HEART OF ALGEBRA: LINEAR EQUATIONS AND FUNCTIONS

SAT HEART OF ALGEBRA DOMAIN	
Content Dimension	**Description**
Linear equations in one variable	1. Create and use linear equations in one variable to solve problems in a variety of contexts. 2. Create a linear equation in one variable, and when in context interpret solutions in terms of the context. 3. Solve a linear equation in one variable, making strategic use of algebraic structure. 4. For a linear equation in one variable, a. interpret a constant, variable, factor, or term in a context; b. determine the conditions under which the equation has no solution, a unique solution, or infinitely many solutions. 5. Fluently solve a linear equation in one variable.
Linear functions	Algebraically, a linear function can be defined by a linear expression in one variable or by a linear equation in two variables. In the first case, the variable is the input and the value of the expression is the output. In the second case, one of the variables is designated as the input and determines a unique value of the other variable, which is the output. 1. Create and use linear functions to solve problems in a variety of contexts. 2. Create a linear function to model a relationship between two quantities. 3. For a linear function that represents a context, a. interpret the meaning of an input/output pair, constant, variable, factor, or term based on the context, including situations where seeing structure provides an advantage; b. given an input value, find and/or interpret the output value using the given representation; c. given an output value, find and/or interpret the input value using the given representation, if it exists. 4. Make connections between verbal, tabular, algebraic, and graphical representations of a linear function by a. deriving one representation from the other; b. identifying features of one representation given another representation; c. determining how a graph is affected by a change to its equation. 5. Write the rule for a linear function given two input/output pairs or one input/output pair and the rate of change.
Linear equations in two variables	A linear equation in two variables can be used to represent a constraint or condition on two-variable quantities in situations where neither of the variables is regarded as an input or an output. A linear equation can also be used to represent a straight line in the coordinate plane. 1. Create and use a linear equation in two variables to solve problems in a variety of contexts. 2. Create a linear equation in two variables to model a constraint or condition on two quantities. 3. For a linear equation in two variables that represents a context, a. interpret a solution, constant, variable, factor, or term based on the context, including situations where seeing structure provides an advantage; b. given a value of one quantity in the relationship, find a value of the other, if it exists. 4. Make connections between tabular, algebraic, and graphical representations of a linear equation in two variables by a. deriving one representation from the other; b. identifying features of one representation given the other representation; c. determining how a graph is affected by a change to its equation. 5. Write an equation for a line given two points on the line, one point and the slope of the line, or one point and a parallel or perpendicular line.

SAT HEART OF ALGEBRA DOMAIN	
Content Dimension	**Description**
Systems of two linear equations in two variables	1. Create and use a system of two linear equations in two variables to solve problems in a variety of contexts. 2. Create a system of linear equations in two variables, and when in context interpret solutions in terms of the context. 3. Make connections between tabular, algebraic, and graphical representations of the system by deriving one representation from the other. 4. Solve a system of two linear equations in two variables, making strategic use of algebraic structure. 5. For a system of linear equations in two variables, a. interpret a solution, constant, variable, factor, or term based on the context, including situations where seeing structure provides an advantage; b. determine the conditions under which the system has no solution, a unique solution, or infinitely many solutions. 6. Fluently solve a system of linear equations in two variables.
Linear inequalities in one or two variables	1. Create and use linear inequalities in one or two variables to solve problems in a variety of contexts. 2. Create linear inequalities in one or two variables, and when in context interpret the solutions in terms of the context. 3. For linear inequalities in one or two variables, interpret a constant, variable, factor, or term, including situations where seeing structure provides an advantage. 4. Make connections between tabular, algebraic, and graphical representations of linear inequalities in one or two variables by deriving one from the other. 5. Given a linear inequality or system of linear inequalities, interpret a point in the solution set.

Algebra is the language of much of high school mathematics, and it is also an important prerequisite for advanced mathematics and postsecondary education in many subjects. The redesigned SAT focuses strongly on algebra and recognizes in particular the essentials of the subject that are most essential for success in college and careers. Heart of Algebra will assess students' ability to analyze, fluently solve, and create linear equations and inequalities. Students will also be expected to analyze and fluently solve equations and systems of equations using multiple techniques.

