Evidence Statement Tables

Grade 3 Mathematics

Evidence Statement Keys

Evidence statements describe the knowledge and skills that an assessment item/task elicits from students. These are derived directly from the Common Core State Standards for Mathematics (the standards), and they highlight the advances of the standards, especially around their focused coherent nature. The evidence statement keys for grades 3 through 8 will begin with the grade number. High school evidence statement keys will begin with "HS" or with the label for a conceptual category.

An Evidence Statement might:

1. **Use exact standard language** – For example:

   - 8.EE.1 - Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. This example uses the exact language as standard 8.EE.1

2. **Be derived by focusing on specific parts of a standard** – For example: 8.F.5-1 and 8.F.5-2 were derived from splitting standard 8.F.5:

   - 8.F.5-1 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).
   - 8.F.5-2 Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

   Together these two evidence statements are standard 8.F.5:

   Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or 2 decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

3. **Be integrative (Int) –** Integrative evidence statements allow for the testing of more than one of the standards on a single item/task without going beyond the standards to create new requirements. An integrative evidence statement might be integrated across all content within a grade/course, all standards in a high school conceptual category, all standards in a domain, or all standards in a cluster. For example:

   - **Grade/Course – 4.Int.2**[1] (Integrated across Grade 4)
   - **Conceptual Category – F.Int.1**[1] (Integrated across the Functions Conceptual Category)
   - **Domain – 4.NBT.Int.1**[1] (Integrated across the Number and Operations in Base Ten Domain)
   - **Cluster** – **3.NF.A.Int.1**[1] (Integrated across the Number and Operations – Fractions Domain, Cluster A)

4. **Focus on mathematical reasoning–** A reasoning evidence statement (keyed with C) will state the type of reasoning that an item/task will require and the content scope from the standard that the item/task will require the student to reason about. For example:

- 3.C.2[1] -- Base explanations/reasoning on the relationship between addition and subtraction or the relationship between multiplication and division.
  - Content Scope: Knowledge and skills are articulated in 3.OA.6
- 7.C.6.1[1] – Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.
  - Content Scope: Knowledge and skills are articulated in 7.RP.2

Note: When the focus of the evidence statement is on reasoning, the evidence statement may also require the student to reason about securely held knowledge from a previous grade.

5. **Focus on mathematical modeling –** A modeling evidence statement (keyed with D) will state the type of modeling that an item/task will require and the content scope from the standard that the item/task will require the student to model about. For example:

- 4.D.2[1] – Solve multi-step contextual problems with degree of difficulty appropriate to Grade 4 requiring application of knowledge and skills articulated in 3.OA.A, 3.OA.8,3.NBT, and/or 3.MD.

Note: The example 4.D.2 is of an evidence statement in which an item/task aligned to the evidence statement will require the student to model on grade level, using securely held knowledge from a previous grade.

- HS.D.5[1] - Given an equation or system of equations, reason about the number or nature of the solutions.
  - Content scope: A-REI.11, involving any of the function types measured in the standards.

[1] The numbers at the end of the integrated, modeling and reasoning Evidence Statement keys are added for assessment clarification and tracking purposes. For example, 4.Int.2 is the second integrated Evidence Statement in Grade 4.

Grade 3 Sub-Claims Structure

To stay on track for college and career readiness, students need to learn a wide range of skills, content knowledge, and critical-thinking abilities at every grade level as set forth in the Standards for Mathematical Content with connections to the Standards for Mathematical Practice.

Sub-Claim A — MAJOR CONTENT

- Solving problems involving multiplication and division, area, measurement, and basic fraction understanding.

Sub-Claim B — ADDITIONAL & SUPPORTING CONTENT

- Solving problems involving perimeter, place value, geometric shapes, and representations of data.

Sub-Claim C — EXPRESSING MATHEMATICAL REASONING

- Creating and justifying logical mathematical solutions and analyzing and correcting the reasoning of others.

Sub-Claim D — MODELING & APPLICATION

- Solving real-world problems, representing and solving problems with symbols, reasoning quantitatively, and strategically using appropriate tools.

