Heart of Algebra
Calculator

1. When a scientist dives in salt water to a depth of 9 feet below the surface, the pressure due to the atmosphere and surrounding water is 18.7 pounds per square inch. As the scientist descends, the pressure increases linearly. At a depth of 14 feet, the pressure is 20.9 pounds per square inch. If the pressure increases at a constant rate as the scientist's depth below the surface increases, which of the following linear models best describes the pressure p in pounds per square inch at a depth of d feet below the surface?

   A) $p = 0.44d + 0.77$

   B) $p = 0.44d + 14.74$

   C) $p = 2.2d - 1.1$

   D) $p = 2.2d - 9.9$

Problem Solving and Data Analysis
Calculator

2. A typical image taken of the surface of Mars by a camera is 11.2 gigabits in size. A tracking station on Earth can receive data from the spacecraft at a data rate of 3 megabits per second for a maximum of 11 hours each day. If 1 gigabit equals 1,024 megabits, what is the maximum number of typical images that the tracking station could receive from the camera each day?

   A) 3

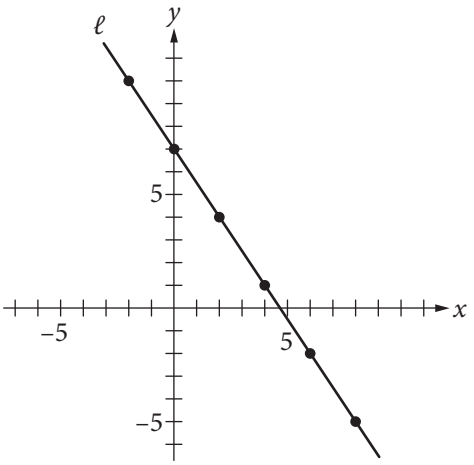
   B) 10

   C) 56

   D) 144

Heart of Algebra
No-Calculator

Line l is graphed in the xy-plane below.



3. If line l is translated up 5 units and right 7 units, then what is the slope of the new line?

A) $-\frac{2}{5}$

B) $-\frac{3}{2}$

C) $-\frac{8}{9}$

D) $-\frac{11}{14}$

Questions 4 and 5 refer to the following information.

A survey was conducted among a randomly chosen sample of U.S. citizens about U.S. voter participation in the November 2012 presidential election. The table below displays a summary of the survey results.

Reported Voting by Age (in thousands)

	VOTED	DID NOT VOTE	NO RESPONSE	TOTAL
18- to 34-year-olds	30,329	23,211	9,468	63,008
35- to 54-year-olds	47,085	17,721	9,476	74,282
55- to 74-year-olds	43,075	10,092	6,831	59,998
People 75 years old and over	12,459	3,508	1,827	17,794
Total	132,948	54,532	27,602	215,082

Problem Solving and Data Analysis
Calculator

4. Of the 18- to 34-year-olds who reported voting, 500 people were selected at random to do a follow-up survey where they were asked which candidate they voted for. There were 287 people in this follow-up survey sample who said they voted for Candidate A, and the other 213 people voted for someone else. Using the data from both the follow-up survey and the initial survey, which of the following is most likely to be an accurate statement?

A) About 123 million people 18 to 34 years old would report voting for Candidate A in the November 2012 presidential election.

B) About 76 million people 18 to 34 years old would report voting for Candidate A in the November 2012 presidential election.

C) About 36 million people 18 to 34 years old would report voting for Candidate A in the November 2012 presidential election.

D) About 17 million people 18 to 34 years old would report voting for Candidate A in the November 2012 presidential election.

Problem Solving and Data Analysis
Calculator

5. According to the table, for which age group did the greatest percentage of people report that they had voted?

A) 18- to 34-year-olds

B) 35- to 54-year-olds

C) 55- to 74-year-olds

D) People 75 years old and over

Heart of Algebra
Calculator

6. The toll rates for crossing a bridge are $6.50 for a car and $10 for a truck. During a two-hour period, a total of 187 cars and trucks crossed the bridge, and the total collected in tolls was $1,338. Solving which of the following systems of equations yields the number of cars, x, and the number of trucks, y, that crossed the bridge during the two hours?

A) $\begin{aligned} x + y &= 1{,}338 \\ 6.5x + 10y &= 187 \end{aligned}$

B) $\begin{aligned} x + y &= 187 \\ 6.5x + 10y &= \frac{1{,}338}{2} \end{aligned}$

C) $\begin{aligned} x + y &= 187 \\ 6.5x + 10y &= 1{,}338 \end{aligned}$

D) $\begin{aligned} x + y &= 187 \\ 6.5 + 10y &= 1{,}338x \end{aligned}$

CONTENT: Heart of Algebra	**CALCULATOR USAGE:** Calculator
KEY: B	

1. When a scientist dives in salt water to a depth of 9 feet below the surface, the pressure due to the atmosphere and surrounding water is 18.7 pounds per square inch. As the scientist descends, the pressure increases linearly. At a depth of 14 feet, the pressure is 20.9 pounds per square inch. If the pressure increases at a constant rate as the scientist's depth below the surface increases, which of the following linear models best describes the pressure p in pounds per square inch at a depth of d feet below the surface?

