓ Connecting prior knowledge to new concepts
- ✓ Building understanding through schema recognition
- ✓ Encouraging student-generated models and representations
- ✓ Formative questioning focused on relationships, not answers
- ✓ Use of rich tasks with multiple entry points

Not Aligned with Evidence

- × Overemphasis on memorization without conceptual grounding
- × Isolated skill practice
- × Single-representation instruction only
- × Premature abstraction without concrete support
- × Emphasis on speed over understanding
- × Tasks with only one correct representation
- × Teacher-only modeling without student construction of ideas
- × Lack of explicit connections between concepts

Specialized Learning Considerations

Multilingual Learners

- ✓ Visual models paired with vocabulary supports
- ✓ Hands-on manipulatives for meaning making
- ✓ Language-rich discussion anchored in visuals

Students with IEPs/504 Plans

- ✓ Chunking concepts and tasks
- ✓ Step-by-step visual scaffolds (graphic organizers, number lines, etc.)
- ✓ Extended time with manipulatives before abstraction

Gifted & Advanced Learners

- ✓ Open-ended "what if" explorations
- ✓ Multiple representations of the same concept
- ✓ Generalization and pattern exploration tasks

Students with Dyscalculia

- ✓ Heavy reliance on concrete models and manipulatives
- ✓ Explicit teaching of number relationships
- ✓ Reduced symbolic overload early in learning

Conceptual understanding

SMPs	Teacher Actions	Student Actions
MP.2 MP.4 MP.7	✓ Make mathematics accessible to students through concrete, pictorial, and abstract representations ✓ Explicitly teach and name problem schemas ✓ Proactively facilitate multiple "access points/modes" for students ✓ Use multiple representations when appropriate while using and eliciting student thinking ✓ Ask students to provide visual proof of their solution	✓ Demonstrate understanding of concepts in multiple ways ✓ Make multiple connections among different models and representations ✓ Justify strategy choices based on problem structure rather than key words ✓ Understand that multiple iterations may be needed to reach an appropriate solution and persevere in the revisions of thinking ✓ Use multiple modes of communication to convey solutions and explain what the solutions mean in context

Assessment Considerations

Assessment of conceptual understanding should focus on revealing how students reason, make connections, and apply ideas—not just whether they can produce the correct answers. Effective assessments include open-ended tasks and rich problems that require explanation, representation, and justification of thinking. Students should be asked to use multiple representations (models, diagrams, symbols, words) to demonstrate understanding of underlying concepts and relationships. Formative assessment plays a critical role, providing ongoing evidence of learning that informs instructional pivots and next steps. Teachers should attend to students' misconceptions and partial understandings, using assessment data to support targeted feedback and instructional responses. Opportunities for student self-assessment and reflection help learners articulate what they understand and where they are still developing. Assessment should be coherent with instruction, aligned to learning goals, and embedded regularly, not reserved only for summative measures. High-quality assessment practices emphasize conceptual growth, reasoning, and sense-making as central indicators of mathematical proficiency.

Progressions

Young Learners: Concrete Foundations

- ✓ Develop number sense through manipulatives and visual models
- ✓ Understand quantity, comparison, and basic operations conceptually

Example: "Show different ways to make 10 using objects or drawings." (K.OA)

Elementary: Expanding Relationships

- ✓ Understand operations as relationships (inverse, equivalence)
- ✓ Use area models, arrays, and number line

Example: "Explain how multiplication is related to repeated addition using an array." (3.OA)

Middle School: Abstract Connections

- ✓ Generalize relationships across numbers, ratios, and expressions
- ✓ Connect algebraic representations to visual models

Example: "Explain why multiplying by a fraction less than 1 reduces a quantity." (6.NS)

High School: Structural Understanding

- ✓ Understand structure of expressions, functions, and transformations
- ✓ Reason about equivalence and generalization

Example: "Interpret the meaning of parameters in a linear function in context." (F-IF)

Procedural fluency

Procedural fluency extends beyond the ability to perform standard algorithms and involves understanding mathematical processes and procedures, knowing when and how to apply them effectively, and being able to execute them with flexibility, accuracy, and efficiency.[14] Procedural fluency also includes the ability to accurately estimate and check for reasonableness, which closely aligns with the development of conceptual understanding. A key component of procedural fluency is automaticity, the ability to recall and execute basic facts and procedures with little cognitive effort, which supports efficient problem solving and frees students' working memory to focus on reasoning and underlying concepts.[15] A student's ability to perform mental mathematics calculations with little to no errors lays the foundation for higher math as well as allows students to focus on the "why" behind the procedure. Procedural fluency builds the foundation for higher order thinking, advanced mathematics skills, and real-world applications such as budgeting, calculating a tip at a restaurant, time management, etc. Ultimately, students who are fluent with procedures can easily apply the appropriate strategies to produce the correct answers or reasonable estimates and understand they must remain flexible in their approach to problem solving, as mathematics strategies are not a one-size-fits-all solution.

Why It Matters

Procedural fluency is essential in mathematics because it enables students to carry out calculations and processes accurately, efficiently, and flexibly. In everyday life, individuals rely on these skills to manage tasks such as calculating totals, determining discounts and tips, adjusting recipes, measuring materials, or interpreting quantities like medication dosages where precision matters. When students develop fluency through meaningful application rather than isolated repetition, they not only perform procedures correctly but also understand when and why to use them. This efficiency reduces cognitive load, allowing individuals to focus on problem solving, reasoning, and making informed decisions rather than getting stuck in computation. Recognizing patterns and structures such as using place value, properties of operations, or mental math strategies further streamlines thinking and increases flexibility across contexts. As a result, procedural fluency becomes a practical, transferable skill that supports accuracy, confidence, and independence in daily life. Ultimately, it ensures mathematics functions as a reliable and efficient tool for real-world decision-making.

- ☑ Enables efficient and accurate problem solving in everyday contexts
- ☑ Frees cognitive capacity for reasoning, modeling, and decision-making
- ☑ Reduces errors and supports precision in real-world tasks
- ☑ Strengthens flexibility through pattern recognition and strategic thinking
- ☑ Supports confident, independent use of mathematics in daily situations

Procedural fluency

Instructional Practices and Strategies[16]

Aligned with Evidence

- ✓ Number talks and mental math routines
- ✓ Emphasis on strategy comparison before formal algorithms
- ✓ Explicit connection between concept and procedure
- ✓ Incorporate spaced retrieval and fluency practice
- ✓ Use of error analysis tasks
- ✓ Encouraging multiple solution pathways
- ✓ Spiral review embedded in meaningful contexts
- ✓ Visual models to support procedures
- ✓ Timely, specific feedback on reasoning
- ✓ Tasks that require selecting appropriate strategies

Not Aligned with Evidence

- × High stakes timed tests as primary fluency measure
- × Memorization without conceptual grounding
- × Isolated skill drills without context
- × Teaching procedures before conceptual understanding
- × Single-method instruction only
- × Emphasis on speed over accuracy or reasoning
- × Lack of student explanation of procedures
- × No opportunity for strategy comparison
- × Procedures taught as “rules” or “steps to follow” without meaning

Specialized Learning Considerations

Multilingual Learners

- ✓ Visual step charts and bilingual supports
- ✓ Modeling procedures with language scaffolds

Gifted & Advanced Learners

- ✓ Efficiency comparisons and optimization tasks
- ✓ Exploration of non-standard methods

Students with IEPs/504 Plans

- ✓ Reduced problem sets with high-quality feedback
- ✓ Step-by-step guided practice

Students with Dyscalculia

- ✓ Manipulatives to anchor procedures
- ✓ Structured repetition with conceptual reinforcement

Procedural fluency

SMPs	Teacher Actions	Student Actions
MP.6 MP.7 MP.8	✓ Provide opportunities for distributed practice of procedures ✓ Provide multiple opportunities using a variety of formats to assess students' understanding ✓ Provide opportunities for students to connect student-generated strategies with efficient procedures ✓ Have students play games that involve strategy and the use of procedural fluency ✓ Recognize the need for various modes of explanation, including written, verbal, and diagrams ✓ Uses precise mathematical language when describing strategies ✓ Provide feedback based on student thinking ✓ Provide opportunities for students to find and analyze errors	✓ Accurately use a procedure on an unfamiliar problem or new number set ✓ Demonstrate flexibility in using strategies and recognize and justify when one strategy is more efficient than others ✓ Identify, use, and communicate multiple pathways to a solution ✓ Find and analyze errors to show an understanding of the procedures and why they work

Procedural fluency

Assessment Considerations

Assessment of mathematical procedural understanding should focus on how accurately, efficiently, and flexibly students carry out mathematical procedures. Effective assessments evaluate not only correct answers but also students' ability to select and apply appropriate procedures in varied contexts. Tasks should examine procedural fluency, including accuracy, efficiency, and consistency over time, while avoiding an exclusive focus on speed. Assessments benefit from including non-routine problems that require students to adapt procedures rather than follow memorized steps. Teachers should use formative checks (such as worked examples, exit tasks, and error analysis) to identify gaps in fluency or misunderstandings that hinder procedural use. Attention to common errors and misconceptions helps distinguish procedural mistakes from deeper conceptual gaps. Feedback should be specific and corrective, supporting refinement and automaticity of procedures.

Progressions

Young Learners: Concrete Foundations

- ✓ Develop efficient counting strategies and number relationships

Example: "Solve 7+5 using a make-ten strategy." (1.OA)

Elementary: Expanding Relationships

- ✓ Use standard algorithms and invented strategies
- ✓ Understand why algorithms work

Example: "Solve 256 x4 using two different strategies." (4.NBT)

Middle School: Abstract Connections

- ✓ Work fluently with rational numbers and expressions

Example: "Solve multi-step equations and justify each step." (7.EE)

High School: Structural Understanding

- ✓ Manipulate algebraic, exponential, and function expressions

Example: "Rewrite expressions to reveal key features of a function." (A-SSE)

Strategic competence

Strategic Competence

Strategic competence is the capacity to formulate, represent, and solve mathematical problems effectively.[17] It includes problem solving as well as problem formulation and recognizes that real-world scenarios often require students to define the problem before applying a mathematical strategy to solve the problem. The development of strategic competence requires practicing problem formulation, cultivating a repertoire of math strategies, and understanding which strategies are appropriate for a given scenario. Strategic competence allows students to become proficient problem solvers and has strong ties to procedural fluency and conceptual understanding.

Why It Matters

Strategic competence is essential in mathematics because it enables students to formulate, represent, and solve problems with purpose and flexibility. In everyday life, individuals draw on this skill when making decisions such as comparing long-term costs, planning travel based on time and budget, or prioritizing expenses to manage finances effectively. Rather than relying on memorized steps, students with strong strategic competence can analyze situations, choose appropriate approaches, and adjust their thinking when initial strategies are unsuccessful. They also learn to use tools intentionally, such as estimation for quick decisions, digital tools to model scenarios, or diagrams and tables to organize and compare information. This adaptability allows individuals to navigate complex, real-world situations where there is no single clear path to a solution. By integrating conceptual understanding and procedural fluency, strategic competence supports thoughtful, efficient problem solving across contexts. Ultimately, it empowers students to use mathematics as a tool for planning, evaluating options, and making informed decisions in daily life.

- ☑ Develops independent and adaptable problem solvers
- ☑ Strengthens transfer of mathematical thinking across contexts
- ☑ Builds decision-making skills in real-world situations
- ☑ Supports effective use of tools and representations
- ☑ Integrates conceptual understanding and procedural fluency for purposeful problem solving

Strategic competence

Instructional Practices and Strategies [18]

Aligned with Evidence

- ✓ Use of non-routine, low-floor/high-ceiling tasks
- ✓ Teaching problem-solving strategies
- ✓ Encouraging multiple representations (table, graph, equation)
- ✓ Thinking routines (read–represent–solve–check)
- ✓ Collaborative problem-solving structures
- ✓ Anchoring tasks in real-world contexts
- ✓ Emphasis on explanation of strategy choice
- ✓ Use of estimation before solving
- ✓ Regular exposure to multi-step problems
- ✓ Reflection on problem-solving process

Not Aligned with Evidence

- × Routine-only problems with predictable structures
- × Teaching "key words" as primary strategy
- × Over-scaffolding
- × Tasks without real-world or conceptual context
- × Single-representation problem solving
- × Focus on answer over process
- × Lack of discussion of strategy selection
- × Minimal student reasoning required

Specialized Learning Considerations

Multilingual Learners

- ✓ Visual problem breakdowns and vocabulary supports
- ✓ Structured partner talk for comprehension

Students with IEPs/504 Plans

- ✓ Chunked problems with guided steps
- ✓ Reduced linguistic complexity

Gifted & Advanced Learners

- ✓ Open-ended or multiple-solution problems
- ✓ Extension through constraints or generalization

Students with Dyscalculia

- ✓ Visual modeling and manipulatives
- ✓ Reduced memory load through structured supports

Strategic competence

SMPs	Teacher Actions	Student Actions
MP.1 MP.5	✓ Develop content knowledge to utilize multiple strategies and progressions ✓ Develop the pedagogical knowledge to allow for multiple ways of solving problems ✓ Provide class discussions on the efficiency of different strategies through questioning ✓ Anticipate barriers or misconceptions that may arise during instruction and create intentional questions to engage and develop strategies ✓ Provide a thinking task or problem that can be solved in multiple ways with a specific instructional goal in mind ✓ Provide feedback on student processes and strategies	✓ Ask peers about strategy selected to compare with own ✓ Reflect on which strategies are best used and when to use them when looking at work ✓ Recognize that if arriving at a solution that does not make sense, there is another strategy to try ✓ Formulate mathematical problems, represent, and solve them ✓ Explain other students' solution strategies and connect to own

Assessment Considerations

Assessment of strategic competence should focus not only on correct solutions but also on how students analyze problems, select appropriate strategies, and adapt their approach when faced with challenges. Tasks aligned to grade-level mathematics standards should require students to determine what mathematics is needed rather than telling students which procedure to apply, ensuring alignment with the intent of the standards. Performance tasks, open-ended problems, and real-world scenarios allow students to demonstrate strategic competence by showing multiple solution paths, creating representations, and justifying their decisions. Rubrics should value strategy selection, evidence of reasoning, and flexibility alongside accuracy. Together, these assessment considerations provide a more complete picture of student understanding and ensure that strategic competence is developed and measured in ways that reflect both rigorous math standards and authentic mathematical thinking.