Grade 3 Evidence Statements
Listing by Type I, Type II, and Type III

The Evidence Statements for Grade 3 Mathematics are provided starting on the next page. The list has been organized to indicate whether items designed are aligned to an Evidence Statement used for Type I items (sub-claims A and B), Type II items (reasoning/sub-claim C), or Type III items (modeling/sub-claim D).

Evidence Statements are presented in the order shown below and are color coded:

Peach – Evidence Statement is applicable to Type I items.

Lavender – Evidence Statement is applicable to the Type II items.

Aqua – Evidence Statement is applicable to the Type III items.

Grade 3 Evidence Statements

Type I **Type II** **Type III**

Sub-Claim	Evidence Statement Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
Sub-claim A (20 of 52 points) & Sub-claim B (10 of 52 points)				
A	3.OA.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	i) Tasks involve interpreting rather than calculating products in terms of equal groups, arrays, area, and/or measurement quantities. (See Table 2 below.) For example, "the total number of books if 5 shelves each have 7 books" can be represented by the expression 5x7 rather than "Marcie placed 7 books on each of 5 shelves. How many books does she have?" ii) Tasks do not require students to interpret products in terms of repeated addition, skip-counting, or jumps on the number line. iii) The italicized example refers to describing a real-world context, but describing a context is not the only way to meet the standard. For example, another way to meet the standard would be to identify contexts in which a total can be expressed as a specified product.	MP.2, MP.4
A	3.OA.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	i) Tasks involve interpreting rather than calculating quotients in terms of equal groups, arrays, area, and/or measurement quantities. (See Table 2 below.) For example, "35 books are placed equally on 7 shelves" can be represented by the expression 35 ÷ 7 rather than "Marcie has 35 books. She placed the same number on each of 7 shelves. How many books did she place on each shelf?" ii) Tasks do not require students to interpret quotients in terms of repeated subtraction, skip-counting, or jumps on the number line. iii) The italicized example refers to describing a real-world context, but describing a context is not the only way to meet the standard. For example, another way to meet the standard would be to identify contexts in which a number of objects can be expressed as a specified quotient. iv) Half the tasks require interpreting quotients as a number of objects in each share and half require interpreting quotients as a number of equal shares.	MP.2, MP.4
A	3.OA.3-1	Use multiplication within 100 (both factors less than or equal to 10) to solve word problems in situations involving equal groups, arrays, or area, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	i) All products come from the harder three quadrants of the times table ($a \times b$ where $a > 5$ and/or $b > 5$). ii) 75% of tasks involve multiplying to find the total number (equal groups, arrays); 25% involve multiplying to find the area. iii) For more information see Table 1 and Table 2 and Table 3 below.	MP.1, MP.4

Grade 3 Evidence Statements

Type I **Type II** **Type III**

Sub-Claim	Evidence Statement Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
A	3.OA.3-2	Use multiplication within 100 (both factors less than or equal to 10) to solve word problems in situations involving measurement quantities other than area, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	i) All products come from the harder three quadrants of the times table ($a \times b$ where $a > 5$ and/or $b > 5$). ii) Tasks involve multiplying to find a total measure (other than area). iii) For more information see Table 1 and Table 2 and Table 3 below.	MP.1, MP.4
A	3.OA.3-3	Use division within 100 (quotients related to products having both factors less than or equal to 10) to solve word problems in situations involving equal groups, arrays, or area, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	i) All quotients are related to products from the harder three quadrants of the times table ($a \times b$ where $a > 5$ and/or $b > 5$). ii) Tasks using this Evidence Statement will be created equally among the following: • dividing to find the number in each equal group or in each equal row/column of an array; • dividing to find the number of equal groups or the number of equal rows/columns of an array; and • dividing an area by a side length to find an unknown side length. iii) For more information see Table 1 and Table 2 and Table 3 below.	MP.1, MP.4
A	3.OA.3-4	Use division within 100 (quotients related to products having both factors less than or equal to 10) to solve word problems in situations involving measurement quantities other than area, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	i) All quotients are related to products from the harder three quadrants of the times table ($a \times b$ where $a > 5$ and/or $b > 5$). ii) Half the tasks involve finding the number of equal pieces and half involve finding the measure of each piece. iii) For more information see Table 1 and Table 2 and Table 3 below.	MP.1, MP.4
A	3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$.	i) Tasks do not have a context. ii) Only the answer is required. iii) All products and related quotients are from the harder three quadrants of the times table ($a \times b$ where $a > 5$ and/or $b > 5$).	-
A	3.OA.6	Understand division as an unknown-factor problem. *For example, find* $32 \div 8$ *by finding the number that makes 32 when multiplied by 8.*	i) All products and related quotients are from the harder three quadrants of the times table ($a \times b$ where $a > 5$ and/or $b > 5$).	-