A) $p = 0.44d + 0.77$

B) $p = 0.44d + 14.74$

C) $p = 2.2d - 1.1$

D) $p = 2.2d - 9.9$

In approaching this problem, students must determine the relationship between the two variables described within the text: the depth and the pressure.

Choice B is correct. To determine the linear model, one can first determine the rate at which the pressure due to the atmosphere and surrounding water is increasing as the depth of the diver increases.

Calculating this gives $\frac{(20.9 - 18.7)}{(14 - 9)} = \frac{2.2}{5}$ or 0.44. Then one needs to determine the pressure due to the atmosphere or, in other words, the pressure when the diver is at a depth of 0. Solving the equation $18.7 = 0.44(9) + b$ gives $b = 14.74$.
Therefore, the model that can be used to relate the pressure and the depth is $p = 0.44d + 14.74$.

Choice A is not the correct answer. The rate is calculated correctly, but the student may have incorrectly used the ordered pair (18.7, 9) rather than (9, 18.7) to calculate the pressure at a depth of 0 feet.

Choice C is not the correct answer. The rate here is incorrectly calculated by subtracting 20.9 and 18.7 and *not dividing* by 5. The student then uses the coordinate pair $d = 9$ and $p = 18.7$ in conjunction with the incorrect slope of 2.2 to write the equation of the linear model.

Choice D is not the correct answer. The rate here is incorrectly calculated by subtracting 20.9 and 18.7 and not dividing by 5. The student then uses the coordinate pair $d = 14$ and $p = 20.9$ in conjunction with the incorrect slope of 2.2 to write the equation of the linear model.

CONTENT: Problem Solving and Data Analysis	**CALCULATOR USAGE:** Calculator
KEY: B	

2. A typical image taken of the surface of Mars by a camera is 11.2 gigabits in size. A tracking station on Earth can receive data from the spacecraft at a data rate of 3 megabits per second for a maximum of 11 hours each day. If 1 gigabit equals 1,024 megabits, what is the maximum number of typical images that the tracking station could receive from the camera each day?

A) 3

B) 10

C) 56

D) 144

In this problem, students must use the unit rate (data-transmission rate) and the conversion between gigabits and megabits as well as conversions in units of time. Unit analysis is critical to solving the problem correctly, and the problem represents a typical calculation that would be done when working with electronic files and data-transmission rates. A calculator is recommended in solving this problem.

Choice B is correct. The tracking station can receive 118,800 megabits each day $\left(\frac{3\text{ megabits}}{1\text{ second}} \times \frac{60\text{ seconds}}{1\text{ minute}} \times \frac{60\text{ minutes}}{1\text{ hour}} \times 11\text{ hours}\right)$, which is about 116 gigabits each day $\frac{118{,}800}{1{,}024}$

If each image is 11.2 gigabits, then the number of images that can be received each day is $\frac{116}{11.2} = 10.4$.

Since the question asks for the maximum number of typical images, rounding the answer down to 10 is appropriate because the tracking station will not receive a complete 11th image in one day.

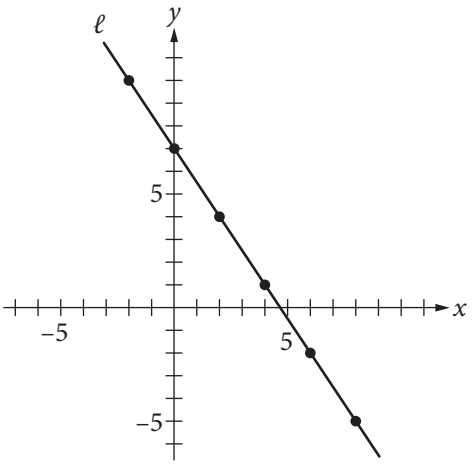
Choice A is not the correct answer. The student may not have synthesized all of the information. This answer may result from multiplying 3 (rate in megabits per second) by 11 (hours receiving) and dividing by 11.2 (size of image in gigabits), neglecting to convert 3 megabits per second into megabits per hour and to utilize the information about 1 gigabit equaling 1,024 megabits.

Choice C is not the correct answer. The student may not have synthesized all of the information. This answer may result from converting the number of gigabits in an image to megabits (11,470), multiplying by the rate of 3 megabits per second (34,410) and then converting 11 hours into minutes (660) instead of seconds.

