Equations

Core Concepts

A An equation is a statement that two expressions are equal.

B The solutions of an equation are the values of the variables that make the resulting numerical statement true.

C The steps in solving an equation are guided by understanding and justified by logical reasoning.

D Equations not solvable in one number system may have solutions in a larger number system.

Core Skills

1. Understand a problem and formulate an equation to solve it.
2. Solve equations in one variable using manipulations guided by the rules of arithmetic and the properties of equality.
3. Rearrange formulas to isolate a quantity of interest.
4. Solve systems of equations.
5. Solve linear inequalities in one variable and graph the solution set on a number line.
6. Graph the solution set of a linear inequality in two variables on the coordinate plane.

Example Tasks

1. **Core Concept A**.
   Are the following equations?
   (a) $y = x^2 + 3x + 2$
   (b) $x^2 + 3x + 2 = 0$
   (c) $x^2 + 3x + 2 = (x + 1)(x + 2)$
   (d) $\left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$
   (e) $x = -2$

2. **Core Concept B**.
   What are the solutions of the equation below?

$$2n(3n - 12) = 0$$

   (a) 0 and 4 (b) 0 and 12
   (c) 2 and 4 (d) 2 and 12

3. **Core Concept B, Core Skill 6**.
   The shaded region inside the triangle ABC is defined by three inequalities. One of these is $x + y < 5\frac{1}{2}$.

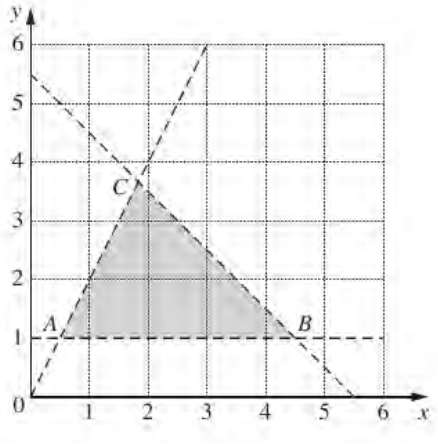

Figure 1

   (a) Write down the other inequalities.
   (b) How many points, with integer coordinates, lie in the shaded region?

4. **Core Concept B**.
   In (a)–(c), does the equation have a solution? Give a reason for your answer that does not depend on solving the equa-

tion.

(a) $\dfrac{t+2}{3+t} = 1$ (b) $\dfrac{3+t}{3-t} = 1$

(c) $\dfrac{t-2}{2-t} = 1$

5. **Core Concept C**.

A student performs the following steps in solving an equation:

$$\frac{x+3}{2x+6} = 1$$
$$x+3 = 2x+6$$
$$x = -3$$

Is the solution correct? If yes, explain why. If no, explain what was wrong with the student's reasoning.

6. **Core Concept C**.

If the equations

$$3x + 2y + 2z = 19$$
$$3x + y + z = 14$$

are true, which of the following is the value of $y + z$?

(a) -5 (b) -4 (c) 0
(d) 4 (e) 5

7. **Core Concept D**.

Write an equation or inequality that has

(a) no real solutions;
(b) infinite numbers of real solutions; and
(c) exactly one real solution.

8. **Core Concept D**.

In (a)–(f), how many solutions are there? Are they rational, real, or complex? Give a reason for your answer that does not depend on solving the equation.

(a) $(x+3)^2 = 9$
(b) $(x-3)^2 = 9$
(c) $-(x-3)^2 = 9$
(d) $16 - (x-3)^2 = 9$
(e) $9(x+3)^2 = 0$
(f) $(x+3)^2 = (x+4)^2$

9. **Core Skill 1**.

One firm offers an investment plan that pays a flat rate of 10% interest each year on the original sum invested. So each dollar grows after n years to

$$(1 + \frac{10n}{100}) \text{ dollars.}$$

Another firm offers a plan that pays 5% interest each year on the previous year's balance. So each dollar grows after n years to

$$(1 + \frac{5}{100})^n \text{ dollars.}$$

Find, using a graphing calculator or a spreadsheet, when the two offers give roughly equal returns. Which is better in the long term?

10. **Equations: Core Skill 1, Core Skill 2; Functions: Core Skill 4**.

Quinn works in Chicago and in New York City. He travels by taxi in each of the two cities.

In Chicago, he pays a fixed taxi fare of \$1.90 per ride, plus \$1.60 per mile traveled.

(a) Write an equation that expresses f, Quinn's total fare for a taxi ride in Chicago, as a function of m, the number of miles traveled.

In New York City, Quinn pays a fixed taxi fare of \$1.50 per ride, plus 25 cents per $\frac{1}{10}$ mile traveled.

(b) Write an equation that expresses f, Quinn's total fare for a taxi ride in New York City, as a function of m, the number of miles traveled.

(c) On a recent trip Quinn noticed that the total number of miles traveled by taxi from the airport to the hotel was the same in each of the two cities. Before tips were added, his taxi fare to the hotel in New York City was \$12.20 more than his taxi fare to the hotel in Chicago. What was the distance from the airport to the hotel in each city? Show or explain how you got your answer.

11. **Core Skill 1, Core Skill 4**.

'Give me 8 sheep and then we will have an equal number' said one shepherd to another. 'No, you give me 8 sheep and then I will have twice as many as you' replied another shepherd. How many sheep did each shepherd have to start with?

12. **Core Skill 1, Core Skill 4**.

(*The Abbot of Canterbury's Puzzle: AD 735–804*)

One hundred bushels of corn were distributed among one hundred people in such a way that each man received three bushels, each woman received two bushels, and each child received half a bushel. Given that there were five times as many women as men, how many children were there?

13. **Core Skill 2**.