Grade 3 Evidence Statements

Type I **Type II** **Type III**

Sub-Claim	Evidence Statement Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
A	3.OA.7-1	Fluently multiply and divide within 25. By end of Grade 3, know from memory all products of two one-digit numbers.	i) Tasks do not have a context. ii) Only the answer is required. iii) Tasks require finding products and related quotients accurately. For example, each 1-point task might require four or more computations, two or more multiplication and two or more division. iv) Tasks are not timed.	-
A	3.OA.7-2	Fluently multiply and divide within 100. By the end of Grade 3, know from memory all products of two one-digit numbers.	i) Tasks do not have a context. ii) Only the answer is required. iii) Tasks require finding products and related quotients accurately. For example, each 1-point task might require four or more computations, two or more multiplication and two or more division. iv) 75% of tasks are from the harder three quadrants of the times table ($a \times b$ where $a > 5$ and/or $b > 5$). v) Tasks are not timed.	-
A	3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	i) Tasks do not require a student to write a single equation with a letter standing for the unknown quantity in a two-step problem, and then solve that equation. ii) Tasks may require students to write an equation as part of their work to find a solution, but students are not required to use a letter for the unknown. iii) Addition, subtraction, multiplication and division situations in these problems may involve any of the basic situation types with unknowns in various positions (see Table 1 and Table 2 and Table 3 below).	MP.1, MP.4
B	3.NBT.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	i) Tasks have no context. ii) Tasks are not timed.	-
B	3.NBT.3	Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.	i) Tasks have no context.	MP.7
A	3.NF.1	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.	i) Tasks do not involve the number line. ii) Fractions equivalent to whole numbers are limited to 0 through 5. iii) Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8.	MP.2

Grade 3 Evidence Statements

Type I **Type II** **Type III**

Sub-Claim	Evidence Statement Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
A	3.NF.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.	i) Fractions may be greater than 1. ii) Fractions equivalent to whole numbers are limited to 0 through 5. iii) Fractions equal whole numbers in 20% of these tasks. iv) Tasks have "thin context"[2] or no context. v) Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8.	MP.5
A	3.NF.3a-1	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size.	i) Tasks do not involve the number line. ii) Fractions equivalent to whole numbers are limited to 0 through 5. iii) Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8. iv) The explanation aspect of 3.NF.3 is not assessed here.	MP.5
A	3.NF.3a-2	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same point on a number line.	i) Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8. ii) Fractions equivalent to whole numbers are limited to 0 through 5. iii) The explanation aspect of 3.NF.3 is not assessed here.	MP.5
A	3.NF.3b-1	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3).	i) Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8. ii) Fractions equivalent to whole numbers are limited to 0 through 5. iii) The explanation aspect of 3.NF.3 is not assessed here.	MP.7
A	3.NF.3c	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*	i) Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8. ii) Fractions equivalent to whole numbers are limited to 0 through 5. iii) The explanation aspect of 3.NF.3 is not assessed here.	MP.3, MP.5, MP.7
A	3.NF.3d	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	i) Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8. ii) Fractions equivalent to whole numbers are limited to 0 through 5. iii) Justifying is not assessed here. For this aspect of 3.NF.3d, see 3.C.3-1 and 3.C.4-4. iv) Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.	MP.7