Choice D is not the correct answer. The student may not have synthesized all of the information. This answer may result from converting 11 hours into seconds (39,600), then dividing the result by 3 gigabits converted into megabits (3,072), and multiplying by the size of one typical image.

CONTENT: Heart of Algebra	**CALCULATOR USAGE:** No-Calculator
KEY: B	

Line l is graphed in the xy-plane below.



3. If line l is translated up 5 units and right 7 units, then what is the slope of the new line?

A) $-\frac{2}{5}$

B) $-\frac{3}{2}$

C) $-\frac{8}{9}$

D) $-\frac{11}{14}$

Students can approach this problem conceptually or concretely. The core skill being assessed here is the ability to make a connection between the graphical form of a relationship and a numerical description of a key feature.

Choice B is correct. The slope of the line is read from the graph as "down 3, over 2." Translating the line moves all the points on the line by the same amount. Therefore, the slope does not change and the answer is $\left(-\frac{3}{2}\right)$.

Choice A is not the correct answer. This value may result from a combination of errors. The student may misunderstand how the negative sign affects the fraction and apply the transformation as $\frac{(-3+5)}{(-2+7)}$.

Choice C is not the correct answer. This value may result from finding the slope of the line and then subtracting 5 from the numerator and 7 from the denominator.

Choice D is not the correct answer. This answer may result from adding $\frac{5}{7}$ to the slope of the line.

Questions 4 and 5 refer to the following information.

A survey was conducted among a randomly chosen sample of U.S. citizens about U.S. voter participation in the November 2012 presidential election. The table below displays a summary of the survey results.

Reported Voting by Age (in thousands)

	VOTED	DID NOT VOTE	NO RESPONSE	TOTAL
18- to 34-year-olds	30,329	23,211	9,468	63,008
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55- to 74-year-olds	43,075	10,092	6,831	59,998
People 75 years old and over	12,459	3,508	1,827	17,794
Total	132,948	54,532	27,602	215,082

CONTENT: Problem Solving and Data Analysis	**CALCULATOR USAGE:** Calculator
KEY: D	

4. Of the 18- to 34-year-olds who reported voting, 500 people were selected at random to do a follow-up survey where they were asked which candidate they voted for. There were 287 people in this follow-up survey sample who said they voted for Candidate A, and the other 213 people voted for someone else. Using the data from both the follow-up survey and the initial survey, which of the following is most likely to be an accurate statement?

A) About 123 million people 18 to 34 years old would report voting for Candidate A in the November 2012 presidential election.

B) About 76 million people 18 to 34 years old would report voting for Candidate A in the November 2012 presidential election.

C) About 36 million people 18 to 34 years old would report voting for Candidate A in the November 2012 presidential election.

D) About 17 million people 18 to 34 years old would report voting for Candidate A in the November 2012 presidential election.

The second question asks students to extrapolate from a random sample to estimate the number of 18- to 34-year-olds who voted for Candidate A: this is done by multiplying the fraction of people in the random sample who voted for Candidate A by the total population of voting 18- to 34-year-olds: $\frac{287}{500} \times 30{,}329{,}000 =$ approximately 17 million, choice D. Students without a clear grasp of the context and its representation in the table might easily arrive at one of the other answers listed.

Choice A is not the correct answer. The student may not have multiplied the fraction of the sample by the correct subgroup of people (18- to 34-year-olds who voted). This answer may result from multiplying the fraction by the entire population, which is an incorrect application of the information.

Choice B is not the correct answer. The student may not have multiplied the fraction of the sample by the correct subgroup of people (18- to 34-year-olds who voted). This answer may result from multiplying the fraction by the total number of people who voted, which is an incorrect application of the information.

Choice C is not the correct answer. The student may not have multiplied the fraction of the sample by the correct subgroup of people (18- to 34-year-olds who voted). This answer may result from multiplying the fraction by the total number of 18- to 34-year-olds, which is an incorrect application of the information.

CONTENT: Problem Solving and Data Analysis	**CALCULATOR USAGE:** Calculator
KEY: C	

5. According to the table, for which age group did the greatest percentage of people report that they had voted?

A) 18- to 34-year-olds

B) 35- to 54-year-olds

C) 55- to 74-year-olds

D) People 75 years old and over

To succeed on these questions, students must conceptualize the context and retrieve relevant information from the table, next manipulating it to form or compare relevant quantities. The first question asks students to select the relevant information from the table to compute the percentage of self-reported voters for each age group and then compare the percentages to identify the largest one, choice C. Of the 55- to 74-year-old group's total population (59,998,000), 43,075,000 reported that they had voted, which represents 71.8% and is the highest percentage of reported voters from among the four age groups.