Solve the equations

(a) $\frac{24}{x-4} = 1$,
(b) $12 - 2(5 - y) = 5y$.

14. **Core Skill 2**.

Solve the following quadratic equations using factoring methods:

(a) $2x^2+x-3 = 0$ (b) $4x^2 + 6x = 0$
(c) $36x^2 - 25 = 0$ (d) $x^2+6x+9 = 0$

15. **Core Skill 2**.

Use the quadratic formula to solve $4x^2 - 2x = 5$.

16. **Core Skill 3**.

The distance d travelled after time t at a steady speed v is given by

$$d = vt.$$

(a) How long would it take to travel 150 miles at 60 miles per hour?
(b) How fast would you have to go to do it in one and a half hours?

17. **Core Skill 3**.

Solve $A = p + prt$ for p.

18. **Core Skill 3**.

The distance D that it takes a car moving at speed v to stop is given by

$$D = rv + \frac{v^2}{2a}$$

where r is the reaction time it takes the driver to hit the brakes and a is the breaking deceleration.

(a) If r is 2 seconds and $a = 5$ meters/sec^2, how far would the car travel when stopping from 50 meters/sec?
(b) Solve the given equation to find an expression for v in terms of D, r and a. If you want to stop within 100 meters, how fast can you safely go?

COMMENTS ON SOLUTION:

Part (b) goes beyond the scope of Core Skill 3, and illustrates the extension of this task into later course work.

19. **Core Skill 4**.

Solve the simultaneous equations

$$\begin{cases} 2x - y = 16 \\ 3x + 2y = 17. \end{cases}$$

20. **Core Skill 4**.

The only coins that Alexis has are dimes and quarters.

- Her coins have a total value of \$5.80.
- She has a total of 40 coins.

Which of the following systems of equations can be used to find the number of dimes, d, and the number of quarters, q, that Alexis has?

(a)

$$\begin{aligned} d + q &= 5.80 \\ 40d + 40q &= 5.80 \end{aligned}$$

(b)

$$\begin{aligned} d + q &= 40 \\ 5.80d + 5.80q &= 40 \end{aligned}$$

(c)

$$\begin{aligned} d + q &= 5.80 \\ 0.10d + 0.25q &= 40 \end{aligned}$$

(d)

$$\begin{aligned} d + q &= 40 \\ 0.10d + 0.25q &= 5.80 \end{aligned}$$

21. **Core Skill 4**.
Solve the following system of equations:

$$\begin{cases} 2x - y - z = 7 \\ 3x + 5y + z = -10 \\ 4x - 3y + 2z = 4. \end{cases}$$

22. **Core Skill 5**.
Solve $|x - 6| \leq 4$ and locate the solution on the number line.

23. **Core Skill 5**.
Solve $8.5 < 3x + 2 \leq 9.7$ and locate the solution on the number line.

24. **Core Skill 1, Core Skill 5**.
Mr. Smith uses the following formula to calculate students' final score C in his Algebra II class: $C = 0.4E + 0.6T$, where E represents the score on the final exam, and T represents the average score of all tests given during the grading period. All tests and the final exam are worth a maximum of 100 points. The minimum passing score on tests, the final exam, and the course is 60. Determine the inequalities that describe the following situation. When necessary, round scores to the nearest tenth.
(a) Is it possible for a student to have a failing test score average (i.e., $T < 60$ points) and still pass the course?
(b) If you answered "yes," what is the minimum test score average a student can have and still pass the course? What final exam score is needed to pass the course with a minimum test score average?
(c) A student has a particular test score average. How can (s)he figure out the minimum final exam score needed to pass the course?

25. **Core Skill 3**.
If oil should ever be spilled into the Columbia River Estuary, the company responsible for the spill would be liable for monetary damages according to a formula. By Washington state law, the formula in 2009 was given by:[1]

$$D = 0.508GS(A + B + C)$$

In this formula, D is the damage liability in dollars; G is the number of gallons spilled; S is a "vulnerability score" in the range from 1 to 5 that takes into account the wildlife characteristics of any given square kilometer of the estuary[2]; and A, B and C are "chemical penalty scores" in the range from 1 to 5 that take into account the toxicity, harmful mechanical properties, and longevity of the material spilled. For example, kerosene has a toxicity score $A = 1.4$, a harmful mechanical property score $B = 2.4$, and a longevity score $C = 1$.[3] Suppose that a company responsible for a kerosene spill in an area of lowest vulnerability is held liable for \$10 million. How many gallons were spilled? How many dollars per gallon was the company charged for the spill? In general, what is a formula for the number of dollars of liability per gallon of spill? What is the maximum possible liability in dollars per gallon?

Sources

1. McCallum, William
2. Massachusetts Department of Elementary and Secondary Education
3. University of Cambridge International Examinations

[1] http://apps.leg.wa.gov/WAC/default.aspx?cite=173-183-840
[2] see http://apps.leg.wa.gov/wac/default.aspx?cite=173-183-500
[3] http://apps.leg.wa.gov/WAC/default.aspx?dispo=true&cite=173-183&full=true.

4. McCallum, William
5. McCallum, William
6. College Board
7. Washington Office of the Superintendent of Public Instruction
8. McCallum, William
9. Shell Centre for Mathematical Education/Mathematics Assessment Resource Service (MARS)
10. Massachusetts Department of Elementary and Secondary Education
11. Gardiner, Tony
12. Gardiner, Tony
13. University of Cambridge International Examinations
14. Washington Office of the Superintendent of Public Instruction
15. Washington Office of the Superintendent of Public Instruction
16. Shell Centre for Mathematical Education/Mathematics Assessment Resource Service (MARS)
17. Washington Office of the Superintendent of Public Instruction
18. Shell Centre for Mathematical Education/Mathematics Assessment Resource Service (MARS)
19. University of Cambridge International Examinations
20. Massachusetts Department of Elementary and Secondary Education
21. Washington Office of the Superintendent of Public Instruction
22. Washington Office of the Superintendent of Public Instruction
23. Washington Office of the Superintendent of Public Instruction
24. Washington Office of the Superintendent of Public Instruction
25. Zimba, Jason

Modeling

Core Concepts

A Mathematical models involve choices and assumptions that abstract key features from situations to help us solve problems.