Progressions

Young Learners: Concrete Foundations

- ✓ Use objects and drawings to solve problems

Example: “Solve: You have 6 apples and get 3 more. How many apples do you have now?” (1.OA)

Elementary: Expanding Relationships

- ✓ Choose and apply strategies for word problems

Example: “Solve a multi-step word problem involving fractions.” (5.NF)

Middle School: Abstract Connections

- ✓ Translate real-world problems into equations and models

Example: “Write an equation that represents a proportional relationship.” (7.RP)

High School: Structural Understanding

- ✓ Build and analyze models for real-world situations

Example: “Model exponential growth and interpret parameters.” (F-LE)

Adaptive reasoning

Adaptive reasoning is the ability to think logically about the connections among concepts and situations, evaluate alternative approaches to problem solving, and justify solutions.[19] In mathematics, it serves as the guiding principle that integrates facts, procedures, and ideas into a coherent whole. Students who successfully develop adaptive reasoning apply prior knowledge to justify their answers through reasoning rather than rely on confirmation from teachers or peers.

Why It Matters

Adaptive reasoning is essential in mathematics because it enables students to think logically, reflect on their reasoning, and justify their conclusions. In everyday life, individuals rely on this skill to determine whether a solution is reasonable, reconsider decisions when conditions change, and explain or defend their thinking. Adjusting a budget after an unexpected expense, interpreting data in the news, or modifying plans when outcomes differ from expectations all require adaptive reasoning skills. Students with strong adaptive reasoning do more than arrive at answers; they evaluate the validity of their solutions, recognize errors, and revise their approach when needed. This ability to reflect and adjust strengthens both understanding and problem solving, allowing individuals to respond thoughtfully in situations where information is incomplete or evolving. By connecting concepts and reasoning, adaptive reasoning supports deeper learning and long-term retention. It also promotes equitable access by positioning all students as capable thinkers whose ideas can be examined, refined, and justified. Ultimately, adaptive reasoning ensures that mathematics is not only used but trusted as a tool for making informed decisions.

- ☑ Develops logical and critical thinking skills
- ☑ Supports justification, explanation, and mathematical argumentation
- ☑ Strengthens connections between concepts and procedures
- ☑ Enables evaluation of solutions and identification of errors
- ☑ Builds independence and confidence in mathematical thinking

Adaptive reasoning

Instructional Practices and Strategies[20]

Aligned with Evidence

- ✓ Routine use of "justify your answer" prompts
- ✓ Structured argumentation tasks (claim-evidence-reasoning)
- ✓ Encouraging critique of multiple solution methods
- ✓ Use of error analysis as a reasoning tool
- ✓ Emphasis on explaining "why it works," not just "how"
- ✓ Incorporation of visual and symbolic justification
- ✓ Peer discourse focused on reasoning validation
- ✓ Use of true/false and always/sometimes/never tasks
- ✓ Teacher facilitation that probes reasoning depth

Not Aligned with Evidence

- × Accepting answers without justification
- × Focusing only on procedural correctness
- × Limited opportunities for student explanation
- × Teacher provides reasoning instead of students constructing it
- × Overuse of recall-based questioning
- × Tasks that do not allow for multiple reasoning paths
- × Emphasis on speed over explanation
- × Lack of critique or comparison of methods
- × Reasoning treated as optional rather than expected
- × Minimal use of mathematical language in explanations

Specialized Learning Considerations

Multilingual Learners

- ✓ Sentence frames for justification ("I know this because…")
- ✓ Visual models paired with oral explanation
- ✓ Partner-supported reasoning discussions

Students with IEPs/504 Plans

- ✓ Scaffolded argument structures (sentence starters, templates)
- ✓ Reduced linguistic complexity in reasoning tasks
- ✓ Oral rather than written justification options

Gifted & Advanced Learners

- ✓ Formal proof and generalization tasks
- ✓ "Always/sometimes/never" justification extensions
- ✓ Student-led critique and debate opportunities

Students with Dyscalculia

- ✓ Emphasis on verbal and visual reasoning over symbolic load
- ✓ Use of manipulatives to support logic building
- ✓ Step-by-step reasoning scaffolds

Adaptive reasoning

SMPs	Teacher Actions	Student Actions
MP.2 MP.3	✓ Make connections to prior knowledge to further student thinking ✓ Utilize protocols for students to justify their mathematical claims and respond to those of others ✓ Anticipate student strategies and plan responses ✓ Acknowledge, listen, and respond to student thinking and revisit when needed ✓ Create a classroom culture where sharing initial thinking, think-alouds, and rough drafts are valued	✓ Find and correct errors, recognizing that mistakes should be used to further development of the concept ✓ Identify multiple strategies to approach a problem and justify the approach taken ✓ Justify the change in the initial approach to a problem ✓ Provide informal and formal explanations and justifications ✓ Apply what has been learned from one idea, task, etc., to a new scenario ✓ Ask and answer questions to clarify the task

Assessment Considerations

To capture adaptive reasoning, assessments should intentionally include opportunities to reason abstractly and quantitatively (SMP 2) and construct viable arguments and critique reasoning (SMP 3). Examples include open-ended problems that ask students to explain their thinking, error-analysis tasks where students identify and correct a flawed solution, and “what-if” questions that require students to adjust a solution when conditions change. Performance tasks, such as interpreting real-world data, revising a model based on new constraints, or comparing two solution strategies and justifying which is more reasonable, provide strong evidence of adaptive reasoning. Rubrics should value clarity of explanation, logical coherence, accuracy, and the ability to revise thinking. Together, these assessment approaches ensure that adaptive reasoning is measured authentically, emphasizing flexibility, justification, and reflective thinking as key components of mathematical proficiency.

Progressions

Young Learners: Concrete Foundations

- ✓ Explain thinking using objects, drawings, and language
- ✓ Identify patterns and simple relationships

Example: “Explain why 12 is even using objects or drawings.” (2.OA)

Elementary: Expanding Relationships

- ✓ Use models and properties to explain reasoning
- ✓ Compare and critique simple solution strategies

Example: “Explain why two fractions are equivalent using a visual model.” (4.NF)

Middle School: Abstract Connections

- ✓ Construct and critique mathematical arguments
- ✓ Use definitions, properties, and prior knowledge

Example: “Justify whether a given equation has one, none, or infinitely many solutions.” (8.EE)

High School: Structural Understanding

- ✓ Develop formal proofs and logical arguments
- ✓ Justify assumptions and evaluate validity of claims

Example: “Prove properties of triangles using geometric reasoning.” (G-CO)

Productive disposition

Productive Disposition

Productive disposition refers to a positive belief that mathematics makes sense, is valuable, and can be mastered through perseverance. It also includes confidence in students' capacity to learn and use mathematics correctly.[21] Productive disposition develops alongside all other components of numeracy, as all components work together to create competent, confident mathematics learners. Teachers play a crucial role in developing productive dispositions among students. It is paramount that teachers model productive dispositions and create learning conditions and opportunities that promote students who view themselves as capable problem solvers who understand that mathematics connects to the world around them in infinite ways.

Why It Matters

Productive disposition is essential in mathematics because it reflects a student's belief that mathematics is sensible, useful, and worth the effort. In everyday life, individuals regularly encounter situations that require persistence and flexibility, such as sticking with a budget when costs change, revising plans when initial calculations do not work out, or working through multi-step decisions like planning travel or managing time and resources. When students develop a productive disposition, they are more willing to engage with complex problems, persist through challenges, and view effort as a pathway to understanding rather than a barrier. This mindset supports the development of positive mathematical identity and reduces avoidance behaviors by helping learners see themselves as capable problem solvers. Through meaningful, real-world tasks, students experience mathematics as relevant and useful, which increases motivation to persist even when solutions are not immediately clear. Over time, these experiences build resilience, confidence, and independence in mathematical thinking. Ultimately, productive disposition empowers students to approach both academic and real-world challenges with perseverance and a belief in their ability to succeed.

- Encourages persistence in real-world problem solving and decision-making
- Builds a positive mathematical identity and sense of agency
- Reduces avoidance behaviors and supports confidence with challenge
- Strengthens engagement in complex, multi-step tasks
- Reinforces the belief that effort leads to understanding and success

Productive disposition

Instructional Practices and Strategies[22]

Aligned with Evidence

- ✓ Normalize making mistakes as part of learning
- ✓ Praise effort, strategy, and reasoning (not speed or correctness alone)
- ✓ Use open-ended and low-floor/high-ceiling tasks
- ✓ Provide time for revision and reflection
- ✓ Use collaborative problem-solving structures
- ✓ Explicitly teach persistence strategies ("try a different representation")
- ✓ Provide feedback focused on process, not just answers
- ✓ Build classroom norms around mistake-making as learning
- ✓ Use math discussions to validate multiple approaches

Not Aligned with Evidence

- ✗ Overemphasis on correct answers over process
- ✗ Immediate teacher rescue during struggle moments
- ✗ Rewarding speed over persistence
- ✗ Avoidance of challenging tasks
- ✗ Excessive repetition without cognitive demand
- ✗ Discouraging mistakes or correcting too quickly
- ✗ Limited reflection on learning process
- ✗ Lack of opportunities for revision or rethinking
- ✗ Task design that limits challenge or exploration

Specialized Learning Considerations

Multilingual Learners

- ✓ Encourage oral participation before written output
- ✓ Provide culturally responsive math contexts
- ✓ Use visuals to reduce language barriers

Students with IEPs/504 Plans

- ✓ Break tasks into manageable steps with checkpoints
- ✓ Provide structured encouragement for persistence
- ✓ Allow alternative demonstration of understanding

Gifted & Advanced Learners

- ✓ Provide extended challenge tasks requiring persistence
- ✓ Encourage exploration beyond initial solution
- ✓ Allow student-designed investigations

Students with Dyscalculia

- ✓ Reduce cognitive overload through scaffolding
- ✓ Use manipulatives to support sustained engagement
- ✓ Provide frequent success points to maintain confidence

Productive disposition

SMPs	Teacher Actions	Student Actions
MP. 1	✓ Believe every student is capable of learning mathematics ✓ Develop perseverance in students through complex tasks and encouragement ✓ Provide opportunities for students to grow and show mastery by connecting math learned in the classroom to their community and the world ✓ Utilize protocols for students to share their thinking and make sense of others' thinking ✓ Model utilizing mistakes and multiple strategies through think-alouds to demonstrate the learning process ✓ Allow students opportunities to identify and correct errors	✓ Believe that math can be used to solve problems in the community and the world ✓ Persevere in learning new concepts, recognizing that understanding concepts takes time ✓ Accept the challenge of the task and recognize that there is always a place to start when trying to solve a problem ✓ Recognize that there are a variety of strategies to choose from when approaching a math problem ✓ Share own thinking and make sense of others' thinking ✓ Make connections with the math learned in the classroom to the community and the world ✓ Use think-alouds to share how mistakes were identified and corrected as part of understanding concepts

Assessment Considerations

Effective assessment for productive disposition focuses on how students approach challenging tasks, reflect on their learning, and perceive themselves as capable mathematical learners. These assessments should be embedded in daily instruction and aligned with grade-level mathematics standards and the Standards for Mathematical Practice, particularly perseverance and sense-making (SMP 1). Common assessment approaches include student reflection journals, math surveys or self-assessments, teacher observations of persistence and collaboration, goal-setting reflections, and discussion prompts or exit tickets that capture how students overcome challenges. Together, these tools provide insight into students' mathematical mindset and support the development of confidence, resilience, and a positive disposition toward learning mathematics.

Progressions

Young Learners: Concrete Foundations

- ✓ Develop confidence through success with manipulatives and games
- ✓ See math as exploration and sense-making

Example: "Find different ways to make 10 and explain your thinking." (K.OA)

Elementary: Expanding Relationships

- ✓ Engage in multi-step problems with support
- ✓ Develop positive attitudes toward challenge

Example: "Solve a multi-step word problem and explain why you used your selected strategy." (5.NBT)

Middle School: Abstract Connections

- ✓ Persist through complex problems and revise thinking
- ✓ Reflect on mistakes

Example: "Analyze the error and explain how to solve the problem correctly." (8.EE)

High School: Structural Understanding

- ✓ Engage independently with complex, open-ended tasks
- ✓ Develop ownership of mathematical reasoning and persistence

Example: "Construct a quadratic model to represent a real-world scenario and justify your assumptions and revisions." (F-LE)

Discourse

Discourse

Mathematical discourse is the purposeful communication of mathematical ideas, including the tools and practices that make thinking visible. Discourse can take place verbally, visually, or in written formats. It involves students articulating their reasoning, engaging with peers' ideas, and collaboratively constructing mathematical understanding. Mathematical discourse promotes students' understanding of mathematical concepts and procedures and can make a positive impact on productive disposition.[23] Through discussion and justification of mathematical ideas, teachers gain insight into students' conceptual understanding, problem-solving strategies, and misconceptions, enabling timely and targeted instructional adjustments.[24]

Why It Matters

Mathematical discourse is essential because it is the primary means through which students develop, test, and refine their understanding of mathematics. In everyday life, individuals must communicate quantitative ideas clearly, justify decisions, and interpret the reasoning of others. Examples include explaining a financial choice, discussing data presented in the media, or collaborating to solve a problem at work. Through purposeful discussion and writing, students learn to articulate their thinking, question ideas, and make sense of multiple approaches, deepening both understanding and reasoning. This process not only strengthens conceptual understanding but also builds the ability to evaluate arguments and communicate with clarity and precision. When classrooms prioritize discourse, they create equitable environments where all students' ideas are valued, and learning is driven by sense-making rather than speed or memorization. Errors become opportunities for growth, and understanding is constructed collaboratively through dialogue and reflection. Ultimately, discourse positions mathematics as something students actively do, share, and apply in real-world contexts.

