Passport to Advanced Math

Calculator

1. The function f is defined by $f(x) = 2x^3 + 3x^2 + cx + 8$, where c is a constant. In the xy-plane, the graph of f intersects the x-axis at the three points $(-4, 0)$, $\left(\frac{1}{2}, 0\right)$, and $(p, 0)$. What is the value of c?

A) –18

B) –2

C) 2

D) 10

Students could tackle this problem in many different ways, but the focus is on their understanding of the zeros of a polynomial function and how they are used to construct algebraic representations of polynomials.

Choice A is correct. The given zeros can be used to set up an equation to solve for c. Substituting –4 for x and 0 for y yields $-4c = 72$, or $c = -18$.

Alternatively, since -4, $\frac{1}{2}$, and p are zeros of the polynomial function $f(x) = 2x^3 + 3x^2 + cx + 8$, it follows that $f(x) = (2x - 1)(x + 4)(x - p)$.

Were this polynomial multiplied out, the constant term would be $(-1)(4)(-p) = 4p$. (We can see this without performing the full expansion.) Since it is given that this value is 8, it goes that $4p = 8$ or rather, $p = 2$. Substituting 2 for p in the polynomial function yields $f(x) = (2x - 1)(x + 4)(x - 2)$, and after multiplying the factors one finds that the coefficient of the x term, or the value of c, is –18.

Choice B is not the correct answer. This value is a misunderstood version of the value of p, not c, and the relationship between the zero and the factor (if a is the zero of a polynomial, its corresponding factor is $x - a$) has been confused.

Choice C is not the correct answer. This is the value of p, not c. Using this value as the third factor of the polynomial will reveal that the value of c is –18.

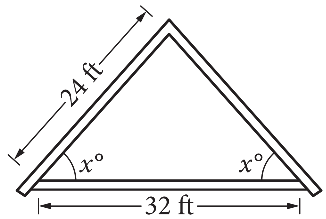
Choice D is not the correct answer. This represents a sign error in the final step in determining the value of c.

Additional Topics in Math

Calculator

Student-Produced Response Problem

2.



Note: Figure not drawn to scale.

What is the value of cos x?

This problem requires students to make use of properties of triangles to solve a problem.

Because the triangle is isosceles, constructing a perpendicular from the top vertex to the opposite side will bisect the base and create two smaller right triangles. In a right triangle, the cosine of an acute angle is equal to the length of the side adjacent to the angle divided by the length of the hypotenuse. This gives $\cos x = \frac{16}{24}$, which can be simplified to $\cos x = \frac{2}{3}$.

Passport to Advanced Math

No calculator

3. What is one possible solution to the equation $\frac{24}{x+1} - \frac{12}{x-1} = 1$?

Students should look for the best solution methods for solving rational equations before they begin. Looking for structure and common denominators will prove very useful at the onset and will help prevent complex computations that do not lead to a solution.

In this problem, multiplying both sides of the equation by the common denominator $(x + 1)(x - 1)$ yields $24(x - 1) - 12(x + 1) = (x + 1)(x - 1)$. Multiplication and simplification then yields

$12x - 36 = x^2 - 1$, or

$x^2 - 12x + 35 = 0$.

Factoring the quadratic gives $(x - 5)(x - 7) = 0$, so the solutions occur at $x = 5$ and $x = 7$, both of which should be checked in the original equation to ensure that they are not extraneous. In this case, both values are solutions.

Additional Topics in Math

No Calculator

4. Which of the following is equal to $\sin\left(\frac{\pi}{5}\right)$?

A) $-\cos\left(\frac{\pi}{5}\right)$

B) $-\sin\left(\frac{\pi}{5}\right)$

C) $\cos\left(\frac{3\pi}{10}\right)$

D) $\sin\left(\frac{7\pi}{10}\right)$

This question is solved most efficiently when a student is fluent with radian measure and has a conceptual understanding of the relationship between the sine and cosine functions.

Choice C is correct. Sine and cosine are related by the equation:

$\sin(x) = \cos\left(\frac{\pi}{2} - x\right)$.

Therefore, $\sin\left(\frac{\pi}{5}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$, which reduces to $\cos\left(\frac{\pi}{10}\right)$

Choice A is not the correct answer. This answer may result from a misunderstanding about trigonometric relationships. A student may think that cosine is the opposite function of sine, and therefore think that the negative of the cosine of an angle is equivalent to the sine of that angle.

Choice B is not the correct answer. This answer may result from a misunderstanding of the unit circle and how it relates to trigonometric expressions. A student may think that, on a coordinate grid, the negative sign only changes the orientation of the triangle formed, not the value of the trigonometric expression.

Choice D is not the correct answer. The student mistakenly remembers the relationship between sine and cosine and adds $\frac{\pi}{2}$ to the angle measure instead of subtracting the angle measure from $\frac{\pi}{2}$.