B Even very simple models can be useful.

Core Skills

1. Model numerical situations.
2. Model physical objects with geometric shapes.
3. Model situations with equations and inequalities.
4. Model situations with common functions.
5. Model situations using probability and statistics.
6. Interpret the results of applying a model and compare models for a particular situation.

The Modeling standard needs discussion in relation to the other standards. For example, what is the difference between a *geometry task* versus *a modeling task that uses geometry*? What is the difference between a contextualized algebra problem and a modeling problem that uses equations to describe a situation?

In these standards, a task is considered to belong more in Modeling, the more it is the case that:

- The math techniques to be used are not stated explicitly in the problem.
  - However, beginning/developmental modeling tasks can walk the student through the techniques, as a way to show their use.
- Various assumptions must be imposed by the student to apply the techniques; these assumptions are not explicitly stated in the problem; and differing sets of assumptions could all be considered reasonable.
- The task involves making a decision about something.
- The task involves an optimization of some kind.
- The context is not a pretext. While the task inevitably teaches mathematics, its primary focus is the situation or phenomenon at hand. See the figure below (after a diagram by Malcom Swann, in Muller and Burkhardt, 2006).

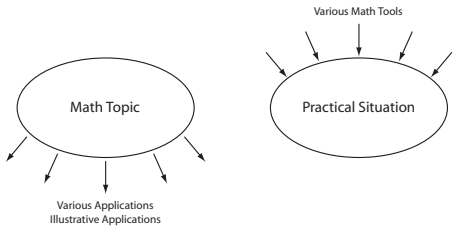

Figure 1

- The phenomenon or situation is interesting or worthwhile beyond the academic discourse of the classroom.

A list of criteria such as this one does not define a hard-and-fast rule that could be used to unambiguously classify tasks. The distinction between *geometry* and *modeling with geometry* is a heuristic one. Moreover, few, if any, of the following Modeling tasks satisfy all of the listed criteria. But each task fulfills one or more of these criteria to a great enough extent to be considered an example of Modeling.

Reference

E. Muller and H. Burkhardt, Application and Modelling for Mathematics—Overview. In W. Blum and P.L. Galbraith (Eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study*, Springer, 2006. http://books.google.com/books?id=XICuY-BFaHYC.

Example Tasks

1. **Core Concept A; Core Skill 2**.

   If everyone in the world went swimming in Lake Michigan, what would happen to the water level? (Would Chicago be flooded?)

2. **Core Concept A; Core Skill 2; Core Skill 7**.

   The Federal Communications Commission (FCC) needs to assign radio frequencies to seven new radio stations located on the grid in the accompanying figure. Such assignments are based on several considerations including the possibility of creating interference by assigning the same frequency to stations that are too close together. In this simplified situation it is assumed that broadcasts from two stations located within 200 miles of each other will create interference if they broadcast on the same frequency, whereas stations more than 200 miles apart can use the same frequency to broadcast without causing interference with each other. The FCC wants to determine the smallest number of frequencies that can be assigned to the six stations without creating interference.

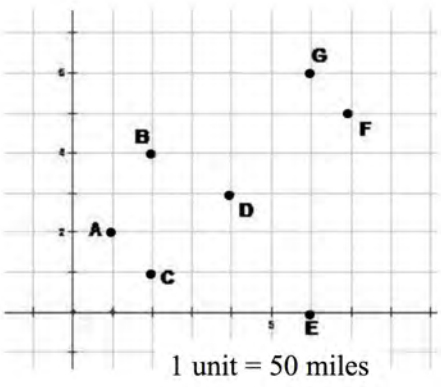

Figure 2

- Student 1 began thinking about the problem by drawing a circle of radius 200 miles centered on each radio station.
- Student 2 began thinking about the problem by drawing line segments to connect pairs of radio stations whenever the two radio stations are within 200 miles of one another.
- Student 3 began thinking about the problem by drawing line segments to connect pairs of radio stations whenever the two radio stations are more than 200 miles from one another.

Which approach seems most promising to you? Use this approach to determine the smallest number of frequencies that can be assigned to the six stations without creating interference. Justify your final answer.

3. **Core Concept A; Core Skill 2; Core Skill 3; Core Skill 5; Equations Core Concept A; Equations Core Skill 3**.

Picture a tightly rolled spool of paper. Everyday examples might include a roll of bathroom tissue or a roll of cash-register tape. See the accompanying figure.