- ☑ Deepens conceptual understanding through articulation and revision of thinking
- ☑ Builds reasoning and problem solving through explanation and argumentation
- ☑ Improves transfer of learning across contexts and representations
- ☑ Supports equity by making thinking visible and accessible to all learners
- ☑ Strengthens procedural fluency through explanation, not memorization alone

Discourse

Instructional Practices and Strategies[25]

Aligned with Evidence

- ✓ Structured math talk routines (think-pair-share, number talks, debate protocols)
- ✓ Explicit teaching and modeling of mathematical vocabulary
- ✓ Teacher facilitation that prioritizes student-to-student discourse over teacher explanation
- ✓ Formative assessment through listening and analyzing student reasoning
- ✓ Norm-setting for respectful disagreement and revision of thinking

Not Aligned with Evidence

- × Heavy reliance on teacher-led explanation with limited student talk
- × Tasks that have only one correct method with no opportunity for justification
- × Discourse limited to "answer sharing" rather than reasoning explanation
- × Correction of language before meaning is developed (focus on grammar over reasoning)
- × Group work without accountability for mathematical talk
- × Limited wait time after posing questions

Specialized Learning Considerations

Multilingual Learners

- ✓ Use visual models, gestures, and structured talk frames
- ✓ Provide bilingual glossaries and cognate support
- ✓ Allow translanguaging to develop conceptual understanding
- ✓ Pair oral reasoning with diagrams and manipulatives

Gifted & Advanced Learners

- ✓ Extend discourse into justification of multiple strategies or proofs
- ✓ Require critique of efficiency and generalization
- ✓ Encourage student-led facilitation of math discussions
- ✓ Introduce "always/sometimes/never" justification tasks

Students with IEPs/504 Plans

- ✓ Provide sentence starters and guided discourse scaffolds
- ✓ Reduce linguistic load while maintaining cognitive demand
- ✓ Offer alternative expression modes (oral, visual, recorded responses)
- ✓ Pre-teach vocabulary and concepts before discourse tasks

Students with Dyscalculia

- ✓ Emphasize verbal reasoning supported by visual representations
- ✓ Reduce memory load through manipulatives and reference tools
- ✓ Use structured talk routines to externalize thinking
- ✓ Prioritize conceptual explanation over procedural recall

Discourse

SMPs	Teacher Actions	Student Actions
MP.3 MP.6	✓ Explicitly teach what math discourse looks and sounds like ✓ Provide adequate wait time before calling on students ✓ Use think-pair-share, turn and talk, or small group structures before whole class discussion ✓ Plan sequenced tasks to encourage comparison of strategies ✓ Ask probing questions such as: Why does that work? How do you know? What would happen if…? ✓ Ask students to share their thinking visually, verbally, or in writing ✓ Connect students' contributions to lesson goals or concepts ✓ Model precise mathematical language in authentic contexts ✓ Ask students to reflect on how their thinking changed ✓ Listen carefully to student talk to inform instructional decisions	✓ Justify an answer using reasoning or evidence ✓ Connect methods to a model, design, or equation ✓ Ask clarifying questions about vocabulary, strategies, a peer's reasoning, etc. ✓ Rephrase a strategy to check for understanding ✓ Agree, disagree, or expand on a peer's idea/thinking ✓ Use appropriate mathematic vocabulary ✓ Analyze, critique, evaluate, and argue to defend or define reasoning, errors, misconceptions, etc. ✓ Reflect, revise, and summarize thinking

Assessment Considerations

Discourse assessment is embedded within meaningful tasks and supports deeper conceptual understanding, multiple solution pathways, equity, and students' capacity to construct and critique mathematical arguments using evidence. Formative assessments of mathematical discourse are ongoing and include strategies such as observing student discussions, collecting exit tickets with explanations, using turn-and-talks, and analyzing annotated work samples to inform instruction. Summative assessments evaluate cumulative understanding through tasks like written mathematical arguments, performance-based problem solving with explanations, discourse portfolios, or oral defenses. Together, these approaches ensure that assessment measures not only procedural accuracy but also the quality, clarity, and depth of students' mathematical thinking and communication.

Progressions

Young Learners: Concrete Foundations

- ✓ Orally explain thinking using everyday language and emerging math vocabulary
- ✓ Use manipulatives and drawings to show reasoning
- ✓ Participate in structured talk

Example: "Explain how you know 8 is more than 5 using objects or drawings." (K.CC)

Elementary: Expanding Relationships

- ✓ Use sentence frames ("I noticed...," "I agree because...")
- ✓ Begin justifying strategies using operations and models
- ✓ Compare and critique solution methods

Example: "Compare two strategies for 347 + 129 and explain which is more efficient and why." (4.NBT)

Middle School: Abstract Connections

- ✓ Use increasingly precise mathematical language and reasoning
- ✓ Justify and critique strategies using properties, representations, and relationships among quantities
- ✓ Build on, question, and revise ideas

Example: "Compare two different methods for solving a multi-step ratio and explain which method is more efficient or reliable." (6.RP)

High School: Structural Understanding

- ✓ Construct formal proofs and multi-step reasoning
- ✓ Defend assumptions and model choices using precise notation
- ✓ Engage in academic discourse using discipline-specific language

Example: "Prove or disprove that the product of two irrational numbers is always irrational." (N-RN)

Connecting Content, Practices, and Components of Numeracy

Connecting content, the SMPs, and the Six Components of Numeracy is essential for coherent mathematics instruction. The Illinois Learning Standards define what students should know and be able to do by the end of each grade level, while the SMPs describe how students engage with the learning. Through sustained engagement with both, students develop numeracy skills including conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition, and discourse skills. These elements are not separate, nor are they sequential; they work together within and across lessons to support meaningful and connected learning. Figure 5 illustrates how these components interact to support student understanding over time.

Teachers intentionally weave together content and practices through tasks, discussions, and assessments. Students engage actively, make connections, and build understanding over time.

Illinois Learning Standards for Mathematics | *The What*

Grade-level mathematical content expectations

Examples of Content Domains

- Number and Operations in Base Ten
- Numbers and Operations — Fractions
- Ratios and Proportional Relationships
- Algebra
- Geometry
- Statistics and Probability

Standards for Mathematical Practice | *The How*

Cross-cutting practices that are applied to all content

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Six Components of Numeracy | *The Outcome*

Interacting strands that define mathematical proficiency.

Conceptual Understanding
Comprehending mathematical concepts, operations, and relations

Procedural Fluency
Skill in carrying out procedures flexibly, accurately, efficiently

Strategic Competence
Formulating, representing, and solving problems

Adaptive Reasoning
Explaining and justifying mathematical thinking

Productive Disposition
Seeing mathematics as sensible and useful

Discourse
Purposeful communication of mathematical ideas

Content *(The What)* + **Practices** *(The How)* + **Proficiency** *(The Outcomes)* = **Mathematical success for every student**

Figure 5: Relationships Among Content, Practices, and Components

Instructional Considerations

Considerations for All Learners

Ensuring that every student develops numeracy skills requires a commitment to access, equity, and differentiation. Evidence shows that when instruction is designed to meet diverse learning needs, all students are more likely to engage meaningfully with mathematics and to see themselves as capable problem solvers.[26] This section emphasizes that numeracy is not reserved for some students—it is a right for all learners.

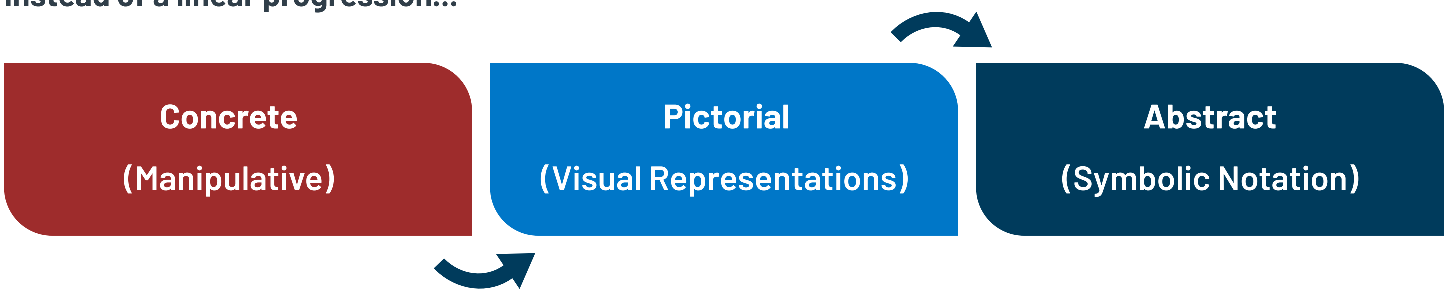
Student Engagement and Mindset

Engagement, curiosity, and mindset shape whether students persist with challenging problems, see value in mathematics, and develop confidence in their ability to learn. High-quality mathematics instruction should deliberately cultivate positive mathematical mindsets and mathematical curiosity by:

- Building relevance and wonder: Connecting mathematics to students' interests, cultural contexts, and community issues increases engagement and demonstrates the usefulness of numeracy beyond the classroom. Presenting open-ended problems or puzzles sparks curiosity and invites students to ask "what if" questions.
- Normalizing productive struggle: Framing mistakes as opportunities for learning helps students see challenge as part of the process, not as a sign of inability. Curiosity grows when students feel safe to explore and experiment and are provided the time to do so.
- Encouraging voice, agency, and discourse: Providing opportunities for students to explain their reasoning, share strategies, ask questions, and engage in rich mathematical discussions fosters ownership of learning. The intentional use of precise mathematical vocabulary by both teachers and students is essential for developing clear communication and deep conceptual understanding.
- Strengthening belonging: Classrooms that emphasize collaboration, respect, and inclusion counteract stereotypes and help students feel they belong in mathematics.
- Low-floor, high-ceiling tasks: Instruction should also include low-floor, high-ceiling tasks, which provide multiple entry points for learners of varying abilities while allowing all students opportunities to progress and deepen their understanding.[27] Tasks should be authentic and, while often messy, reflect real-world contexts, current interests, or future possibilities. Authentic tasks challenge students to reason, model, problem solve, and engage in meaningful discourse, which fosters and promotes curiosity through exploration.
- Valuing diverse ways of knowing: Inviting multiple approaches and representations allows students to see their thinking as valid and important and encourages exploration of alternative solutions. Students in all grades benefit from the use of manipulatives in transitioning from concrete to abstract representations. Students need to have ongoing access to manipulatives over extended periods of time (not just for a single lesson or unit) to support sustained development of conceptual understanding. Learning progressions are not linear; rather, they are integrated and overlapping as students make transitions between levels of understanding. Teachers must guide students in making connections between manipulatives, prior knowledge, and the current concept to promote conceptual understanding.[28] See Figures 7 through 9 on pages 54 and 55 depicting examples of CPA progressions.

Engagement is not only about motivation but also about creating conditions for active participation, exploration of curiosity, and collaborative discourse. When students are engaged in asking questions, investigating patterns, and discussing, defending, and critiquing mathematical ideas, they may make connections to other subjects, civic life, and future careers. Fostering mathematical confidence, curiosity, persistence, and discourse ensures that students leave school with the skills needed to navigate the 21st century.[29]

Instead of a linear progression...



...Plan for an integrated and overlapping progression in which students may move flexibly between representations:

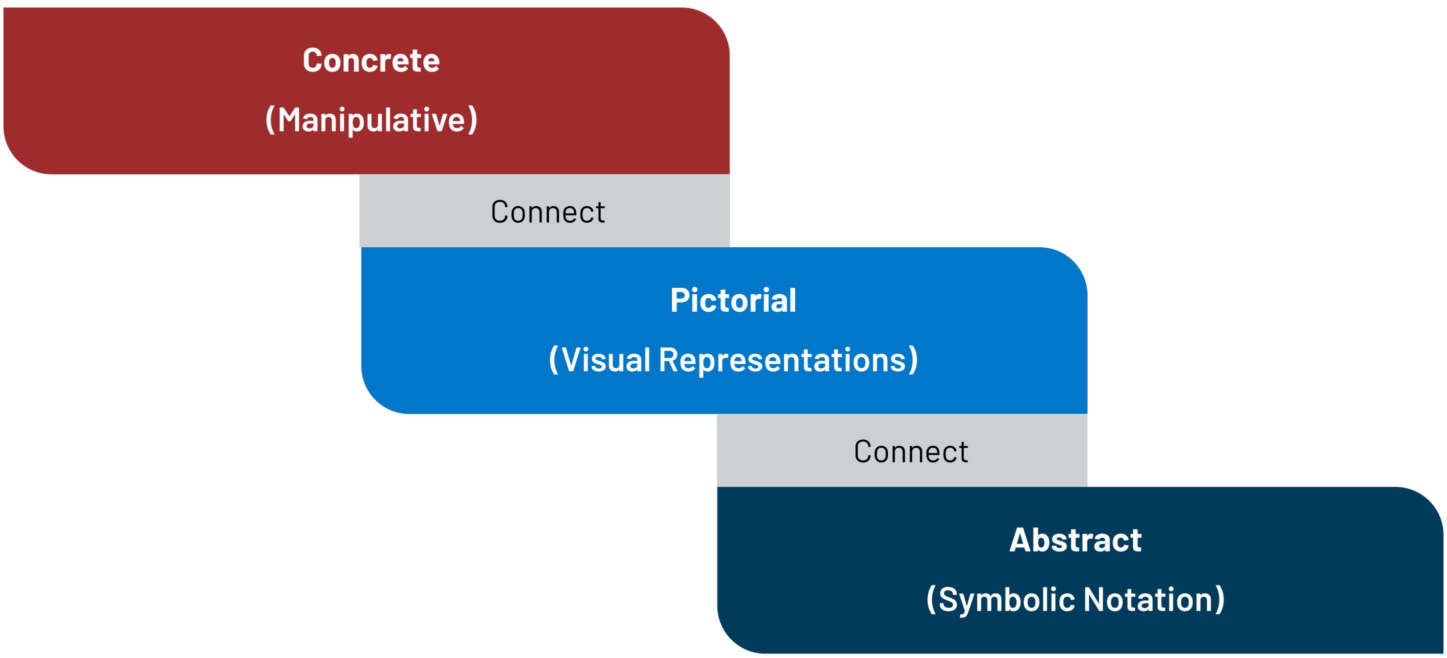
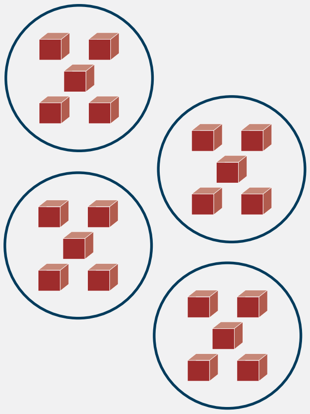


Figure 6: Integration of CPA Progressions

Problem: 4x5=20

Concrete:



Pictorial

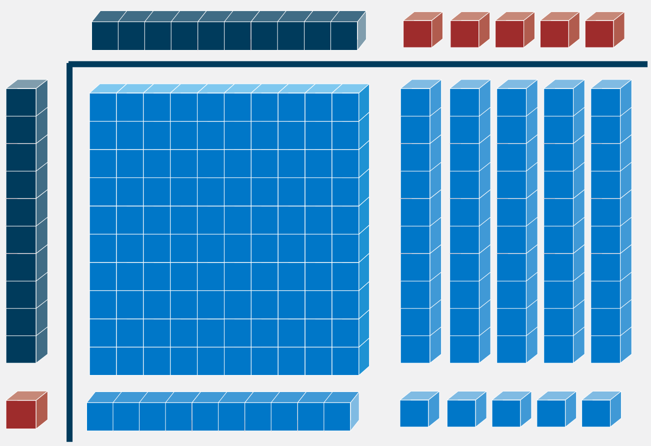
Abstract:

$4 \times 5 = 20$

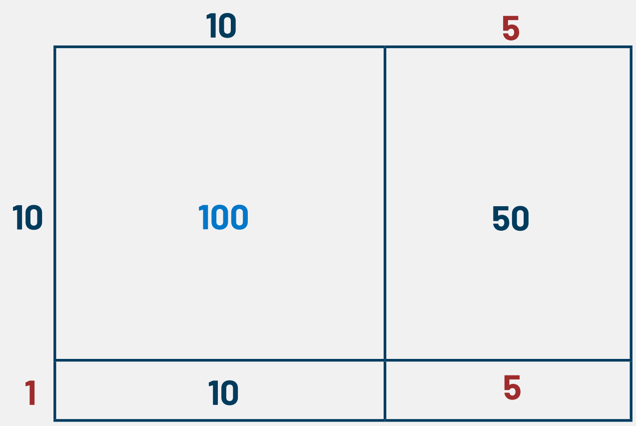
Figure 7: CPA Progressions of Single-Digit Multiplication

Problem: 15x11

Concrete:



Pictorial:



$100 + 50 + 10 + 5 = 165$

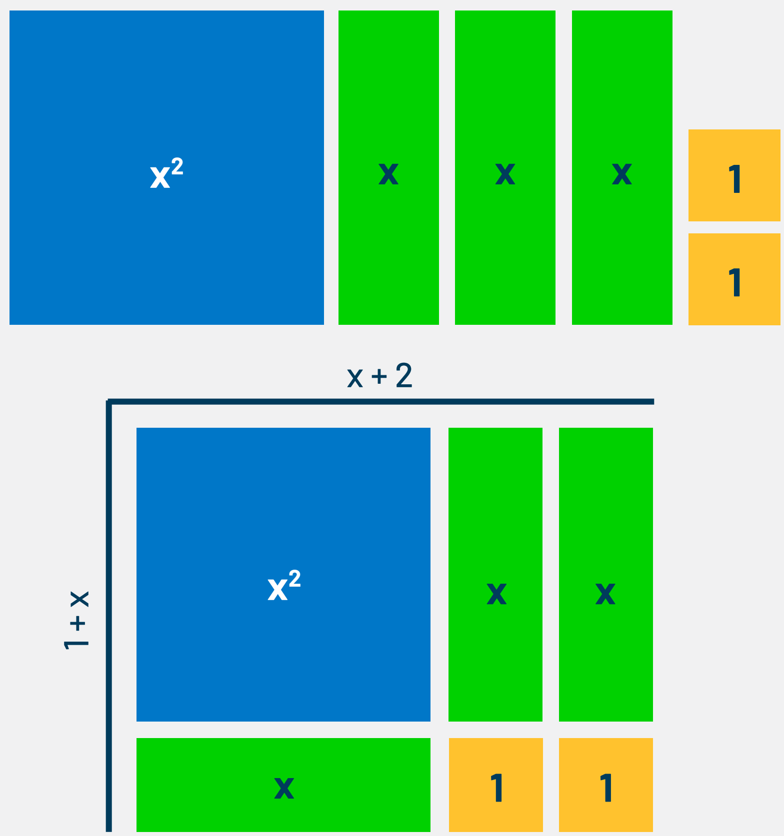
Abstract:

$$\begin{array}{r} 15 \\ \times\ 11 \\ \hline 15 \\ +\ 150 \\ \hline 165 \end{array}$$

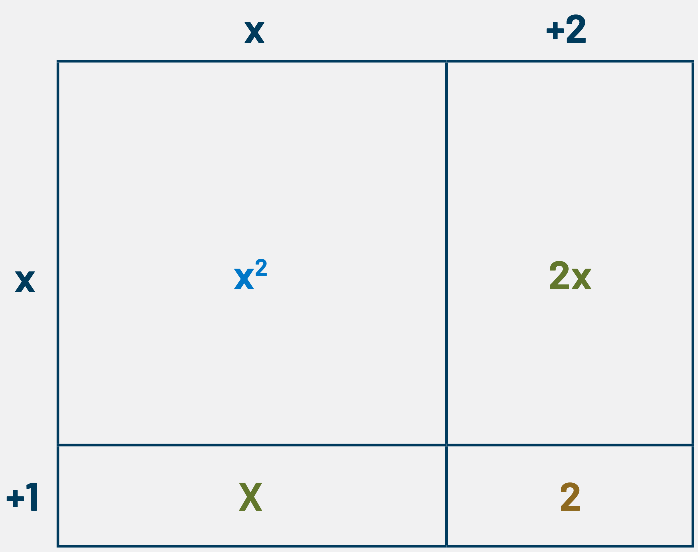
Figure 8: CPA Progressions of Multi-Digit Multiplication

Factor the polynomial: x^2+3x+2

Concrete:



Pictorial:



(x+1) (x+2)

Abstract:

$$x^2+3x+2$$

$$(x+\underline{1})(x+\underline{2})$$

2	
1	2

Figure 9: CPA Progressions of Factoring Polynomials

	Teacher Actions	Student Actions
Introduction of a Concept	Presents a real-world problem (e.g., sharing snacks, calculating discounts, designing a garden) and models the concept using concrete materials, visual representations, and symbolic notation. Uses Universal Design for Learning (UDL) principles to provide multiple entry points (visual, verbal, kinesthetic, digital). *View additional information regarding UDL on page 70.*	Engages with the concept through manipulatives (concrete), diagrams or drawings (pictorial), and numerical symbols or equations (abstract).
Problem Solving	Facilitates a group or individual problem-solving activity. Encourages students to try multiple strategies, such as drawing diagrams or models, using manipulatives, applying mental math or algorithms, and exploring patterns and generalizations. Supports collaboration and discussion while scaffolding for diverse learners.	Works with partners or independently to explore solutions using concrete, representational, and abstract methods. Tries multiple strategies, justifies choices, and demonstrates flexibility in finding solutions.
Consolidating Learning	Collaboratively creates anchor charts, graphic organizers, or digital representations with student input. Highlights key strategies, patterns, and connections across representations.	Contributes ideas, examples, manipulatives, diagrams, and equations to the shared reference, reinforcing understanding across CPA stages.
Check for Understanding	Uses formative assessments, such as exit tickets, quick sketches, oral explanations, or short performance tasks, to evaluate understanding across CPA stages.	Responds with hands-on materials, drawings, or symbolic work. Demonstrates understanding and identifies areas needing clarification.
Deepening Understanding	Challenges students to explain reasoning, compare multiple approaches, justify solutions, and connect ideas across CPA levels and conceptual, procedural, and application dimensions of the standards as written. Promotes rich discourse and reflection on mathematical thinking.	Explains thought processes, critiques peer strategies, applies reasoning to new problems, and demonstrates mastery through concrete, representational, or abstract methods.
Extending & Enrichment	Provides parallel tasks that allow all students to engage meaningfully while offering opportunities to extend reasoning or complexity for advanced learners. Connects tasks to real-world contexts and future applications.	Engages in tasks at their level of readiness, explores advanced or creative strategies, and applies concepts in novel or complex situations.

Figure 10: Actions to Build Mathematical Confidence

Differentiation

Effective differentiation in mathematics instruction recognizes the assets students bring to the classroom, including their cultural and linguistic resources, while also addressing gaps that may arise from systemic inequities. Instructional planning should consider all learners, not just the "average" student, ensuring that outliers, both students who need additional support and those who require enrichment, have meaningful access and challenge.[30]

Universal Design for Learning (UDL) instruction is designed to provide multiple means of engagement, representation, and expression. In practice, UDL encourages teachers to present information through diverse modalities, such as visuals, text, audio, manipulatives, or interactive simulations, so students with varying learning preferences can grasp concepts.[31] It also allows students to express their learning in multiple ways, including writing, speaking, drawing, or using digital tools, and provides varied opportunities for engagement through choice, real-world applications, collaboration, and appropriately challenging tasks. This approach ensures that all students can access, participate in, and demonstrate understanding of grade-level content, regardless of learning profile or background. *View additional information regarding UDL on page 70.*

Low-floor, high-ceiling tasks are intentionally designed to provide an accessible entry point for all students (low floor), while offering opportunities for extended reasoning and challenge (high ceiling). Such tasks allow students to engage at their level, explore multiple solution strategies, and deepen conceptual understanding.[32]

Additional differentiation strategies include:

- Offering varied entry points to mathematical concepts through visual, verbal, kinesthetic, or digital modalities
- Providing scaffolds and extensions such as intervention, remediation, or enrichment to learning so that tasks remain accessible yet rigorous for all learners
- Encouraging multiple-solution strategies and representations to honor diverse ways of reasoning
- Embedding opportunities for student choice and voice, which builds ownership of learning and affirms students' mathematical identities

For example:

Students are asked to design a rectangular 24-square-meter school garden, using whole-number side lengths. They must determine all possible dimensions and explain their reasoning. Students who are developing basic multiplication skills can use grid paper or manipulatives to count squares and explore simple length × width combinations, such as 4 × 6 or 3 × 8, providing a low-floor entry point. Advanced students can extend the task by generalizing patterns, calculating perimeter and fencing costs, considering fractional or mixed-number dimensions for irregular plots, or modeling the problem algebraically. They might also opt for additional variables, such as sunlight exposure or plant spacing, creating a high-ceiling challenge that encourages rich mathematical reasoning and multiple-solution strategies.

Equity in differentiation requires more than adjusting instruction within classrooms. It also involves ensuring that all students, regardless of background, prior achievement, or learning profile, have access to grade-level content, advanced coursework, and enrichment opportunities. Systemic barriers that track or limit students must be dismantled to create a truly equitable numeracy experience across Illinois schools.[33]

By intentionally planning for outliers and integrating UDL principles with low-floor and high-ceiling tasks, educators can meet students where they are, provide rigorous pathways forward, and ensure that all learners can engage meaningfully in mathematics.

Scaffolding

Scaffolding is an effective teaching method whereby teachers provide targeted supports for students to learn new concepts by building on prior knowledge. Scaffolding is different from differentiation. Scaffolding provides students with supports they may need to complete a task, while differentiation involves creating different tasks for different students based upon student needs. Effective instruction balances support and independence. Scaffolds should be deliberately removed as students develop competence, ensuring they move confidently through concrete, representational, and abstract stages while building mathematical resilience and problem-solving skills.[34]

✓ Do

- ✓ Personalize scaffolds based upon students' zone of proximal development
- ✓ Guide student thinking through correcting, modeling, cluing, or prompting
- ✓ Gradually fade supports as students become more independent
- ✓ Attach scaffolds to grade level and standards-aligned tasks

✗ Don't

- × Provide all students with the same supports
- × Provide students with answers
- × Remove supports too soon or keep them indefinitely
- × Use remedial work in place of rigorous learning tasks

Student is most independent

Definition	Visual	Verbal	Written
Self-Scaffolding Students independently draw on prior knowledge, strategies, and reasoning to plan, monitor, and evaluate their approach to a task. Educators observe, give space for thinking, and intervene only when necessary.	Students independently create visual reminders or organizers (e.g., number lines, arrays, coordinate grids, diagrams, geometric models) that help them consolidate learning.	Students generate questions to support metacognition or next steps (e.g., "What do I know about this type of problem?" "How else could I represent this?" "Does my answer make sense?").	Students independently create success criteria, checklists, or self-evaluation tools to monitor their work. (e.g., multi-step word problems, fraction comparisons).
Prompting Educators provide prompts that encourage students to apply what they already know without directing them toward a specific method. Prompts nudge students toward productive strategies and independence.	Educators encourage students to identify classroom visuals (e.g., fact families, multiplication charts, geometric vocabulary walls, fraction models) that may support decision-making.	Educators offer reflective or planning questions such as: "What strategy could you try first?" "Where have you seen a problem like this?" "Which representation might help you understand this better?"	Educators encourage students to revisit written scaffolds they have developed previously (e.g., steps for solving equations, writing frames for explaining reasoning, structures for math arguments).
Cueing Educators provide targeted hints that point students toward key information or conceptual connections while still requiring them to think and problem-solve.	Visual cues highlight essential concepts or representations (e.g., equal groups, symmetry, place value shifts, slope triangles, partitioned fraction bars) to support understanding.	Educators offer verbal clues connecting to strategies previously used successfully (e.g., "Think about how the numbers are related," "Check the units," "What operation fits this context?")	Educators provide partially completed examples, sentence starters, word banks that help students engage with a task independently (e.g., labeled number lines, incomplete tables, partially filled area models)

Definition	Visual	Verbal	Written
Modeling Educators model a process, strategy, or representation so students can observe expert thinking. Students are expected to apply the modeled strategy shortly afterward.	Educators demonstrate the use of manipulatives, diagrams, or models for students to reference when completing their own work.	Educators model high-quality discourse or reasoning (e.g., "If I were justifying my solution, I might start by saying…").	Educators offer worked examples as references, ensuring students apply, but not copy, the structure and reasoning shown.
Correcting Educators identify errors or misconceptions and guide students in understanding the correction. Correcting should be used sparingly and only when conceptual clarity is necessary for learning to continue.	Educators annotate student work or highlight representations that need revision without completing the work for the student.	Educators verbally explain misconceptions (e.g., "This step doesn't follow because… Let's examine the relationship between these quantities again.").	Educators provide accurate models, spellings, or symbolic notation for students to reference or rewrite when appropriate.