Grade 3 Evidence Statements

Type I **Type II** **Type III**

Sub-Claim	Evidence Statement Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
A	3.NF.A.Int.1	In a contextual situation involving a whole number and two fractions not equal to a whole number, represent all three numbers on a number line diagram, then choose the fraction closest in value to the whole number.	i) Fractions equivalent to whole numbers are limited to 0 through 5. ii) Fraction denominators are limited to 2, 3, 4, 6 and 8.	MP.2, MP.4, MP.5
A	3.MD.1-1	Tell and write time to the nearest minute and measure time intervals in minutes.	i) Time intervals are limited to 60 minutes. ii) No more than 20% of items require determining a time interval from clock readings having different hour values. iii) Acceptable interval: Start time 1:20, end time 2:10 – time interval is 50 minutes. Unacceptable interval: Start time 1:20, end time 2:30 – time interval exceeds 60 minutes.	-
A	3.MD.1-2	Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	i) Only the answer is required. ii) Tasks do not involve reading start/stop times from a clock nor calculating elapsed time.	MP.1, MP 2, MP.4, MP.5
A	3.MD.2-1	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).	i) Estimates are the result of reading a scale.	-
A	3.MD.2-2	Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.	i) Only the answer is required (methods, representations, etc. are not assessed here). ii) Units of grams (g), kilograms (kg), and liters (l).	MP.1, MP.2, MP.4, MP.5
A	3.MD.2-3	Measure or estimate liquid volumes or masses of objects using standard units of grams (g), kilograms (kg), and liters (l), then use the estimated value(s) to estimate the answer to a one-step word problem by using addition, subtraction, multiplication, or division. Content Scope: 3.MD.2	-	MP.5, MP.6 (in the case of measuring)
B	3.MD.3-1	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	i) Tasks involve no more than 10 items in 2-5 categories. ii) Categorical data should not take the form of a category that could be represented numerically (e.g. ages of students). iii) Tasks do not require students to create the entire graph, but might ask students to complete a graph or otherwise demonstrate knowledge of its creation.	MP.2

Grade 3 Evidence Statements

Type I **Type II** **Type III**

Sub-Claim	Evidence Statement Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
B	3.MD.3-3	Solve a put-together problem using information presented in a scaled bar graph, then use the result to answer a "how many more" or "how many less" problem using information presented in the scaled bar graph. Content Scope: 3.MD.3	i) Tasks do not require computations beyond the Grade 3 expectations.	MP.4
B	3.MD.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	-	MP.2, MP.5
A	3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.	-	MP.7
A	3.MD.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	-	MP.7
A	3.MD.7b-1	Relate area to the operations of multiplication and addition. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems.	i) Products are limited to the 10x10 multiplication table. ii) This ES is different from 3.OA.3-1 in the following ways: • 3.MD.7b-1 emphasizes application/skill while the emphasis of 3.OA.3-1 is on demonstration of understanding of multiplication using not only area but also equal groups and arrays by modeling. • 3.MD.7b-1 permits mathematical problems while 3.OA.3-1 is restricted to word problems. • 3.MD.7b-1 allows for factors less than or equal to 5 while the factors used in 3.OA.3-1 are restricted to the harder three quadrants.	MP.4, MP.5

Grade 3 Evidence Statements

Type I **Type II** **Type III**

Sub-Claim	Evidence Statement Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
A	3.MD.7d	Relate area to the operations of multiplication and addition. d. Recognize area as additive. Find areas of rectilinear[3] figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	-	MP.7
B	3.MD.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	-	MP.2, MP.4, MP.5
B	3.G.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	-	-
B	3.G.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	-	-
A	3.Int.1	Given a two-step problem situation with the four operations, round the values in the problem, then use the rounded values to produce an approximate solution. Content Scope: 3.OA.8, 3.NBT.1, 3.NBT.2, 3.NBT.3	i) Tasks must be aligned to the first standard and 1 or more of the subsequent standards listed in the content scope. ii) Tasks do not require computations beyond the Grade 3 expectations. iii) Tasks do not require a student to write a single equation with a letter standing for the unknown quantity in a two-step problem, and then solve that equation. iv) Tasks may require students to write an equation as part of their work to find a solution, but students are not required to use a letter for the unknown. v) Addition, subtraction, multiplication and division situations in these problems may involve any of the basic situation types with unknowns in various positions (see Table 1 and Table 2 and Table 3 below).	MP.4, MP.6