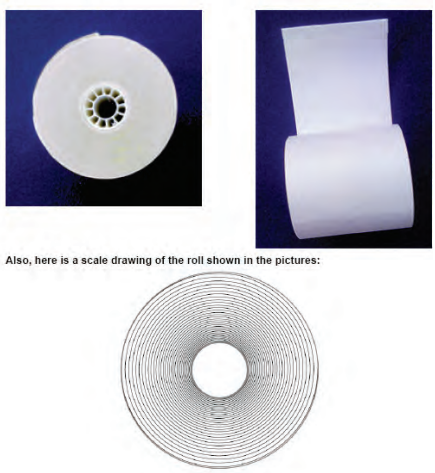

Figure 3

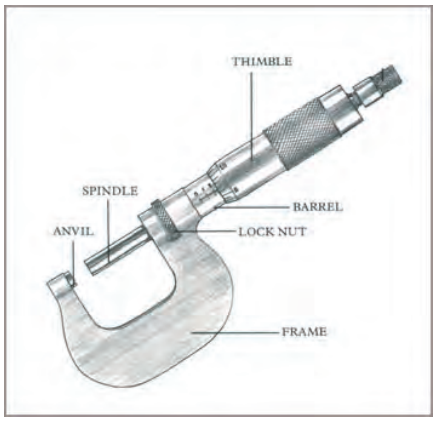

Figure 4

(a) Assuming the paper in the roll is very thin, what is the relationship between the thickness of the paper, the inner and outer diameters of the roll, and the length of the paper in the roll? Express your answer as an algebraic formula involving the four listed variables.

(b) A roll of masking tape is another example of a tightly rolled spool. In one classroom, layers of masking tape of various thicknesses were measured using a micrometer, a tool for measuring small distances (see the photo; image from http://www.design-technology.org/CDT10micrometer.htm). The table below shows the micrometer readings.

Table 1

Number of Tape Layers	0	1	2	3	4	5	6	7
Thickness (millimeters)	0.1	0.24	0.37	0.49	0.6	0.71	0.81	0.92

When you look at the first column of the table, you may wonder why zero layers of tape have a thickness of 0.1 millimeters! The reason is that these measurements were made by sticking pieces of tape to a sheet of paper (see the photo below). So the first value in the table simply represents the thickness of the piece of paper.

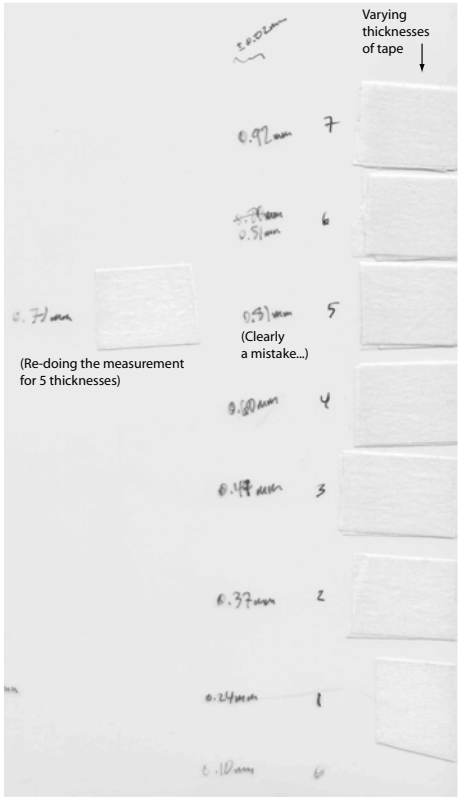

Figure 5

(i) Estimate the thickness of a single layer of tape by subtracting values in the first and second rows of Table 1.

(ii) Make a scatterplot of the data in Table 1.

(iii) Draw a straight line that you think best describes the pattern in your scatter plot.

(iv) Find the equation of this line. Explain how you determined your equation.

(v) Use your equation to estimate the thickness of a single piece of tape.

(vi) Which estimate of the thickness of a single piece of tape do you think is more accurate, the one you found in Question (i) or the one you found in Question (v)? Why?

(vii) What is your best estimate of the thickness of the paper in this experiment?

(viii) In a roll of masking tape, the inner radius is 60 millimeters and the outer radius is 80 millimeters. Using the thickness value determined in Question (v), how long is this roll of tape?

4. **Core Concept A; Core Concept B; Core Skill 1**.

- In a country with 300 million people, about how many high school math teachers will be needed? Try to estimate a sensible answer using your own everyday knowledge about the world. Write an explanation of your answer, stating any assumptions you make.
- Likewise, estimate the number of people born each day on planet earth.
- Likewise, estimate the percentage of Americans who are pregnant at any given time. Also estimate the percentage of elephants who are pregnant at any given time.

5. **Core Concept B; Core Skill 1**.
   Suppose you wrote check #556 on November 5, 1995, and check #953 on September 26, 1997. What is a good guess for when you wrote check #678?

6. **Core Skill 1**.
   The accompanying figure shows a conversation that occurred during a cross-country road trip.