Student is least independent

Figure 11: Scaffolding Framework[35]

Addressing Misconceptions

A critical component of maintaining a coherent learning continuum is the intentional identification and instructional use of student misconceptions. Misconceptions should not be viewed simply as errors to correct but as evidence of students' current reasoning and partial understandings. Effective mathematics instruction requires teachers to anticipate likely misconceptions, elicit student thinking through purposeful questioning and formative assessment, and use that information to make real-time instructional decisions that refine and extend understanding.[36]

Rather than immediately correcting errors, teachers should create opportunities for students to examine misconceptions through discussion, comparison of strategies, and engagement with multiple representations. This approach positions misconceptions as productive entry points for deeper learning and strengthens students' conceptual understanding over time.

In some cases, misconceptions can be unintentionally reinforced through instruction that emphasizes procedural "tricks" or shortcuts without conceptual grounding. For example, teaching students to "move the decimal" when multiplying by powers of 10 can obscure understanding of place value, and suggesting that an inequality symbol "points to the direction to shade on the number line" can lead to errors when graphing or interpreting solutions. While these shortcuts may produce correct answers in limited situations, they do not build number sense or transferable understanding and can break down in more complex contexts.

If misconceptions are not addressed with intention, they can solidify and disrupt students' ability to access future content. For example, when first learning fractions, students may believe that a larger denominator indicates a larger fraction (e.g., thinking 1/8 > 1/4). Addressing this misconception requires more than correction; it involves engaging students with visual models, number lines, and reasoning tasks that help them reconcile the relationship between unit size and quantity. This type of targeted instructional response supports stronger understanding of fraction magnitude, which is foundational for later work with equivalence, operations with rational numbers, and proportional reasoning.[37]

Collaboration

Collaboration is essential for maintaining a cohesive continuum of learning. Educators need structured opportunities to examine standards and progressions across grade levels, identify where students may struggle, and design instructional supports. Families also play an important role in reinforcing numeracy development across a child's lifetime, especially in early years, and should be viewed as educational partners by educators and administrators.[38]

Collaboration and communication between general education and special education staff are crucial to supporting numeracy growth. Successful inclusive instruction requires structured, ongoing communication about what is being taught, how it is taught, and the strategies and accommodations being applied, as well as considerations for advanced learners. When general education teachers bring strong content knowledge and special education teachers contribute expertise in individualized supports and executive functioning, a co-planning model can maximize access for all students.

Shared ownership includes:

- Co-planning for differentiation and accessibility in advance of instruction
- Structured communication protocols between general education teachers, special education teachers, and paraeducators, including discussion of goals, strategies, assessments, classroom observations, and data
- Intentional scaffolding of mathematical discourse to ensure all students can participate, build understanding, and develop a positive mathematical identity

Additionally, integrated learning opportunities for students can increase numeracy learning. Mathematics teachers are not solely responsible for numeracy instruction as all core content areas can support numeracy skills. When students see that mathematical skills can be applied across all content areas, they are more likely to realize that mathematics is necessary and relevant in their everyday lives rather than just a skill they need to pass a test or learn to prepare for the next grade level. Integrated curriculums have been shown to instill a positive mindset among students as well as increase student motivation and achievement.[39] Collaboration among grade-level teachers can promote more opportunities for integrated learning of all subjects to better support student achievement, especially in areas in which there is already overlap or close alignment of skills or practice standards.

Specialized Learning Considerations

Students with specialized learning needs, including those with disabilities, multilingual learners, and students requiring additional supports, as well as those who are gifted and talented all benefit from evidence-based practices grounded in the principles of UDL.[40]

Key considerations include:

- Accessibility: Tasks and assessments should be designed to reduce barriers and allow all students to demonstrate mathematical understanding. This includes providing multiple means of representation (visual models, manipulatives, etc.), offering options for how students express their thinking (verbally, in writing, or visual representations) and ensuring tasks maintain mathematical rigor while removing unnecessary linguistic or procedural barriers. Educators should audit tasks and activities during the planning process to identify potential barriers to learning and proactively plan supports that increase access to meeting the learning targets.
- Scaffolding and Supports: Scaffolds include using tools such as manipulatives, technology, visual aids, and structured peer collaboration. Effective scaffolding involves modeling strategies, cueing conceptual connections, and prompting students to apply prior knowledge to solve problems while maintaining high cognitive demand. Worked examples, structured discourse routines, and intentional use of representations further support access to complex material. Educators should intentionally fade supports over time based on student understanding to promote independence and transfer.
- Flexible Pacing and Pathways: It's important to recognize that students may progress through numeracy development at different rates and require personalized supports. This may include using formative assessments to adjust instruction in real time, offering varied entry points to tasks, and providing opportunities for extension or additional practice based on student need. Options for flexibility such as choice boards or parallel tasks should be included in lessons to engage students at an appropriate instructional level.

- Strength-Based Approaches: Teachers must identify and build on students' assets. Instruction should intentionally leverage students' prior knowledge, cultural and linguistic resources, and problem-solving strategies to position students as capable learners and promote positive mathematics identities. Teachers can incorporate opportunities for students to share and compare strategies, highlighting diverse ways of thinking as an asset to learning.
- Collaboration: General educators, special educators, and support staff must work together to deliver coherent and aligned instruction. Collaborative planning, co-teaching models, and data-driven instruction support consistency in expectations and teaching methods, leading to more inclusive and effective environments. Establishing regular structures for collaboration such as common planning times or data meetings helps ensure support is coordinated and responsive to student needs.

When classrooms implement these considerations, students with specialized needs, including those who require intervention or enrichment, not only access the same numeracy content as their peers but also engage in meaningful problem solving, reasoning, and communication. By adopting this approach, Illinois schools can ensure all students, regardless of ability, background, or need, have equitable opportunities to thrive mathematically.

Considerations for Learners with Specialized Education Needs

Professional learning grounded in evidence-based practices, such as those outlined in [Institute of Education Sciences Practice Guides], is essential for equipping educators with strategies that are effective for students with learning disabilities and beneficial for all learners. Learners with specialized learning needs must be exposed to grade level content as well as receive instruction that supports the Six Components of Numeracy. Students should be supported with appropriate learning accommodations. Collaborative efforts between general education teachers and special education teachers must be prioritized to ensure learners with specialized education needs are provided with equitable access to rigorous mathematics instruction. Co-teaching, when possible, is the ideal solution to ensure this collaboration as well as to account for the common differences in educator preparation among special education and general education teachers.[41]

Early screening is important to identify students at risk and those with specialized education needs who may need evidence-based supports. Identification takes place via screening and assessment, and learning supports for individuals are determined through Individualized Education Programs. Explicit and systematic approaches are especially important for specialized education needs.

Considerations for Learners with Dyscalculia

Dyscalculia is a learning disability that affects how students learn, understand, and retain mathematical concepts.[42] It is a specific learning disorder that impacts number sense, magnitude, and the ability to make sense of numerical relationships.[43]

Students with dyscalculia may experience difficulty with:

- Understanding number relationships and magnitude
- Counting and number sense development
- Math fact retrieval and fluency
- Multistep computation and procedures
- Interpreting math symbols
- Understanding place value and number structure

Effective instruction for students with dyscalculia requires intentional design that prioritizes clarity, structure, and conceptual understanding.[44] Key considerations include:

- Use clear, direct language and break instruction into manageable steps. Check for understanding throughout the process and create time and space for students to ask for clarification.
- Use visual supports, checklists, and think-alouds to make procedures transparent and support independent problem solving over time.
- Consistently use and provide access to concrete and visual supports. Manipulatives, number lines, visual models, and reference tools help students develop number sense and should remain available across grade levels and tasks when needed.
- Use visual step-by-step modeling and guided practice. Structured checklists and worked examples help students retain processes, increasing independence over time.
- Point out patterns and connections in mathematics for students. Students with dyscalculia benefit from explicit support in recognizing structure in mathematics (e.g., place value patterns, algebraic structure, or relationships in operations). Visual cues and schema-based learning can assist students in identifying structures.
- Support students' emotional well-being and confidence in mathematics. Students with dyscalculia may experience increased math anxiety, frustration, or avoidance due to repeated struggle. Instruction should intentionally normalize mistakes, provide frequent opportunities for success, and create a supportive environment that builds confidence and reduces fear of failure in mathematics.

Considerations for Multilingual Learners

To support multilingual learners (MLs) as they become competent and confident problem solvers and contributors of mathematics knowledge while engaging in a mathematical discourse community, Celedón-Pattichis & Ramirez suggest the following guiding principles for teaching mathematics:

Challenging mathematical tasks	Linguistically sensitive social environments	Support for learning English while learning mathematics	Mathematical tools and modeling as resources	Cultural and linguistic differences as intellectual resources

Effective teachers of MLs address these principles by using the following practices and actions in their classrooms[45]:

- At the beginning stages of ML development, limit the use of various academic terms for calculations in word problems. Wait until MLs have shown mastery before introducing new terms.
- Use challenging problems and tasks and assess MLs' prior knowledge of the problem's background and context. This includes culturally relevant problems or tasks, planning the use of multiple tools, representations, and models, and focusing on what mathematical and everyday language demands of the problem or task.
- Model making sense of written instructions prior to using these instructions on exams so students are not seeing these instructions for the first time on an exam.

- Form groups based on the cognitive demand of the task or problem and awareness of how students may use their native language to process and reason their way to a solution, particularly as the complexity of the problem increases. Try to avoid calling attention to a student who appears to need additional support.
- Sequence problems and tasks carefully to allow MLs to develop competence with mathematics while developing proficiency with language, both mathematical and English.
- Facilitate and model mathematical discourse (e.g., provide sentence starters, visuals, use topics that are familiar, provide models of how to respond or ask questions) in class so that students can develop conceptual understanding and procedural fluency while increasing their language proficiency.
- Provide support through multiple modalities (e.g., visual, auditory, native language) to address complexity of language demands in problems and tasks.
- Allow processing time while recognizing the mental requirements students have with speaking, reading, listening, and writing in a new language.
- Provide opportunities for MLs to read and write about their learning of mathematics, receive feedback that purposefully addresses mathematical language development, and allow students to revise their writing to support students in their learning of language. Include opportunities for problems and tasks with rich mathematical contexts and sophisticated language to help students advance their language development and mathematical learning.
- Provide space in the classroom where mathematical vocabulary and meaning are visible, referred to often during instruction, and frequently used in students' oral and written communications.
- Allow for the use of multiple tools and representations to enhance mathematical discourse and understanding. Intentionally teach the use of these tools to MLs. Based on background and/or prior experience, students may be unfamiliar with calculators, math apps, or other manipulatives. Provide students the opportunity to practice with the tools after teaching their use.
- Make visual connections to mathematical models and representations found in the classroom.
- Call attention to the peculiarities between meanings of words (e.g., words with multiple meanings such as yard, area, times, and others, as well as homophones like eight and ate) used in both everyday language and mathematical language.
- Meet the needs of students by listening to what they have to say while providing supporting guidance and mentoring and advocating for their rights as students and individuals.
- Learn about students' culture and community, language, dialects, and ways of knowing and understanding to enhance the teacher-student experience. Be cognizant of cultural differences in mathematical representations and notations, writing, and problem solving.
- Model acceptance and interest in students' language and culture to show MLs that they are valued for what they have to offer the class. This can have a positive effect on students' learning and identity.
- Communicate and collaborate regularly with your school or district's ESL or bilingual teacher(s).
- Identify the background knowledge a student may need to solve a word problem. Unfamiliarity with the topic of the word problem may be a barrier to the student demonstrating their understanding of the mathematical concepts being assessed. For example, in a mathematics problem about receiving a coupon for a free t-shirt at a football game, the student may be distracted trying to figure out what a coupon is, and whether football and soccer are the same sport.

Considerations for Advanced Learners

Advanced learners are present in every district and across all levels of socioeconomic status, and educators must be equipped with the skills and knowledge to support them. The following strategies and best practices can be used to create a supportive, equitable learning environment for advanced learners.

Strategy	Definition	Additional Information
Acceleration	Moving through the curriculum at a pace that challenges students appropriately, preventing boredom and stagnation	Examples of acceleration include skipping a grade level, early entrance into kindergarten or college, and entrance into dual credit or Advanced Placement courses.
Curriculum Compacting	Modifying the regular curriculum to skip previously mastered learned material to focus on more challenging numeracy tasks	Educators must begin curriculum compacting by identifying the intended instructional outcomes of a unit. A pre-assessment can then be used to determine which skills students have previously mastered and which skills must still be developed. Based upon results, educators will then provide acceleration or enrichment.
Grouping	Placing students with similar ability levels together for more advanced instruction and rapid growth	Grouping should be flexible to allow for variation and avoid tracking with the goal of matching student readiness with instruction.
Community Partnerships	Collaborating with local businesses, museums, universities, community members, etc.	External partners can provide resources and supports and offer students opportunities for real-world applications.
Enrichment	Extending learning experiences beyond the standard curriculum and invite students to explore numeracy more deeply	Deepening knowledge occurs when students engage with current content through complex problem solving, investigation, multiple-solution strategies, etc., strengthening conceptual understanding and the ability to apply learning across contexts. Enrichment can occur in the classroom or through pull-out programs.

Figure 12: Considerations for Advanced Learners[46]

Multi-Tiered Systems of Support

A strong Multi-Tiered System of Support (MTSS) is foundational to ensuring that every student in Illinois develops numeracy. This system is designed to enhance the core learning environment, identify struggling students early and those who would benefit from extension; offering timely interventions and enrichment opportunities that encompass various aspects of a child's development, including academics, behavior, social and emotional needs, and attendance. At its heart, MTSS is about delivering high-quality mathematics instruction, monitoring progress, and engaging all students in meaningful, grade-level learning.