To assess full command of the material, these problems will vary significantly in form and appearance. Problems may be straightforward fluency exercises or may pose challenges of strategy or understanding, such as interpreting the interplay between graphical and algebraic representations or solving as a process of reasoning. Students will be required to demonstrate both procedural skill and a deeper understanding of the concepts that undergird linear equations and functions to successfully exhibit a command of the Heart of Algebra.

Mastering linear equations and functions has clear benefits to students. The ability to use linear equations to model scenarios and to represent unknown quantities is powerful across the curriculum in the postsecondary classroom as well as in the workplace. Further, linear equations and functions remain the bedrock upon which much of advanced mathematics is built. Consider, for example, that derivatives in calculus are used to approximate curves by straight lines and to approximate nonlinear functions by linear ones. Without a strong foundation in the core of algebra, much of this advanced work remains inaccessible.

PROBLEM SOLVING AND DATA ANALYSIS: PROPORTIONAL RELATIONSHIPS, PERCENTAGES, COMPLEX MEASUREMENTS, AND DATA INTERPRETATION AND SYNTHESIS

SAT PROBLEM SOLVING AND DATA ANALYSIS DOMAIN	
Content Dimension	**Description**
Ratios, rates, proportional relationships, and units	Items will require students to solve problems by using a proportional relationship between quantities, calculating or using a ratio or rate, and/or using units, derived units, and unit conversion. 1. Apply proportional relationships, ratios, rates, and units in a wide variety of contexts. Examples include but are not limited to scale drawings and problems in the natural and social sciences. 2. Solve problems involving a. derived units, including those that arise from products (e.g., kilowatt-hours) and quotients (e.g., population per square kilometer); b. unit conversion, including currency exchange and conversion between different measurement systems. 3. Understand and use the fact that when two quantities are in a proportional relationship, if one changes by a scale factor, then the other also changes by the same scale factor.
Percentages	1. Use percentages to solve problems in a variety of contexts. Examples include, but are not limited to, discounts, interest, taxes, tips, and percent increases and decreases for many different quantities. 2. Understand and use the relationship between percent change and growth factor (5% and 1.05, for example); include percentages greater than or equal to 100%.
One-variable data: distributions and measures of center and spread	1. Choose an appropriate graphical representation for a given data set. 2. Interpret information from a given representation of data in context. 3. Analyze and interpret numerical data distributions represented with frequency tables, histograms, dot plots, and boxplots. 4. For quantitative variables, calculate, compare, and interpret mean, median, and range. Interpret (but don't calculate) standard deviation. 5. Compare distributions using measures of center and spread, including distributions with different means and the same standard deviations and ones with the same mean and different standard deviations. 6. Understand and describe the effect of outliers on mean and median. 7. Given an appropriate data set, calculate the mean.
Two-variable data: models and scatterplots	1. Using a model that fits the data in a scatterplot, compare values predicted by the model to values given in the data set. 2. Interpret the slope and intercepts of the line of best fit in context. 3. Given a relationship between two quantities, read and interpret graphs and tables modeling the relationship. 4. Analyze and interpret data represented in a scatterplot or line graph; fit linear, quadratic, and exponential models. 5. Select a graph that represents a context, identify a value on a graph, or interpret information on the graph. 6. For a given function type (linear, quadratic, exponential), choose the function of that type that best fits given data. 7. Compare linear and exponential growth. 8. Estimate the line of best fit for a given scatterplot; use the line to make predictions.

SAT PROBLEM SOLVING AND DATA ANALYSIS DOMAIN	
Content Dimension	**Description**
Probability and conditional probability	Use one- and two-way tables, tree diagrams, area models, and other representations to find relative frequency, probabilities, and conditional probabilities. 1. Compute and interpret probability and conditional probability in simple contexts. 2. Understand formulas for probability and conditional probability in terms of frequency.
Inference from sample statistics and margin of error	1. Use sample mean and sample proportion to estimate population mean and population proportion. Utilize, but do not calculate, margin of error. 2. Interpret margin of error; understand that a larger sample size generally leads to a smaller margin of error.
Evaluating statistical claims: observational studies and experiments	1. With random samples, describe which population the results can be extended to. 2. Given a description of a study with or without random assignment, determine whether there is evidence for a causal relationship. 3. Understand why random assignment provides evidence for a causal relationship. 4. Understand why a result can be extended only to the population from which the sample was selected.