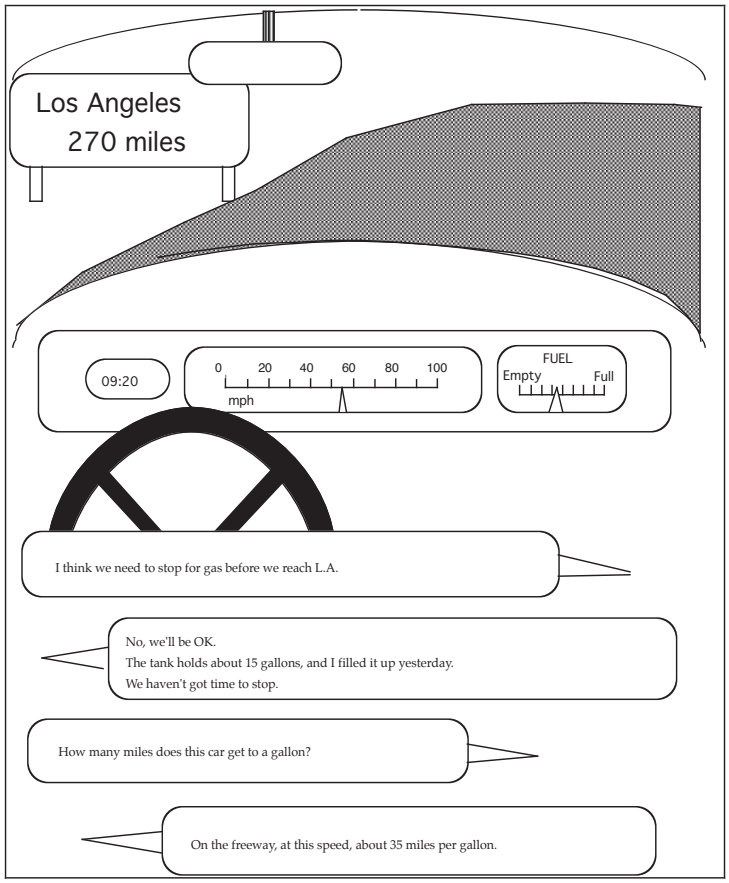

Figure 6

(a) Do they have to stop for gas? Explain your reasoning.
(b) Suppose they decide to stop for 30 minutes. At what time will they reach Los Angeles?

7. **Core Skill 1; Core Skill 2; Core Skill 3**.
   A team of haymakers was assigned the task of scything two meadows, one twice the size of the other. The team worked half a day on the larger meadow. Then it split into two equal groups. The first group remained in the larger meadow and finished it by evening. The second group scythed the smaller meadow, but by evening there still remained a portion to do. This portion was scythed the next day by one haymaker in a single day's work. How many haymakers were there in the team?

8. **Core Skill 1; Core Skill 3**.
   A car does y miles a year, averaging m miles per gallon.
   (a) Write a formula for g, the number of gallons used in a year.

(b) If the average price of gas is $\$p$ per gallon, write a formula for the total cost $\$C$.
(c) You are thinking of changing your 15 mpg gas guzzler for a 40 mpg car. If you drive 20,000 miles each year, how much money would you save on gas at \$3 per gallon? How much is this savings per week?

9. **Core Skill 1; Core Skill 3**.

Funds totaling \$191,000 are designated for four schools. The distribution of the funds is to be in proportion to the number of students in each school. Student populations of the four schools are: School A, 386; School B, 1691; School C, 2109; School D, 817.

(a) Figure out how much money each school gets.
(b) Draw some sort of diagram to scale that helps show visually how the money is divided up among the four schools.
(c) Find general formulas for deciding how much money each of the four schools gets in terms of the populations of the schools and the total amount of money to be distributed.
(d) Add your formulas algebraically. What is the result? Does this make sense?

10. **Core Skill 2**.

Ann is moving to Gridville. She's looking for a house to live in. The location of her house-to-be should satisfy the following conditions:

- Ann wants to live close to the office where she works, so the distance from her house to the office should not be more than 2.5 km.
- Because of the terrible smell of the pet food factory, the distance from her house to the factory must be at least 3 km.

(a) Both conditions mention distance. Is distance measured in the same way in both situations? If yes, why? If no, why not?
(b) On the grid provided, graph the area that meets both conditions.

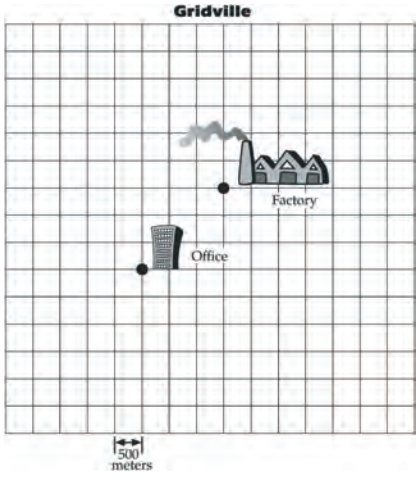

Figure 7

11. **Core Skill 2**.

A poster manufacturing company is considering different ways of making and shipping its posters. The figure below shows two kinds of tubes for posters: square tubes and hexagonal tubes.

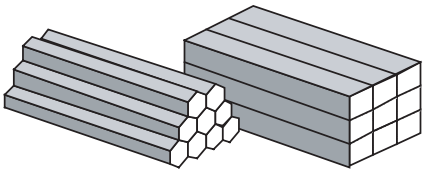

Figure 8

The square tubes are 7.5×7.5 cm, with a length of 75 cm. The hexagonal tubes have the same length of 75 cm but they have a diameter of 9 cm (see figure below).

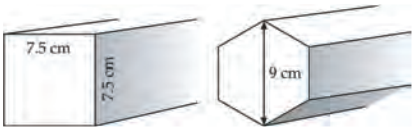

Figure 9

From the customer's perspective, a more tightly rolled poster is harder to flatten and hang on a wall. So a tube that allows the poster to be more loosely rolled is more desirable than a tube that requires the poster to be more tightly rolled. How tightly a poster is rolled is based on the amount of space measured by the largest circle that fits into the tube. See the figure below.

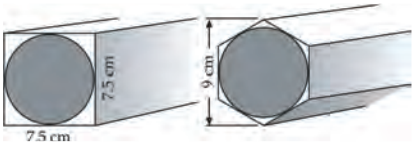

Figure 10

(a) Calculate the area of the circle in the square tube and the area of the circle in the hexagonal tube. Explain your answers.
(b) Calculate the efficiency of both tubes in terms of percentage of space used by the circular poster roll and its interior. Explain your answers.
(c) Based on the information you have now, in which tube do you think the company should pack posters? Why?
(d) The individual hexagonal tubes are packed in a $20 \times 20 \times 75$ cm box (see the Figure below). Calculate the efficiency in terms of the percentage of the box's front that is used by the hexagons.