Grade 3 Evidence Statements

Type I Type II Type III

Sub-Claim	Evidence Statement Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
A	3.Int.2	Solve two-step word problems using the four operations requiring a substantial addition, subtraction, or multiplication step, drawing on knowledge and skills articulated in 3.NBT. Content Scope: 3.OA.8, 3.NBT.2, and 3.NBT.3	i) Tasks must be aligned to the first standard and 1 or more of the subsequent standards listed in the content scope. ii) Tasks do not require a student to write a single equation with a letter standing for the unknown quantity in a two-step problem, and then solve that equation. iii) Tasks may require students to write an equation as part of their work to find a solution, but students are not required to use a letter for the unknown. iv) Addition, subtraction, multiplication and division situations in these problems may involve any of the basic situation types with unknowns in various positions (see Table 1 and Table 2 and Table 3 below). Substantial (def.) – Values should be towards the higher end of the numbers identified in the standards.	MP.1, MP.4
B	3.Int.3	Solve real world and mathematical problems involving perimeters of polygons requiring a substantial addition, subtraction, or multiplication step, drawing on knowledge and skills articulated in 3.NBT. Content Scope: 3.MD.8, 3.NBT.2, and 3.NBT.3	i) Tasks must be aligned to the first standard and 1 or more of the subsequent standards listed in the content scope. Substantial (def.) – Values should be towards the higher end of the numbers identified in the standards.	MP.1 (if the problem has a real world context), MP.4
B	3.Int.4	Use information presented in a scaled bar graph to solve a two-step "how many more" or "how many less" problem requiring a substantial addition, subtraction, or multiplication step, drawing on knowledge and skills articulated in 3.NBT. Content Scope: 3.MD.3, 3.NBT.2, and 3.NBT.3	i) Tasks must be aligned to the first standard and 1 or more of the subsequent standards listed in the content scope. Substantial (def.) – Values should be towards the higher end of the numbers identified in the standards.	MP.1, MP.2, MP.4
A	3.Int.5	Add, subtract, or multiply to solve a one-step word problem involving masses or volumes that are given in the same units, where a substantial addition, subtraction, or multiplication step is required drawing on knowledge and skills articulated in 3.NBT, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Content Scope: 3.MD.2, 3.NBT.2, and 3.NBT.3	i) Tasks must be aligned to the first standard and 1 or more of the subsequent standards listed in the content scope. Substantial (def.) – Values should be towards the higher end of the numbers identified in the standards.	MP.1, MP.2, MP.4

Sub-Claim	Evidence Statement Key	Evidence Statement (ES) Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
Sub-claim C (10 of 52 points)				
C	3.C.1-1	Base explanations/reasoning on the properties of operations. Content Scope: Knowledge and skills articulated in 3.OA.5	i) Students need not use technical terms such as *commutative*, *associative*, *distributive*, or *property*. ii) Products and related quotients are limited to the 10x10 multiplication table. iii) These tasks may not exceed the content limits of Grade 3. For example, 2 x 4 x 5, would be acceptable as students can use the associative property to rewrite the expression as 8 x 5 which falls within the content limits of Grade 3. The problem 7 x 4 x 5 would exceed the content limits of Grade 3 because any use of the associative property would result in a 2-digit multiplier.	MP.3, MP.6, MP.7
C	3.C.1-2	Base explanations/reasoning on the properties of operations. Content Scope: Knowledge and skills articulated in 3.OA.9	i) Students need not use technical terms such as *commutative*, *associative*, *distributive*, or *property*.	MP.3, MP.6, MP.7, MP.8
C	3.C.1-3	Base explanations/reasoning on the properties of operations. Content Scope: Knowledge and skills articulated in 3.MD.7	i) Tasks may include those with and without real-world contexts. ii) Students need not use technical terms such as *commutative*, *associative*, *distributive*, or *property*.	MP.3, MP.5, MP.6, MP.7
C	3.C.2	Base explanations/reasoning on the relationship between multiplication and division. Content Scope: Knowledge and skills articulated in 3.OA.6	i) Products and related quotients are limited to the 10 x 10 multiplication table.	MP.3, MP.6, MP.7
C	3.C.3-1	Base arithmetic explanations/reasoning on concrete referents such as diagrams (whether provided in the prompt or constructed by the student in her response), connecting the diagrams to a written (symbolic) method. Content Scope: Knowledge and skills articulated in 3.NF.3b, 3.NF.3d	i) Tasks may present realistic or quasi-realistic images of a contextual situation (e.g., a drawing of a partially filled graduated cylinder). However, tasks do not provide the sort of abstract drawings that help the student to represent the situation mathematically (e.g., a number line diagram or other visual fraction model). ii) Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8. iii) For fractions equal to a whole number, values are limited to 0 through 5.	MP.3 MP.5 MP.6