The redesigned SAT's Math Test has responded to the research evidence identifying what is essential for college readiness and success by focusing significantly on problem solving and data analysis: the ability to create a representation of a problem, consider the units involved, attend to the meaning of quantities, and know and use different properties of operations and objects. Problems in this category will require significant quantitative reasoning about ratios, rates, and proportional relationships and will place a premium on understanding and applying unit rate.

Interpreting and synthesizing data are widely applicable skills in postsecondary education and careers. In the redesigned SAT's Math Test, students will be expected to identify quantitative measures of center, the overall pattern, and any striking deviations from the overall pattern and spread in one or two different data sets. This includes recognizing the effects of outliers on the measures of center of a data set. In keeping with the need to stress widely applicable prerequisites, the redesigned SAT emphasizes applying core concepts and methods of statistics, rather than covering broadly a vast range of statistical techniques.

Finally, the redesigned SAT's Math Test emphasizes students' ability to apply math to solve problems in rich and varied contexts and features problems that require the application of problem solving and data analysis to solve problems in science, social studies, and career-related contexts.

PASSPORT TO ADVANCED MATH: ANALYZING ADVANCED EXPRESSIONS

SAT PASSPORT TO ADVANCED MATH DOMAIN	
Content Dimension	**Description**
Equivalent expressions	1. Make strategic use of algebraic structure and the properties of operations to identify and create equivalent expressions, including a. rewriting simple rational expressions; b. rewriting expressions with rational exponents and radicals; c. factoring polynomials. 2. Fluently add, subtract, and multiply polynomials.
Nonlinear equations in one variable and systems of equations in two variables	1. Make strategic use of algebraic structure, the properties of operations, and reasoning about equality to a. solve quadratic equations in one variable presented in a wide variety of forms; determine the conditions under which a quadratic equation has no real solutions, one real solution, or two real solutions; b. solve simple rational and radical equations in one variable; c. identify when the procedures used to solve a simple rational or radical equation in one variable lead to an equation with solutions that do not satisfy the original equation (extraneous solutions); d. solve polynomial equations in one variable that are written in factored form; e. solve linear absolute value equations in one variable; f. solve systems of linear and nonlinear equations in two variables, including relating the solutions to the graphs of the equations in the system. 2. Given a nonlinear equation in one variable that represents a context, interpret a solution, constant, variable, factor, or term based on the context, including situations where seeing structure provides an advantage. 3. Given an equation or formula in two or more variables that represents a context, view it as an equation in a single variable of interest where the other variables are parameters and solve for the variable of interest. 4. Fluently solve quadratic equations in one variable, written as a quadratic expression in standard form equal to zero, where using the quadratic formula or completing the square is the most efficient method for solving the equation.

SAT PASSPORT TO ADVANCED MATH DOMAIN	
Content Dimension	**Description**
Nonlinear functions	1. Create and use quadratic or exponential functions to solve problems in a variety of contexts. 2. For a quadratic or exponential function, a. identify or create an appropriate function to model a relationship between quantities; b. use function notation to represent and interpret input/output pairs in terms of a context and points on the graph; c. for a function that represents a context, interpret the meaning of an input/output pair, constant, variable, factor, or term based on the context, including situations where seeing structure provides an advantage; d. determine the most suitable form of the expression representing the output of the function to display key features of the context, including i. selecting the form of a quadratic that displays the initial value, the zeros, or the extreme value; ii. selecting the form of an exponential that displays the initial value, the end-behavior (for exponential decay), or the doubling or halving time; e. make connections between tabular, algebraic, and graphical representations of the function by i. given one representation, selecting another representation; ii. identifying features of one representation given another representation, including maximum and minimum values of the function; iii. determining how a graph is affected by a change to its equation, including a vertical shift or scaling of the graph. 3. For a factorable or factored polynomial or simple rational function, a. use function notation to represent and interpret input/output pairs in terms of a context and points on the graph; b. understand and use the fact that for the graph of $y = f(x)$, the solutions to $f(x) = 0$ correspond to x-intercepts of the graph and $f(0)$ corresponds to the y-intercept of the graph; interpret these key features in terms of a context; c. identify the graph given an algebraic representation of the function and an algebraic representation given the graph (with or without a context).