Choice A is not the correct answer. The question is asking for the age group with the largest percentage of self-reported voters. This answer reflects the age group with the smallest percentage of self-reported voters. This group's percentage of self-reported voters is 48.1%, or 30, 329, which is less than that of the 55- to 74-year-old group.

Choice B is not the correct answer. The question is asking for the age group with the largest percentage of self-reported voters. This answer reflects the age group with the largest number of self-reported voters, not the largest percentage. This group's percentage of self-reported voters is 63.4%, or 47, 085, which is less than that of the 55- to 74-year-old group.

Choice D is not the correct answer. The question is asking for the age group with the largest percentage of self-reported voters. This answer reflects the age group with the smallest number of self-reported voters, not the largest percentage. This group's percentage of self-reported voters is 70.0%, or 12, 459, which is less than that of the 55- to 74-year-old group.

CONTENT: Heart of Algebra	**CALCULATOR USAGE:** Calculator
KEY: C	

6. The toll rates for crossing a bridge are $6.50 for a car and $10 for a truck. During a two-hour period, a total of 187 cars and trucks crossed the bridge, and the total collected in tolls was $1,338. Solving which of the following systems of equations yields the number of cars, x, and the number of trucks, y, that crossed the bridge during the two hours?

A) $x + y = 1{,}338$
$6.5x + 10y = 187$

B) $x + y = 187$
$6.5x + 10y = \frac{1{,}338}{2}$

C) $x + y = 187$
$6.5x + 10y = 1{,}338$

D) $x + y = 187$
$6.5x + 10y = 1{,}338 \times 2$

This question assesses student's ability to create a system of linear equations that represents a real-world situation. Students will have to make sense of the situation presented, choose and define two variables to use, and set up the equations based on the relationships from the information given.

Choice C is correct. If x is the number of cars that crossed the bridge during the two hours and y is the number of trucks that crossed the bridge during the two hours, then $x + y$ represents the total number of cars and trucks that crossed the bridge during the two hours and $6.5x + 10y$ represents the total amount collected in the two hours. Therefore, the correct system of equations is $x + y = 187$ and $6.5x + 10y = 1{,}338$.

Choice A is not the correct answer. The student may have mismatched the symbolic expressions for total cars and trucks and total tolls collected with the two numerical values given. The expression $x + y$ represents the total number of cars and trucks that crossed the bridge, which is 187.

Choice B is not the correct answer. The student may have attempted to use the information that the counts of cars, trucks, and tolls were taken over a period of two hours, but this information is not needed in setting up the correct system of equations. The expression $6.5x + 10y$ represents the total amount of tolls collected, which is $1,338, not $\$\frac{1{,}338}{2}$.

Choice D is not the correct answer. The student may have attempted to use the information that the counts of cars, trucks, and tolls were taken over a period of two hours, but this information is not needed in setting up the correct system of equations. The expression $6.5x + 10y$ represents the total amount of tolls collected, which is $1,338, not $1,338 × 2.

- Provide students with explanations and/or equations that incorrectly describe a graph. Ask students to identify the errors and provide corrections, citing the reasoning behind the change.
- Students can organize information to present data and answer a question or show a problem solution in multiple ways. Ask students to create pictures, tables, graphs, lists, models, and/or verbal expressions to interpret text and/or data to help them arrive at a solution.
- Ask students to solve problems that require multiple steps to arrive at the solution.
- As students work in small groups to solve problems, facilitate discussions in which they communicate their own thinking and critique the reasoning of others as they work toward a solution. Ask open-ended questions. Direct their attention to real-world situations to provide context for the problem.
- Help students strengthen their skills in problem solving and data analysis by reading and understanding graphs in many contexts. Ask them to find a chart/graph/table from a periodical and write a series of questions about the graphic to be discussed in class. Challenge them to dig deep into the data and the purpose of the graphic, then ask meaningful questions about it. Ask them to present purposefully incorrect interpretations and ask the class to correct their analyses.
- The SAT Math Test emphasizes students' ability to apply math to solve problems in rich and varied contexts, and features items that require problem solving and data analysis to solve problems in science, social studies, and career-related contexts. Students must see how the math problems they solve are generated from questions in science, social studies, economics, psychology, health, and other career content areas. Give them many opportunities to practice in all of their classes.
- Use "Guess and Check" to explore different ways to solve a problem when other strategies for solving are not obvious. Students first guess the solution to a problem, then check that the guess fits the information in the problem and is an accurate solution. They can then work backward to identify proper steps to arrive at the solution.
- Assign math problems for students to solve without the use of a calculator. Assign problems for which the calculator is actually a deterrent to expedience and give students the choice whether to utilize the calculator. Discuss how to solve both ways, and which method is more advantageous.