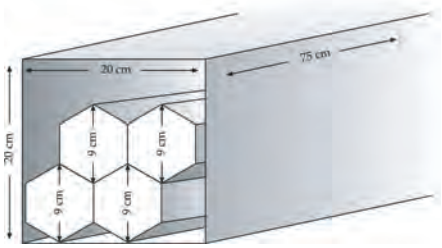

Figure 11

(e) How many of the square tubes will fit in the $20 \times 20 \times 75$ cm box? (You may want to make a sketch to help you decide.)
(f) Calculate the efficiency in terms of the percentage of the box's front that is used by the squares.
(g) Based on the information you have now, in which tube do you think the company should stack posters? Why?
(h) The poster company is considering a larger box for shipping: $45 \times 30 \times 75$ cm.
  (i) How many hexagonal tubes fit in the box?
  (ii) How many square tubes fit in the box?
  (iii) For both the hexagonal tubes and square tubes, find the efficiency in terms of the percentage of the box's front that is used by the hexagons or squares.
(i) Based on the information you have now, in which tube do you think the company should stack posters? Why?

12. **Core Skill 2**.
Based on enrollment predictions school officials have decided that, for the next school year, the classroom trailers will be moved to the current practice football field and a new practice field will be located behind the school parking lot. Including end zones, the practice field will be 120 yards by 53 yards in order to closely approximate a standard field. However, the owner of a local nursery has donated enough grass seed to plant 81,000 square feet. Since they have more than enough grass seed for the practice field, school officials would like to plant a uniform border around the field. What are the dimensions of the 81,000 square feet rectangular area that should be planted for the practice field and uniform border?

13. **Core Skill 2**.
The accompanying figure shows the plan of a city. All curved and straight lines are streets.

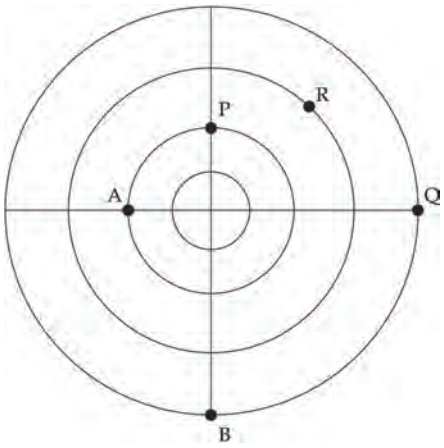

Figure 12

(a) On the figure, draw the shortest route from P to Q and the shortest route from P to R.
(b) On the figure, draw three points that are approximately equidistant from A and B (using streets).
(c) Compare and constrast this type of distance with other types of distance you have studied.

14. **Core Skill 2**.
The accompanying figure is a diagram of a miniature golf hole. Players must start their ball from one of the three tee positions. A wall separates the tees from the hole.

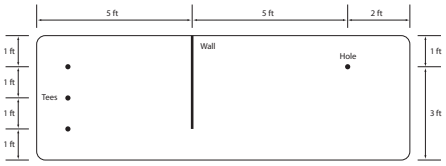

Figure 13

At which tee should the ball be placed to create the shortest "hole in one" path? What is this shortest distance? Explain your reasoning.

15. **Core Skill 2; Core Skill 3**.

All states have building codes. Many such codes can be interpreted as mathematical inequalities, since they establish limits on what can be done.

Most states have codes related to staircase construction (see the photo).

Figure 14

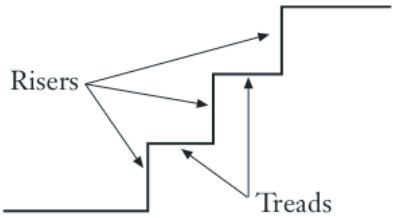

Figure 15

The most basic dimensions of a set of stairs are *riser height* and *tread depth*. A *riser* is the vertical front of a stair. The surface that you step on is called a *tread*. (See the diagram.)

(a) The Massachusetts State Building Code includes this statement:

> Maximum riser height shall be seven inches (178 mm) and minimum riser height shall be four inches (102 mm).

This statement contains two requirements for riser height. Write these requirements in two ways: as a pair of simple inequalities, and also as a compound inequality.

(b) Here is another statement from the Massachusetts State Building Code:

> Minimum tread depth shall be 11 inches (279 mm), measured horizontally between the vertical planes of the foremost projection of adjacent treads at a right angle to the tread's leading edge.

Write this requirement as an inequality.

(c) In addition, most staircases conform to this design guideline:

> The depth of a tread plus two times the height of a riser should have a total value of from 24 to 25 inches.

Write this guideline using inequalities.

(d) Label a suitable coordinate grid. Graph the solution to the system of inequalities regarding staircase riser height and tread depth.

(e) Choose an appropriate point to check your answer.

16. **Core Skill 2; Core Skill 3; Core Skill 4; Quantity; Equations, Core Skill 3**.

A manufacturer wants to design a cylindrical soda can that will hold 500 milliliters of soda. The manufacturer's research has determined that an optimal can radius is between $3\frac{1}{4}$ and 4 centimeters. What is the corresponding range of possible height measurements for the can? Explain your reasoning.

17. **Core Skill 3**.

A group of art students has formed an Arts Collective to organize an arts and crafts exhibition (see photo).

Figure 16

In order to publicize the event, they intend to use two methods. They can print one-page flyers for distribution at schools and malls. The flyers will cost only eight cents each to produce. Another method is to mail postcards about the event to selected people. This option is more expensive: twelve cents for each card plus eighteen cents per card for bulk postage. Postcards are expected to be more effective than flyers. But in order to get the twelve cent rate, they must order at least 1000 cards. The total budget for publicity is $1200.

The students must decide how many flyers to print and how many cards to order. But there are upper and lower limits on the number of each.

(a) What is the lower limit on the number of cards they will order if they want the quantity discount?