An effective MTSS framework for numeracy includes:

- Tier 1 (Core Instruction): High-quality, evidence-based instruction delivered to all students. This tier is culturally and linguistically responsive, differentiated to meet diverse needs, and aligned across grade levels. Tier 1 instruction emphasizes reasoning, problem solving, and application, which are essential components of numeracy.
- Tier 2 (Targeted Supports): Small-group or supplemental instruction is provided for students who need additional support as well as opportunities for extension and enrichment. These supports are guided by assessment data and include reteaching of concepts, structured practice, and scaffolded tasks that reinforce mathematical reasoning and confidence while addressing prerequisite skill gaps necessary for grade-level mastery. In addition, students who demonstrate readiness are provided with enrichment experiences such as problem-based learning, mathematical investigations, and opportunities to apply concepts in more complex or accelerated contexts to deepen understanding. All supports and extensions are aligned to Tier 1 core instruction to ensure coherence and continuity of learning.
- Tier 3 (Intensive Interventions or Acceleration): Individualized, intensive supports are provided for students with significant, persistent prerequisite skill gaps. These interventions are evidence-based, tailored to student strengths, and aligned to Tier 1 core instruction, with a focus on closing foundational gaps and building readiness for grade-level learning. Extension and enrichment opportunities are provided for students who demonstrate advanced readiness or are identified as gifted. These opportunities include complex problem-solving, higher-level tasks, and experiences that deepen and extend learning through advanced or accelerated applications, while remaining aligned to Tier 1 core instruction.
- Engagement and Belonging: MTSS also addresses student engagement, persistence, and sense of belonging in mathematics classrooms. When students feel safe, valued, and included, they are more likely to participate fully in learning opportunities and develop positive mathematical identities.
- Collaboration: Effective MTSS requires collaboration across general educators, interventionists, special educators, administrators, and families. Working together ensures that supports are coherent, timely, and aligned to each student's developmental needs.

By emphasizing both a strong core and tiered supports, MTSS helps prevent gaps in numeracy development while ensuring that interventions are available and equitable. It ensures that every student, including those with specialized needs, MLs, and those historically marginalized in mathematics, has access to high-quality instruction and the opportunity to thrive.[47]

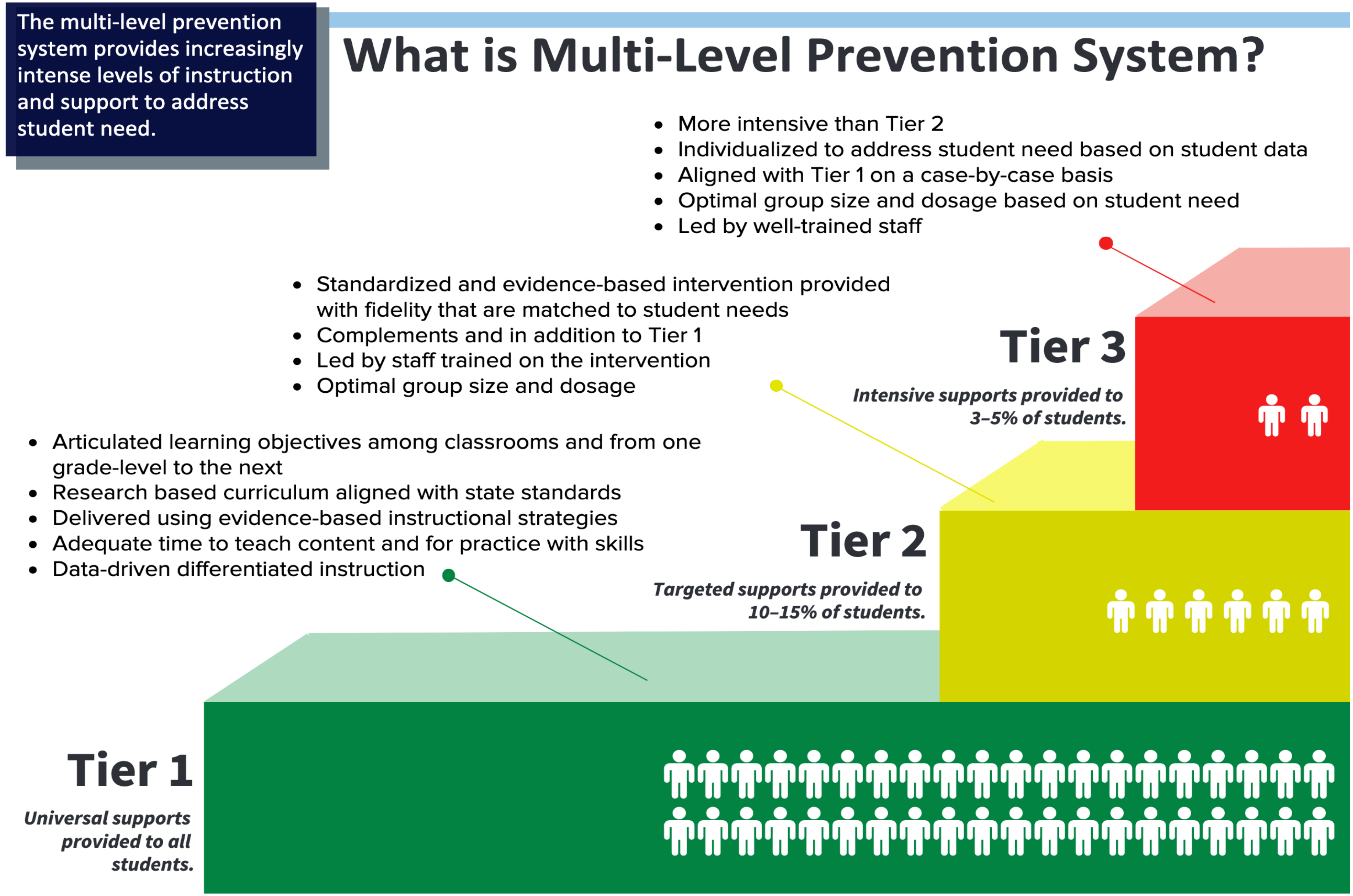


Figure 13: MTSS Tiers

From National Center on Intensive Intervention at American Institutes for Research. [Essential Components of MTSS: Infographic Collection](). Washington, DC: NCII.

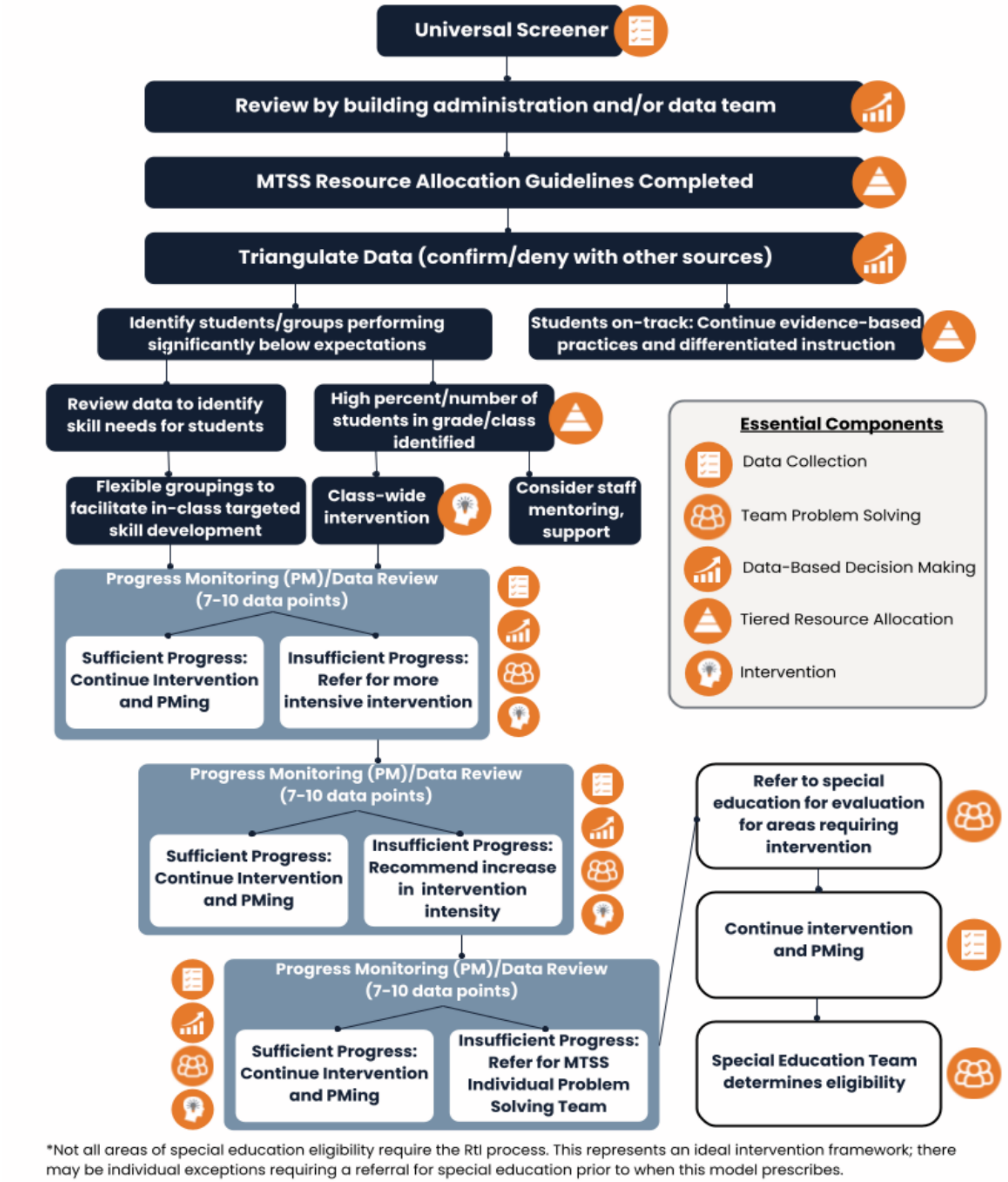


Figure 14: MTSS Intervention Framework

From Illinois Specific Learning Disability Support Project. [The Dyscalculia Handbook](), 2026.

Assessments to Support Numeracy

Assessment is the primary tool for informing instructional decision-making in mathematics, including planning, differentiation, intervention, placement, grading, and progression. It shapes decisions about student learning needs, curriculum implementation, and, in some instances, funding allocation.

ISBE's performance level descriptors (PLDs) as well as samples to success can assist educators in supporting student learning and assessment. PLDs are designed to bridge the state assessment to classroom instruction and the systems of formative assessments to inform instruction as well as individual student need.

Universal Design for Learning

Universal Design for Learning (UDL) is a framework that promotes equitable, rigorous learning for all students. According to the Center for Applied Special Technology (CAST), "UDL aims to change the design of the environment rather than to situate the problem as a perceived deficit within the learner."[48]

The UDL framework consists of three guidelines:

Engagement *(the why of learning)*	Representation *(the what of learning)*	Action and expression *(the how of learning)*

By embedding UDL into assessment practices, educators ensure that every learner has equitable opportunities to demonstrate what they know and can do. Traditional assessments often assume that all students can access content, instructions, and response formats in the same way, but this approach unintentionally creates barriers. UDL challenges that assumption by proactively designing assessments that provide multiple pathways for access and multiple ways to express understanding. When assessments are designed through the UDL lens, students encounter content that is accessible from the start. This might mean offering questions with visual supports, read-aloud options, translations, or flexible text formats so that the focus remains on the construct being measured rather than on a student's ability to navigate inaccessible text. Similarly, students are given opportunities to respond in ways that match their strengths. For some, this may include using speech-to-text tools, drawing or modeling solutions, or creating digital artifacts to represent their thinking. These flexible approaches do not dilute rigor; instead, they ensure that the assessment is measuring the intended skill or concept rather than a barrier unrelated to the learning target.[49]

Application of UDL in the Mathematics Classroom[50]:

- Use formative assessment data regularly (e.g., exit tickets, observations, student work) to make immediate instructional adjustments such as regrouping, reteaching, or extending tasks based on evidence of understanding
- Design engaging, relevant assessment tasks with multiple entry points and allow students to show understanding using varied representations (e.g., models, equations, diagrams, explanations, or manipulatives)
- Look at student thinking, not just answers, to identify partial understanding and guide next steps in instruction
- Provide feedback that focuses on mathematical reasoning and strategies, not only correctness
- Plan instruction based on the idea that students will approach mathematics in different ways, rather than expecting a single method for all learners
- Provide structured opportunities for students to collaborate with peers during problem solving, allowing them to share strategies and build understanding through mathematical discourse
- Intentionally create a supportive learning environment that acknowledges and builds on student strengths, normalizes mistakes, and provides low-risk opportunities for participation to help reduce math anxiety and increase confidence in engaging with challenging tasks

CAST Universal Design for Learning Guidelines

The goal of UDL is **learner agency** that is purposeful & reflective, resourceful & authentic, strategic & action-oriented.

	Design Multiple Means of **Engagement**	Design Multiple Means of **Representation**	Design Multiple Means of **Action & Expression**
Access	Design Options for **Welcoming Interests & Identities** • Optimize choice and autonomy • Optimize relevance, value, and authenticity • Nurture joy and play • Address biases, threats, and distractions	Design Options for **Perception** • Support opportunities to customize the display of information • Support multiple ways to perceive information • Represent a diversity of perspectives and identities in authentic ways	Design Options for **Interaction** • Vary and honor the methods for response, navigation, and movement • Optimize access to accessible materials and assistive and accessible technologies and tools
Support	Design Options for **Sustaining Effort & Persistence** • Clarify the meaning and purpose of goals • Optimize challenge and support • Foster collaboration, interdependence, and collective learning • Foster belonging and community • Offer action-oriented feedback	Design Options for **Language & Symbols** • Clarify vocabulary, symbols, and language structures • Support decoding of text, mathematical notation, and symbols • Cultivate understanding and respect across languages and dialects • Address biases in the use of language and symbols • Illustrate through multiple media	Design Options for **Expression & Communication** • Use multiple media for communication • Use multiple tools for construction, composition, and creativity • Build fluencies with graduated support for practice and performance • Address biases related to modes of expression and communication
Executive Function	Design Options for **Emotional Capacity** • Recognize expectations, beliefs, and motivations • Develop awareness of self and others • Promote individual and collective reflection • Cultivate empathy and restorative practices	Design Options for **Building Knowledge** • Connect prior knowledge to new learning • Highlight and explore patterns, critical features, big ideas, and relationships • Cultivate multiple ways of knowing and making meaning • Maximize transfer and generalization	Design Options for **Strategy Development** • Set meaningful goals • Anticipate and plan for challenges • Organize information and resources • Enhance capacity for monitoring progress • Challenge exclusionary practices

Figure 15: Universal Design for Learning Guidelines

From CAST. Universal Design for Learning Guidelines (Version 3.0): Graphic Organizer. Wakefield, MA: CAST, n.d.