Sub-Claim	Evidence Statement Key	Evidence Statement (ES) Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
C	3.C.3-2	Base explanations/reasoning on concrete referents such as diagrams (whether provided in the prompt or constructed by the student in her response). Content Scope: Knowledge and skills articulated in 3.MD.5, 3.MD.6, 3.MD.7	i) Tasks may include those with and without real-world contexts. ii) Tasks with a context may present realistic or quasi-realistic images of a contextual situation (e.g., a drawing of a meadow). However, tasks do not provide the sort of abstract drawings that help the student to represent the situation mathematically (e.g., a tiling of the meadow).	MP.3, MP.5, MP.6
C	3.C.4-1	Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed 'student' reasoning is presented and the task is to correct and improve it.) Content Scope: Knowledge and skills articulated in 3.OA.5	i) Students need not use technical terms such as *commutative*, *associative*, *distributive*, or *property*. ii) Products and related quotients are limited to the 10x10 multiplication table.	MP.3, MP.6, MP.7
C	3.C.4-2	Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed 'student' reasoning is presented and the task is to correct and improve it.) Content Scope: Knowledge and skills articulated in 3.OA.6	i) Products and related quotients are limited to the 10x10 multiplication table.	MP.3, MP.6
C	3.C.4-3	Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed 'student' reasoning is presented and the task is to correct and improve it.) Content Scope: Knowledge and skills articulated in 3.OA.8	i) Tasks do not require a student to write a single equation with a letter standing for the unknown quantity in a two-step problem, and then solve that equation. ii) Tasks may require students to write an equation as part of their work to find a solution, but students are not required to use a letter for the unknown. iii) Addition, subtraction, multiplication and division situations in these problems may involve any of the basic situation types with unknowns in various positions (see Table 1 and Table 2 and Table 3 below).	MP.3, MP.5, MP.6
C	3.C.4-4	Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed 'student' reasoning is presented and the task is to correct and improve it.) Content Scope: Knowledge and skills articulated in 3.NF.3b, 3.NF.3d	i) Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8. ii) For fractions equal to a whole number, values are limited to 0 through 5.	MP.3, MP.5, MP.6
C	3.C.4-5	Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed 'student' reasoning is presented and the task is to correct and improve it.) Content Scope: Knowledge and skills articulated in 3.MD.7	i) Tasks may include those with and without real-world contexts.	MP.3, MP.5, MP.6