(b) Let C represent the number of cards ordered. Write the lower limit as a mathematical inequality.
(c) What is the lower limit on the number of flyers that can be produced?
(d) Let F represent the number of flyers produced. Write the lower limit as an inequality.
(e) One statement in the problem description implies an upper limit on a quantity. Explain.
(f) Write a sentence that describes this upper limit in more detail. Include the phrases "money spent on flyers" and "money spent on cards."
(g) Rewrite your answer to Question (f) as a mathematical inequality in the variables C and F.
(h) Your answers to Questions (b), (d), and (g) taken together form a system of inequalities in C and F. The solution of this system represents the range of publicity options available to the Arts Collective. The solution can be visualized with a graph. Should the boundaries of the solution region be solid lines or dashed lines? Explain.
(i) The inequalities in Questions (b) and (d) should each contain only one variable, so they are probably easier to graph. Label the axes of a two-dimensional coordinate grid with the variables C and F. Then graph the inequalities from Questions (b) and (d).
(j) Now consider the budget limit. If no flyers are produced, how many cards can be bought and mailed?
(k) If the minimum of 1000 cards is used, how many flyers can be made?
(l) Your answers to Questions (j) and (k) can each be represented by a point on the graph. Use these points to graph the boundary that represents the budget limit. Then shade only the region that satisfies all three inequalities.
(m) Check your answer. Choose any point that is inside the shaded region, and verify that its coordinates satisfy all three inequalities.

18. **Core Skill 3**.

A coffee shop sells several kinds of coffee. The shop also uses some of its coffees to make its own custom blends. Coffee A sells for \$6 a pound. Coffee B sells for \$10 a pound. The shop's manager wants to create a blend of the two types that sells for \$7 a pound. The manager wants to make 10 pounds of this blend. How many pounds of each type of coffee should be in the 10 pounds of blend?

19. **Core Skill 3**.

When summer approaches, Desmond and Farid look forward to baseball games, movies, and weekends at the local amusement park. Because all of these things cost money, Desmond and Farid decide to start a lawn service to earn money.

Desmond can devote 10 hours a week to this new venture, while Farid can devote 4 hours. The boys realize it takes less time to trim the edge than to mow, so they decide that Desmond can do all the mowing and Farid can do the edging. After surveying the neighborhood, they determine that their clients will fall into one of two categories: standard-sized interior lots or large corner lots.

From working on their own yards the boys know it will take about an hour to mow and half an hour to edge a standard-sized yard. A larger yard will take 45 minutes to edge and 2 hours to mow.

Based on research he has done in his neighborhood, Desmond wants to set prices at \$20 per large yard and \$15 for each standard-sized yard. Each price is the total price that includes edging and mowing.

On the one hand, the boys might decide to do more large yards, each of which brings more money. Or they might decide to do more standard yards, each of which takes less time. It's not obvious which approach will earn them the most money. The goal of this problem is to find the best approach.

(a) Use the table below to organize the information in the situation:
(b) Using the information in the table to assist you, write a system of inequalities to represent all the constraints that define the boys' problem situation.
(c) Farid and Desmond's goal is to maximize their earnings from the lawn service. Find a formula that expresses their weekly earnings P in terms of x and y.
(d) Graph the system of inequalities representing the constraints and shade the feasible region—the region representing all of the values of x and y that obey all of the constraints.

	x (# of standard yards)	y (# of large yards)	Constraint
Number of hours mowing			
Number of hours edging			

Table 2: Desmond and Farid's situation

(e) What are the vertices of your feasible region? Explain how you found them.
(f) Use the vertices (from Question (e)) and the earnings formula to find the maximum Farid and Desmond can earn in one week with the given constraints.
(g) How many and which size yards will Farid and Desmond need to mow and edge to maximize their earnings?

20. **Core Skill 3**.

This problem is difficult and would merit an extended treatment in class; it is not meant to be one in a series of routine "word problems" on a worksheet.

Two boats (boat 1 and boat 2) start off from opposite sides of a lake, each heading for the starting point of the other. They pass each other 800 yards from the starting point of boat 1. They continue to the starting point of the other boat, turn around, and return. This time, they pass each other 300 yards from the (original) starting point of boat 2. How wide is the lake? (Each boat travels at a fixed speed. Ignore turnaround times.)

21. **Core Skill 3**.

An academic team is going to a state mathematics competition. There are 30 people going on the trip. There are 5 people who can drive and 2 types of vehicles, vans and cars. A van seats 8 people, and a car seats 4 people, including drivers. How many vans and cars does the team need for the trip? Is more than one option available? Explain your reasoning.

22. **Core Skill 4**.

Clara purchased a used car for \$8400. She estimates that each year she owns the car it will depreciate (lose value) by 12% of its value the previous year.

(a) According to Clara's assumption, the car's value after 1 year from the purchase date will be \$7392. Check this yourself.
(b) What will be the car's value 2, 3, and 4 years from the date of purchase?
(c) Clara plans to keep the car until its value reaches \$4000. By Clara's assumption, what is the minimum number of years from the date of purchase that the car's value will be less than \$4000? Show your work.

23. **Core Skill 4**.

If it is never refreshed, the water in a swimming pool becomes polluted after some time. Urea that comes into the water via body secretions is one of the substances that pollute swimming pools.

In this problem, presume an increase in the amount of urea of 500 g per day, caused by 1000 swimmers a day. Suppose the amount of urea present in the water is represented as u.