Assessment Types

To effectively support teaching and learning, educators should use a combination of equitable, high-quality, and standards-aligned assessment types. It is important to recognize that not all assessments are created equal; educators must consider the purpose of assessments as well as the appropriate time to administer them.

Type	Purpose	Key Components	Best Practices
Universal Screening	Universal screening assists in identifying students who are thriving, at risk, or in need of acceleration through a systematic evaluation of all students within a class, grade, school building, or school district that focuses on critical academic and social-emotional indicators.	• Administered to all students at the beginning of the year • Concise assessment	• Use reliable, valid tools • Establish goals and methods of screening prior to administering • Develop a protocol for how data will be used to assess curriculum, instruction, educational environments, etc.
Benchmarking	Benchmark assessment is a cyclical process that involves using a screening tool multiple times throughout the school year to monitor students' responses to core instruction.	• Administered three times throughout the school year • Defines expected skill levels for students at each grade level at specific times throughout the year	• Use data to analyze common learning trends • Use results to inform interventions • Include students in goal setting and communicate benchmark results with parents in a digestible manner
Diagnostic Screening	Diagnostic screeners pinpoint specific pre-grade and grade-level skill gaps to guide appropriate Tier 2 supports and Tier 3 intensive interventions that address unfinished learning while aligning with Tier 1 instruction. These screeners also identify students who are ready for accelerated learning.	• Must identify and measure student needs and strengths	• Use results to inform interventions and acceleration • Use results to inform specific pre-grade and grade-level skills deficits

Type	Purpose	Key Components	Best Practices
Progress Monitoring	Progress monitoring measures the impact of assigned interventions by tracking progress toward grade-level mastery throughout the intervention period.	• Regularly monitors progress toward mastery of pre-grade and grade-level skill gaps identified through a student's diagnostic screener • Administered at least every two weeks	• Use results to immediately guide coherent, individualized instructional decisions in alignment with each tier of MTSS
Formative Assessments	Formative assessments are used during the learning process to gather evidence about student understanding and to guide immediate instructional adjustments.	• Can take place multiple times throughout a lesson in forms such as observation, exit tickets, quizzes, discourse, etc.	• Provide timely feedback and use results to guide instruction throughout the remainder of the learning unit • Promote student reflection
Summative Assessments	Summative assessments evaluate student learning at the conclusion of an instructional period, often for comparison or accountability purposes.	• Standards-aligned • Can include a unit test, projects, performance tasks, student portfolios, etc.	• Students are aware of assessment criteria • Data informs future instruction
State Assessment	State assessments measure the learning of state standards and are often used as accountability measures.	• Standardized, large-scale assessment	• Provide students with appropriate test-taking strategies • Focus on standards-aligned instruction throughout entirety of school year

Figure 16 Assessment Types

Assessment is only meaningful when it informs and changes instructional decision-making. The purpose of assessment is not simply to collect data but to guide what happens next in teaching and learning. Evidence of student thinking should lead to intentional instructional adjustments, such as modifying pacing, revisiting concepts, grouping students strategically, or selecting different tasks and representations. There is a critical distinction between assessment as a tool for decision-making and assessment as mere data collection; without a clear instructional response, assessment does not improve student outcomes. When used effectively, assessment becomes an ongoing process that drives responsive instruction and ensures all students have access to meaningful mathematical learning.

Section Summary

The Framework for the Evidence-Based Development of Numeracy skills establishes the instructional foundation of the Illinois Comprehensive Numeracy Plan. It emphasizes that developing numeracy requires intentional, evidence-based instruction that connects content, practices, and student thinking across a cohesive continuum of learning. By integrating the Six Components of Numeracy with high-quality instructional practices, educators create opportunities for students to build understanding, develop fluency, and communicate and apply their knowledge in meaningful ways. Attention to instructional considerations ensures that all learners have access to rigorous mathematics through inclusive approaches, while assessment practices provide ongoing evidence to guide and refine instruction. Together, these elements illustrate what high-quality, evidence-based numeracy instruction entails and how it can be developed and sustained across Illinois classrooms.

Additional Resources
Section 1

These resources are recommended based on feedback and insights gathered through public engagement that demonstrate strong alignment to this section of the Numeracy Plan. They provide opportunities to explore the topics in greater depth and offer additional context to support the work. Their inclusion does not constitute an endorsement by ISBE.

Books

- *5 Practices for Orchestrating Productive Mathematics Discussions* by Margaret Schwan Smith and Mary Kay Stein
- *Adding it Up: Helping Children Learn Mathematics* by the National Research Council
- The Dyscalculia Handbook by the Illinois Specific Learning Disability Support Project
- *Figuring Out Fluency* series by Jennifer M. Bay-Williams and John J. SanGiovanni
- *The Impact of Identity in K–8 Mathematics: Rethinking Equity-Based Practices* by Julia Aguirre, Karen Mayfield-Ingram, and Danny Martin
- *Making Number Talks Matter: Developing Mathematical Practices and Deepening Understanding* by Cathy Humphreys and Ruth Parker
- *Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention* by Jennifer Bay-Williams and Gina Kling
- *More Than Counting: Math Activities for Preschool and Kindergarten* by Sally Moomaw and Brenda Hieronymus
- *Number Sense Routines series* by Jessica F. Shumway
- *Number Talks* series by Sherry Parrish
- *Principles to Actions: Ensuring Mathematical Success for All* by DeAnn Huinker, Melissa H. Clements, Douglas Brahier, and Steve Leinwand
- *Rethinking Disability and Mathematics: A UDL Math Classroom Guide for Grades K-8* by Rachel Lambert
- *Rough Draft Math: Revising to Learn* by Amanda Jansen
- *Where's the Math? Books, Games, and Routines to Spark Children's Thinking* by Laura Grandau and Mary Hynes-Berry

Additional Resources
Section 1

Podcasts

- *Adding It All Up* from NCTM
- *Making Math Moments That Matter* hosted by Kyle Pearce and Jon Orr
- *Math Ed Podcast* hosted by Samuel Otten
- *Math Teacher Lounge* hosted by Bethany Lockhart Johnson and Dan Meyer
- *There's Power in Teaching* from PDK International
- *This Teacher Life* hosted by Monica Genta

Tools

- Inside Mathematics
- Learning Trajectories
- Math concept progression videos by Graham Fletcher
- Mathematics instructional routines
- Mathematics vocabulary cards
- Number strings resources
- Problem Strings

Goal 1: Workbook

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

Implementation Considerations

- Classroom environments should promote mathematical reasoning, discourse, and flexible problem solving.
- Students should have access to high-quality instructional materials that build conceptual understanding and procedural fluency.
- Instruction should incorporate multiple representations, including concrete, representational, and abstract models.
- Assessment practices should guide instruction and help identify gaps in conceptual understanding.
- Teachers need ongoing support to implement evidence-based mathematics practices with consistency across grade levels.

Notes

Next Steps

Use high-quality instructional materials and tasks that promote reasoning and conceptual understanding.

Incorporate mathematical discourse routines to help students communicate thinking clearly.

Apply the CRA progression to support students in building strong numeracy foundations.

Collaborate with your grade-level team to strengthen coherence across units and topics.

Monitor student progress using formative assessments and adjust instruction to address misconceptions.

Reflection Questions

1. What does a numeracy-rich classroom environment look and sound like in my grade level?

2. How do I ensure all students access grade-level mathematics content with appropriate scaffolds?

3. What strategies do I use to identify and address common mathematical misconceptions?

4. How do I ensure my assessment practices capture both conceptual understanding and procedural skills?

5. What obstacles impact my ability to implement evidence-based numeracy instruction with fidelity?

Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

Implementation Considerations

- Classroom leaders should help educators strengthen instruction using evidence-based numeracy practices.
- Collaboration across grade levels supports coherent progressions and reduces instructional fragmentation.
- Strong data-analysis routines help identify instructional needs and trends across classrooms.
- Curriculum and resources must align with the components of numeracy and grade-level standards.
- Teacher leaders should model and reinforce the use of multiple representations and mathematical discourse.

Notes

Next Steps

Support educators in implementing high-quality tasks that promote reasoning and problem solving.

Facilitate vertical team discussions to strengthen alignment across grade bands.

Help teachers analyze student work to identify strengths, misconceptions, and instructional needs.

Communicate instructional successes and needs with administrators to inform professional learning.

Guide teachers in using data to adjust instruction and monitor student numeracy development.

Reflection Questions

1. How are teachers supported in using evidence-based mathematics instruction practices?

2. How do I help maintain alignment and coherence across grade-level teams?

3. What trends do I observe in student understanding across classrooms?

4. How do I best support teachers in identifying and addressing misconceptions?

5. What barriers exist in helping teachers implement high-quality numeracy instruction?

Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

Implementation Considerations

- Effective numeracy instruction requires scheduling structures, materials, and training that support strong implementation.
- Equitable access to high-quality mathematics curriculum is essential for student success.
- School-level assessment systems should provide timely data to guide instruction and interventions.
- Collaborative planning time strengthens coherence and instructional quality.
- School leaders need to foster a culture where all educators and students see themselves as capable mathematical thinkers.

Notes

Next Steps

Engage teacher teams in reviewing student data to understand numeracy strengths and gaps.

Evaluate current mathematics curriculum and intervention materials and identify areas for improvement.

Strengthen MTSS processes to identify students needing additional numeracy support.

Build assessment literacy to ensure staff administer and interpret data effectively.

Provide ongoing professional learning aligned to evidence-based mathematics instruction.

Reflection Questions

1. Are all students receiving high-quality, evidence-based numeracy instruction?

2. Does the school's curriculum align to the components of numeracy described in the Illinois Comprehensive Numeracy Plan?

3. How effectively does the school use data to guide instruction and interventions?

4. What systems support teacher collaboration and coherence in mathematics instruction?

5. How do school structures promote equitable mathematics learning opportunities?

Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

Implementation Considerations

- District numeracy plans should reflect student data and instructional needs across schools.
- Curriculum adoption processes should prioritize high-quality, standards-aligned mathematics materials.
- Assessment systems must support monitoring of conceptual understanding, procedural skills, and problem solving.
- Vertical coherence is critical from early childhood through high school mathematics pathways.
- Professional learning should be aligned to evidence-based mathematics instruction and accessible across the district.

Notes

Next Steps

Prioritize selection and implementation of high-quality mathematics instructional materials.

Analyze districtwide data to assess achievement and identify areas requiring targeted support.

Strengthen assessment plans to ensure appropriate measures are available across grade levels.

Guide schools in building strong MTSS structures for numeracy.

Identify and reduce inequities in access to resources, coursework, and instructional opportunities.

Reflection Questions

1. What does district data reveal about numeracy outcomes and equity across schools?

2. How well does the district's curriculum support evidence-based numeracy instruction?

3. What instructional and assessment practices are currently successful?

4. Which grade levels or student populations require additional support?

5. How does the district ensure coherence from early numeracy through advanced coursework?

Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

Implementation Considerations

- Regional support should reflect the varied needs of districts and communities. Regional leaders play a critical role in providing professional learning aligned with evidence-based numeracy practices.
- Curriculum review cycles can be strengthened through regional collaboration.
- Data across districts can illuminate regional patterns and resource needs.
- Regional efforts should prioritize equitable access to high-quality mathematics instruction.

Notes

Next Steps

- Conduct regional needs assessments to guide professional learning offerings.
- Support districts in analyzing numeracy data and addressing instructional gaps.
- Facilitate opportunities for districts to review mathematics curriculum and share best practices.
- Provide guidance on selecting and implementing evidence-based instructional materials.
- Offer targeted support for districts serving high-poverty or high-need student populations.

Reflection Questions

1. How will regional leaders identify and respond to district-level numeracy needs?

2. What structures can strengthen regional collaboration around mathematics instruction?

3. What professional learning supports are most needed across the region?

4. How can regional leaders ensure equitable access to high-quality resources?

5. What barriers limit regional support for numeracy improvement?

Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

Implementation Considerations

- Preparation programs play a foundational role in developing teachers' mathematical content and pedagogical knowledge.
- Candidates must understand numeracy as outlined in the Illinois Comprehensive Numeracy Plan, including reasoning, fluency, and real-world application.
- Coursework should model evidence-based mathematics instruction and UDL principles.
- Early field experiences should reflect high-quality, coherent mathematics practices.
- Preparation programs help shape candidates' beliefs about mathematical identity and capability.

Notes

Next Steps

- Align coursework with the Illinois Comprehensive Numeracy Plan's numeracy definition and components.
- Provide clinical experiences that reflect evidence-based mathematics instruction.
- Strengthen candidate learning around diagnosing and addressing misconceptions.
- Incorporate instruction on mathematical discourse, multiple representations, and problem-solving frameworks.
- Build partnerships with districts to align expectations and practice.

Reflection Questions

1. How do educator preparation programs introduce candidates to numeracy development?

2. What opportunities do candidates have to observe and practice evidence-based mathematics instruction?

3. How are field placements aligned to high-quality mathematics teaching?

4. How do programs prepare candidates to support diverse learners in mathematics?

5. What additional supports are needed to strengthen candidates' mathematical knowledge for teaching?

Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

Implementation Considerations

- State leaders must ensure equitable access to high-quality mathematics instruction across Illinois.
- Statewide data should guide resource allocation and policy decisions.
- High-quality instructional materials and professional learning should be prioritized.
- State initiatives should promote coherence across districts and regions.
- Policies must reflect the Illinois Comprehensive Numeracy Plan's commitment to equity and evidence-based mathematics instruction.

Notes

Next Steps

Communicate the numeracy plan and support stakeholders with aligned resources.

Provide tools for evaluating high-quality mathematics curriculum and interventions.

Monitor statewide numeracy data to identify trends and inequities.

Allocate resources to support schools and districts with the greatest need.

Ensure professional learning reflects the most current evidence-based mathematics practices.

Reflection Questions

1. How will state leaders promote equitable access to high-quality mathematics instruction?

2. What statewide data trends require immediate attention?

3. How can state policies remove barriers to effective numeracy instruction?

4. How can state leaders support districts in selecting high-quality mathematics materials?

5. What obstacles may limit statewide implementation of the Illinois Comprehensive Numeracy Plan?

Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

Implementation Considerations

- Families and communities contribute to building positive mathematical identities.
- Access to community-based numeracy resources can strengthen learning beyond school.
- Transparent communication about student progress fosters partnership.
- Community organizations can support real-world mathematics experiences.
- Equitable access to numeracy opportunities varies across communities and should be addressed.