As a test that provides an entry point to postsecondary education and careers, the redesigned SAT's Math Test will include topics that are central to the ability of students to progress to later, more advanced mathematics. Problems in Passport to Advanced Math will cover topics that have great relevance and utility for college and career work.

Chief among these topics is the understanding of the structure of expressions and the ability to analyze, manipulate, and rewrite these expressions. This includes an understanding of the key parts of expressions, such as terms, factors, and coefficients, and the ability to interpret complicated expressions made up of these components. Students will be able to show their skill in rewriting expressions, identifying equivalent forms of expressions, and understanding the purpose of different forms.

This category also includes reasoning with more complex equations, including solving quadratic and higher-order equations in one variable and understanding the graphs of quadratic and higher-order functions. Finally, this category includes the ability to interpret and build functions, another skill crucial for success in later mathematics and scientific fields.

ADDITIONAL TOPICS IN MATH

SAT ADDITIONAL TOPICS IN MATH DOMAIN	
Content Dimension	**Description**
Area and volume	1. Solve real-world and mathematical problems about a geometric figure or an object that can be modeled by a geometric figure using given information such as length, area, surface area, or volume. a. Apply knowledge that changing by a scale factor of k changes all lengths by a factor of k, changes all areas by a factor of k^2, and changes all volumes by a factor of k^3. b. Demonstrate procedural fluency by selecting the correct area or volume formula and correctly calculating a specified value.
Lines, angles, and triangles	1. Use concepts and theorems relating to congruence and similarity of triangles to solve problems. 2. Determine which statements may be required to prove certain relationships or to satisfy a given theorem. 3. Apply knowledge that changing by a scale factor of k changes all lengths by a factor of k, but angle measures remain unchanged. 4. Know and directly apply relevant theorems such as a. the vertical angle theorem; b. triangle similarity and congruence criteria; c. triangle angle sum theorem; d. the relationship of angles formed when a transversal cuts parallel lines.
Right triangles and trigonometry	1. Solve problems in a variety of contexts using a. the Pythagorean theorem; b. right triangle trigonometry; c. properties of special right triangles. 2. Use similarity to calculate values of sine, cosine, and tangent. 3. Understand that when given one side length and one acute angle measure in a right triangle, the remaining values can be determined. 4. Solve problems using the relationship between sine and cosine of complementary angles. 5. Fluently apply properties of special right triangles to determine side lengths and calculate trigonometric ratios of 30, 45, and 60 degrees.
Circles	1. Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas, to solve problems. 2. Solve problems using a. radian measure; b. trigonometric ratios in the unit circle. 3. Create an equation to represent a circle in the xy-plane. 4. Describe how a. a change to the equation representing a circle in the xy-plane affects the graph of the circle; b. a change in the graph of the circle affects the equation of the circle. 5. Understand that the ordered pairs that satisfy an equation of the form $(x - h)^2 + (y - k)^2 = r^2$ form a circle when plotted in the xy-plane. 6. Convert between angle measures in degrees and radians. 7. Complete the square in an equation representing a circle to determine properties of the circle when it is graphed in the xy-plane, and use the distance formula in problems related to circles.
Complex numbers	1. Apply knowledge and understanding of the complex number system to add, subtract, multiply, and divide with complex numbers and solve problems.

While the overwhelming majority of problems on the redesigned SAT's Math Test fall into the first three domains, the test also addresses additional topics in high school math. In keeping with the approach described in Section II, patterns of selection for these are governed by evidence about their relevance to postsecondary education and work. The additional topics include essential geometric and trigonometric concepts and the Pythagorean theorem, which become powerful methods of analysis and problem solving when connected to other math domains.