Sub-Claim	Evidence Statement Key	Evidence Statement (ES) Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
C	3.C.4-6	Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed 'student' reasoning is presented and the task is to correct and improve it.) Content Scope: Knowledge and skills articulated in 3.OA.9	-	MP.3, MP.6, MP.8
C	3.C.4-7	Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed 'student' reasoning is presented and the task is to correct and improve it.) Content Scope: Knowledge and skills articulated in 2.NBT	i) Tasks may have scaffolding[1], if necessary, in order to yield a degree of difficulty appropriate to Grade 3.	MP.3, MP.6
C	3.C.5-1	Present solutions to two-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as 1 + 4 = 5 + 7 = 12, even if the final answer is correct), or identify or describe errors in solutions to two-step problems and present corrected solutions. Content Scope: Knowledge and skills articulated in 3.OA.8	i) Tasks do not require a student to write a single equation with a letter standing for the unknown quantity in a two-step problem, and then solve that equation. ii) Tasks may require students to write an equation as part of their work to find a solution, but students are not required to use a letter for the unknown. iii) Addition, subtraction, multiplication and division situations in these problems may involve any of the basic situation types with unknowns in various positions (see Table 1 and Table 2 and Table 3 below).	MP.2, MP.3, MP.5, MP.6
C	3.C.5-2	Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as 1 + 4 = 5 + 7 = 12, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions. Content Scope: Knowledge and skills articulated in 3.MD.7b, 3.MD.7d	i) Tasks may include those with and without real-world contexts. ii) Multi-step problems have at least 3 steps.	MP.2, MP.3, MP.5, MP.6
C	3.C.6-1	Base explanations/reasoning on a number line diagram (whether provided in the prompt or constructed by the student in her response). Content scope: Knowledge and skills articulated in 3.NF.2	i) Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8. ii) Fractions equivalent to whole numbers are limited to 0 through 5.	MP.3, MP.5, MP.6

Sub-Claim	Evidence Statement Key	Evidence Statement (ES) Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
C	3.C.6-2	Base explanations/reasoning on a number line diagram (whether provided in the prompt or constructed by the student in her response). Content scope: Knowledge and skills articulated in 3.MD.1	-	MP.3, MP.5, MP.6
Sub-claim D (12 of 52 points)				
D	3.D.1	Solve multi-step contextual word problems with degree of difficulty appropriate to Grade 3, requiring application of knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements.	i) Tasks may have scaffolding[1]. ii) Multi-step problems must have at least 3 steps.	MP.4
D	3.D.2	Solve multi-step contextual problems with degree of difficulty appropriate to Grade 3, requiring application of knowledge and skills articulated in 2.OA.A, 2.OA.B, 2.NBT, and/or 2.MD.B.	i) Tasks may have scaffolding[1], if necessary, in order to yield a degree of difficulty appropriate to Grade 3. ii) Multi-step problems must have at least 3 steps.	MP.4

[1] Scaffolding in a task provides the student with an entry point into a pathway for solving a problem. In unscaffolded tasks, the student determines his/her own pathway and process. Both scaffolded and unscaffolded tasks will be included in reasoning and modeling items.

[2] "Thin context" is a sentence or phrase that establishes a concrete referent for the quantity/quantities in the problem, in such a way as to provide meaningful avenues for mathematical intuition to operate, yet without requiring any sort of further analysis of the context. For example, a task could provide a reason for being given a set of fractional measurements such as, "The fractions represent lengths of ribbon."

[3] A rectilinear figure is a polygon in which all angles measure 90 or 270 degrees.

Table I

TABLE 1. Common addition and subtraction situations.[6]

from Math Standards.indb (b-cdn.net) P. 88

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2

TABLE 2. Common multiplication and division situations.[7]

from Math Standards.indb (b-cdn.net) P. 89

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3

TABLE 3. Addition and subtraction situations by grade level

from Progressions.pdf (mathematicalmusings.org) P. 18

	Result Unknown	Change Unknown	Start Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A + \square = C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	C apples were on the table. I ate B apples. How many apples are on the table now? $C - B = \square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square - B = A$

	Total Unknown	Both Addends Unknown[1]	Addend Unknown[2]
Put Together /Take Apart	A red apples and B green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	C apples are on the table. A are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	*"How many more?" version.* Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy?	*"More" version suggests operation.* Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have?	*"Fewer" version suggests operation.* Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have?
	"How many fewer?" version. Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*"Fewer" version suggests wrong operation.* Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have? $A + B = \square$	*"More" version suggests wrong operation.* Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

Adapted from the *Common Core State Standards for Mathematics*, p. 88, which is based on *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33. (To improve readability, the order of Both Addends Unknown and Addend Unknown reverses the order shown in the *Standards*.)

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems illustrate the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2. Other descriptions of the situations may use somewhat different names (see Appendix 2).

[1] This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the left help children understand that = does not always mean "makes" or "results in" but always means "is the same number as." Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all of the decompositions of a number and reflecting on the patterns involved.

[2] Either addend can be unknown; both variations should be included.