Notes

Next Steps

Ask questions and engage with schools to understand numeracy expectations.

Encourage mathematical thinking through everyday activities at home and in the community.

Advocate for accessible resources that support numeracy learning for all students.

Participate in or support programs that strengthen family-school partnerships.

Review assessment information to better understand student strengths and needs.

Reflection Questions

1. How can families and community partners support numeracy development at home?

2. What resources would help families better understand mathematics instruction?

3. How can communities create meaningful real-world mathematics experiences for students?

4. How can partnerships between families and schools be strengthened?

5. What barriers limit community engagement in supporting numeracy?

Framework for the Evidence-Based Development of Numeracy Skills End Notes

[1] Lambert, Rachel. Rethinking Disability and Mathematics: A UDL Math Classroom Guide for Grades K–8. 2018.

[2] de Jong, Ton, et al. "Let's Talk Evidence – The Case for Combining Inquiry-Based and Direct Instruction." Educational Research Review 39, no. 100536 (2023).

Sweller, John, et al. "Response to de Jong et al.'s (2023) Paper 'Let's Talk Evidence – The Case for Combining Inquiry-Based and Direct Instruction.'" Educational Research Review 42, no. 100584 (2023).

[3] Osborne, Michael C. "Teacher Instructional Practices and Student Mathematics Achievement." Journal of Educational Research and Practice 11, no. 1 (2021): 345–358.

[4] Professional Development of Mathematics Teachers: An Asian Perspective. Singapore: Springer, 2017.

[5] The New Teacher Project. Coherence by Design: Building Instructional Systems That Help Students Catch Up. Brooklyn, NY: TNTP, March 4, 2026.

[6] Council of Chief State School Officers. Making Sense of Instructional Coherence at the State Level. Washington, D.C.: Council of Chief State School Officers, 2026.

The New Teacher Project. Instructional Coherence: A Key to High-Quality Learning Acceleration for All. Brooklyn, NY: TNPT, June 2022.

[7] Kyeremeh, Patrick, Christopher Yarkwah, and Nduka Wonu. "Vertically Integrated Mathematics Curriculum and the Risks of Leaving Out Parts of the Curriculum." 2022.

[8] Pennsylvania Training and Technical Assistance Network (PaTTAN). Concrete-Representational-Abstract: Instructional Sequence for Mathematics. Harrisburg, PA: PaTTAN, 2017.

Root, Jenny R., Sarah K. Cox, Alicia Saunders, and Deidre Gilley. "Applying the Universal Design for Learning Framework to Mathematics Instruction for Learners with Extensive Support Needs." Remedial and Special Education 41, no. 4 (2020): 194–206.

[9] Common Core State Standards Initiative. "Standards for Mathematical Practice." In Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010.

[10] CCSSI. "Standards for Mathematical Practice," 2010.

[11] National Research Council. Adding It Up: Helping Children Learn Mathematics. Washington, DC: The National Academies Press, 2001.

[12] Douglas, Grace P., et al. "A Meta-Analysis of Mathematics Fact Fluency Interventions for Students With Mathematics Difficulties." Journal of Learning Disabilities 59, no. 3 (2026).

[13] Berger, Anne Maranda. "Using Number Talks to Build Procedural Fluency through Conceptual Understanding." Ohio Journal of School Mathematics 75, no. 1 (2017).

National Council of Teachers of Mathematics. "Procedural Fluency in Mathematics," 2023.

National Council of Teachers of Mathematics. Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM, 2014.

National Council of Teachers of Mathematics. Mathematical Proficiency: The Five Strands.

National Research Council. Adding It Up, 2001.

Schulz, Andreas. "Assessing Student Teachers' Procedural Fluency and Strategic Competence in Operating and Mathematizing with Natural and Rational Numbers." Journal of Mathematics Teacher Education 27 (2024): 981–1008.

Rosenshine, Barak. "Principles of Instruction." American Educator 36, no. 1 (Spring 2012): 12–19, 39.

Institute of Education Sciences. Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools. Washington, DC: U.S. Department of Education, 2009.

[14] National Research Council. Adding It Up, 2001.

[15] Jeyabal, K., M. Vasuki, and A. Dinesh Kumar. "From Algorithms to Conceptual Understanding: A Review of Instructional Strategies in Math." Journal of Engineering Scientific Research and Applications 2, no. 1 (2016): 191–205

National Research Council. Adding It Up, 2001.

[16] National Research Council. Adding It Up, 2001.

Mukuka, Angel, and Benjamin Tatira. "Analysis of Preservice Teachers' Understanding of Solving Trigonometric Equations: A Perspective Through Actions, Processes, Objects, and Schemas Theory." Pythagoras, 2025.

National Council of Teachers of Mathematics. Mathematical Proficiency: The Five Strands.

Institute of Education Sciences. Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools. Washington, DC: U.S. Department of Education, 2009.

National Council of Teachers of Mathematics. Principles to Actions, 2014.

Gilmore, Camilla, et al. "Understanding Arithmetic Concepts: The Role of Domain-Specific and Domain-General Skills." PLoS One 12, no. 9 (2018).

Illinois Specific Learning Disability Support Project. The Dyscalculia Handbook, 2026.

Rosenshine, Barak. "Principles of Instruction." American Educator 36, no. 1 (Spring 2012): 12–19, 39.

Hott, Brittany L., et al. "Using Schema-Based Instruction to Improve the Mathematical Problem Solving Skills of a Rural Student with EBD." Learning Disabilities: A Contemporary Journal 19, no. 2 (2021): 127–142.

[17] National Research Council. Adding It Up, 2001.

[18] National Council of Teachers of Mathematics. Principles to Actions, 2014.

National Council of Teachers of Mathematics. Mathematical Proficiency: The Five Strands.

Suh, J. M. & Seshaiyer, P. "Developing Strategic Competence by Teaching Using the Common Core Mathematical Practices." Annual Perspectives in Mathematics Education, 77–87. 2014.

Hatisaru, V., Richardson, S., & Beckwith, W. "Assessing strategic competence in solving algebraic word problems." For the Learning of Mathematics 45, no. 2 (2025): 49–52.

National Research Council. Adding It Up, 2001.

Schulz, Andreas. "Assessing Student Teachers' Procedural Fluency and Strategic Competence in Operating and Mathematizing with Natural and Rational Numbers." Journal of Mathematics Teacher Education 27 (2024): 981–1008.

Yetkin Özdemir, İ. Elif, and Stephen J. Pape. "Supporting Students' Strategic Competence: A Case of a Sixth-Grade Mathematics Classroom." Math Education Research Journal 24 (April 20, 2012): 153–168.

[19] National Research Council. Adding It Up, 2001.

[20] Ansari, B. I., Taufiq, & Saminan. "The Use of Creative Problem Solving Model to Develop Students' Adaptive Reasoning Ability: Inductive, Deductive, and Intuitive." International Journal on Teaching and Learning Mathematics 3, no. 1 (2020): 23–36.

Darwani, et al. "Adaptive Reasoning and Strategic Competence through Problem Based Learning Model in Middle School." Journal of Physics: Conference Series 1460 (2020): 012019.

National Council of Teachers of Mathematics. Principles to Actions, 2014.

National Research Council. Adding It Up, 2001.

National Council of Teachers of Mathematics. Mathematical Proficiency: The Five Strands.

Nurlita, Maya, et al. "The Essence of Students' Adaptive Reasoning Ability in Mathematics Learning: A Systematic Literature Review." The Eurasia Proceedings of Educational and Social Sciences 47 (December 2025): 95–108.

[21] National Research Council. Adding It Up, 2001.

[22] National Research Council. Adding It Up, 2001.

National Council of Teachers of Mathematics. Principles to Actions, 2014.

Go, Mary Cris J., and Laila S. Lomibao. "Exploring Learners' Productive Disposition Towards Learning Mathematics: A Phenomenological Analysis." Science International (Lahore) 36, no. 3 (2024): 329–334.

Handayani, Ita, et al. "Improving Students' Mathematical Disposition in Applied Graph Learning through the Knowledge Sharing Strategy." EduMatSains: Jurnal Pendidikan, Matematika dan Sains 10, no. 3 (2026): 53–62.

National Council of Teachers of Mathematics. Mathematical Proficiency: The Five Strands.

[23] National Research Council. Adding It Up, 2001.

[24] Gonzalez, Josue M. "Bilingual Education." In Encyclopedia of Bilingual Education, 139. SAGE Publications, 2008.

[25] National Research Council. Adding It Up, 2001.

National Council of Teachers of Mathematics. Principles to Actions, 2014.

National Council of Teachers of Mathematics. Mathematical Proficiency: The Five Strands.

Carpenter, Thomas P., Megan Loef Franke, and Linda Levi. Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School. Portsmouth, NH: Heinemann, 2003.

Chapin, Suzanne, Catherine O'Connor, and Nancy C. Anderson. Talk Moves: A Teacher's Guide for Using Classroom Discussions in Math, Grades K–6. Sausalito, CA: Math Solutions, 2022.

[26] Institute of Education Sciences. Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools. Washington, DC: U.S. Department of Education, 2009.

[27] Furner, Joseph M., and Nancy L. Worrell. "The Importance of Using Manipulatives in Teaching Math Today." Transformations 3, no. 1 (2017): Article 2.

[28] Boaler, Jo. Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching. San Francisco: Jossey-Bass, 2016.

National Council of Teachers of Mathematics. Principles to Actions, 2014.

[29] Mustafa, Ade Nandang. "Transformative Approaches and Challenges in 21st Century Mathematics Education: A Comprehensive Review." World Journal of Advanced Research and Reviews 20, no. 3 (2023): 444–457.

[30] Tomlinson, Carol Ann. The Differentiated Classroom: Responding to the Needs of All Learners. 2nd ed. Alexandria, VA: ASCD, 2014.

[31] CAST. Universal Design for Learning Guidelines, Version 2.2. Wakefield, MA: CAST, 2018.

[32] Boaler, Jo. "Promoting 'Relational Equity' and High Mathematics Achievement through an Innovative Mixed-Ability Approach." British Educational Research Journal 34, no. 2 (2008): 167–194.

Hiebert, James et al., Teaching Mathematics in Seven Countries: Results From the TIMSS 1999 Video Study (2003).

[33] National Council of Teachers of Mathematics (NCTM). Catalyzing Change in High School Mathematics: Initiating Critical Conversations. Reston, VA: NCTM, 2018.

Oakes, Jeannie. Keeping Track: How Schools Structure Inequality. 2nd ed. New Haven, CT: Yale University Press, 2005.

[34] van de Pol, Janneke, Monique Volman, Frans Oort, and Jos Beishuizen. "The Effects of Scaffolding in the Classroom: Support Contingency and Student Independent Working Time in Relation to Student Achievement, Task Effort, and Appreciation of Support." Instructional Science 43 (2015): 615–641.

Lombardi, Paula. Instructional Methods: Strategies and Technologies to Meet the Needs of All Learners. LibreTexts, Granite State College, 2018.

[35] Education Endowment Foundation. Special Educational Needs in Mainstream Schools: Supporting High-Quality Teaching for Pupils with SEND. London: EEF, 2020.

[36] Holmes, Vicki-Lynn, et al. "Data-Driven Intervention: Correcting Mathematics Students' Misconceptions, Not Mistakes." The Mathematics Educator 23, no. 1 (2013): 24–44.

[37] Ratnasari. "Students' Errors and Misconceptions about Operations of Fractions in an Indonesian Primary School." Southeast Asian Mathematics Education Journal 8, no. 1 (2018): 83–97.

[38] Douglas, Ashli-Ann, and Bethany Rittle-Johnson. "Parental Early Math Support: The Role of Parental Knowledge About Early Math Development." Early Childhood Research Quarterly 66 (2024): 124–134.

[39] Loepp, Franzie L. "Models of Curriculum Integration." Journal of Industrial Teacher Education 36, nos. 3–4 (1999): 21–25.

[40] CAST. UDL Guidelines, 2018.

[41] Zigmond, Naomi. "Benefits of Co-Teaching in Secondary Mathematics Classes." Teaching Exceptional Children 37, no. 3 (2005): 32–37.

[42] Dyscalculia Training & Research Institute. "Dyscalculia FAQs for Teachers." Dyscalculia Training & Research Institute.

[43] Hughes, Elizabeth et al. "Unpacking and Understanding Specific Learning Disabilities in Mathematics." Teaching Exceptional Children 56, no. 5 (July 2023).

[44] Mahmud, Muhammad Sofwan, et al. "Dyscalculia: What We Must Know about Students' Learning Disability in Mathematics?" Universal Journal of Educational Research 8, no. 12B (December 2020): 8214–8222.

Illinois Specific Learning Disability Support Project. The Dyscalculia Handbook, 2026.

Understood for All. Dyscalculia Fact Sheet. 2025.

[45] Celedón-Pattichis, Sylvia, and Nichole M. Ramirez, eds. Beyond Good Teaching: Advancing Mathematics Education for ELLs. Reston, VA: National Council of Teachers of Mathematics, 2012.

[46] Brown, Elissa F. "Differentiating Curriculum for Gifted Learners." Teaching for High Potential (November 2021): 3. National Association for Gifted Children.

National Association for Gifted Children. "Gifted Education Strategies." Washington, DC: NAGC.

Steinmeyer, Patricia, and Leodis Scott. Promising Practices for Equity & Inclusion: A Portrait of Six Illinois Schools. Illinois Association for Gifted Children, October 2022.

National Association for Gifted Children. "Grouping for Gifted Learners." Washington, DC: NAGC.

Renzulli, Joseph S., Sally M. Reis, and Carla Brigandi. "Enrichment Theory, Research, and Practice." In Critical Issues and Practices in Gifted Education, 185–199. Routledge, 2021.

[47] Fenning, Pamela, et al. Leveraging MTSS: Building a Better Academic and Behavioral Support System. Springfield, IL: ISBE, 2020.

[48] CAST. UDL Guidelines, 2018.

[49] CAST. UDL Guidelines, 2018.

[50] Gaunt, Lorraine, et al. "Universal Design for Learning for Mathematics Education." (2025).

Lambert, Rachel. "The Magic Is in the Margins: UDL Math." Mathematics Teacher: Learning and Teaching PK–12 114, no. 9 (2020): 660–669.

Lambert, Rachel. Rethinking Disability and Mathematics: A UDL Math Classroom Guide for Grades K–8. 2018.