(a) Find a recursive formula for the quantity of urea in the water (per day).
(b) When there are 4000 g of urea in the water, the swimming pool will be closed to the public because the water has to be cleaned. The cleaning equipment can remove 10% of the amount of urea present in one day, so the first day, 400 g of urea can be removed. The second day, 10% of the remaining amount of urea will be removed, and so on.
    (i) What recursive formula describes this process of cleaning?
    (ii) How many days does it take before 50% of the 4000 g of urea is removed from the water?
(c) A better method of keeping the amount of urea under control is to let the people swim during the day and

to refresh the water during the night. Starting with a clean pool (no urea in the water), swimmers will add 500 g of urea to the water every day. Every night, 10% of the urea in the pool will be removed.

(i) Calculate the amount of urea in the pool at the beginning of day 2 (after one day of swimming and one night of refreshing).

(ii) Calculate the amount of urea at the beginning of day 3.

(iii) Find a recursive formula for the amount of urea at the beginning of a day.

(iv) Will the amount of urea continue to increase without bound day after day, or will the amount of urea stabilize at some point? Explain how you found this.

24. **Core Skill 4**.

At the You're Toast, Dude! toaster company, the weekly cost to run the factory is $1400 and the cost of producing each toaster is an additional $4 per toaster.

(a) Write a function rule representing the weekly cost in dollars, $C(x)$, of producing x toasters.
(b) What is the total cost of producing 100 toasters in one week?
(c) If you produce 100 toasters in one week, what is the total production cost per toaster?
(d) Will the total production cost per toaster always be the same? Justify your answer.
(e) Write a function rule representing the total production cost per toaster $P(x)$ for producing x toasters.
(f) Answer the following questions:

(i) What is the production cost per toaster if 300 toasters are produced in one week? If 500 toasters are produced in one week?

(ii) What happens to the total production cost per toaster as the number of toasters produced increases? Explain your answer.

(iii) How many toasters must be produced to have a total production cost per toaster of $8?

25. **Core Skill 4**.

A recommended adult dosage of the cold medication NoMoreFlu is 16 mL. NoMoreFlu causes drowsiness when there are more than 4 mL in one's system, in which case it is unsafe to drive, operate heavy machinery, etc. The manufacturer wants to print a warning label telling people how long they should wait after taking NoMoreFlu for the drowsiness to pass. The typical metabolic rate is such that one quarter of the NoMoreFlu is lost every four hours.

(a) If a person takes the full dosage, how long should adults wait after taking NoMoreFlu to ensure that there will be

(i) Less than 4 mL of NoMoreFlu in their system?

(ii) Less than 1 mL in their system?

(iii) Less than 0.1 mL in their system?

(b) What do you think the warning label should say? Design the label and explain the thinking behind your design.

26. **Core Skill 4; Core Skill 7**.

Among the many species that have been endangered at one time or another is the desert bighorn sheep. The desert bighorn sheep are sensitive to human-induced problems in the environment and their numbers are therefore a good indicator of land health.

It is estimated that in the 1600s, there were about 1.75 million bighorn sheep in North America. By 1960, the bighorn sheep population in North America had dropped to about 17,000. There appears to have been a similar decline in west Texas, where wildlife biologists have data showing that in 1880, there were around 1,500 bighorn sheep in west Texas and by 1955, the population had dwindled to 25 in that area. Efforts to reintroduce desert bighorn sheep in west Texas began around 1957 and by 1993, there were about 400 desert bighorn sheep in west Texas roaming free or in captivity.

(a) Assume that the annual percentage decrease in the bighorn sheep population in west Texas was fairly constant from 1880 to 1955. Model the bighorn sheep population for any year in this range with an exponential function of the form $P = a \cdot b^t$, where t is the number of years since 1880, a is the population of the bighorn sheep in west Texas in 1880, b is the annual rate of retention, and P is the annual bighorn sheep population in west Texas for the given year.

(b) Assume that, starting in 1957 when reintroduction began, the annual percentage increase in the bighorn sheep population in west Texas was fairly constant. Model the bighorn sheep population for any year since 1957 with the exponential function, $P = a \cdot b^t$, where t is the number of years since 1957, b is the annual rate of growth, P is the annual bighorn sheep population in west Texas for the given year, and a is the bighorn sheep population in 1957.

(c) Describe the mathematical domain for the two functions in Questions (a) and (b). Describe reasonable domain values for the actual situations modeled by these functions. Explain any differences between the two types of domains (mathematical and situational).

(d) From 1880 to 1955, by what percentage was the population decreasing annually? From 1957 to 1993, by what percentage was the population increasing annually?

(e) By what year had the sheep population dropped to 750 or fewer? Use numerical, graphical, or algebraic methods and explain your reasoning.

(f) If the reintroduction program continues, in what year will the bighorn sheep population again be at least 750? Use numerical, graphical, or algebraic methods and explain your reasoning.

(g) In 2001, it was reported that there were 500 bighorn sheep in west Texas. Is this number consistent with the number predicted by the exponential model for reintroduction? Why or why not?

27. **Core Skill 5**.

Our school has to select a girl for the long jump at the regional championship. Three girls are in contention. We have a school jump-off. Their results, in meters, are given in the accompanying table.

Elsa	Miki	Aisha
3.25	3.55	3.67
3.95	3.88	3.78
4.28	3.61	3.92
2.95	3.97	3.62
3.66	3.75	3.85
3.81	3.59	3.73

Table 3: Data from the jump-off; distances are given in meters.

Hans says, "Aisha has the longest average. She should go to the championship."
Do you think Hans is right? Explain your reasoning.

28. **Core Skill 5**.

The figure below shows two graphs: one showing the amount of water consumed in the United States from 1950 through 1980, and another graph showing the number of people living in the U.S. during those years.

T is the total amount of water consumed and B is the population. The vertical scale on the left side of the graph belongs to the graph of T. On the right side you see the vertical scale that belongs to the graph of B.

So, in 1970, the total water consumption was about 1340 billion liters per day and the population size was about 200 million people.

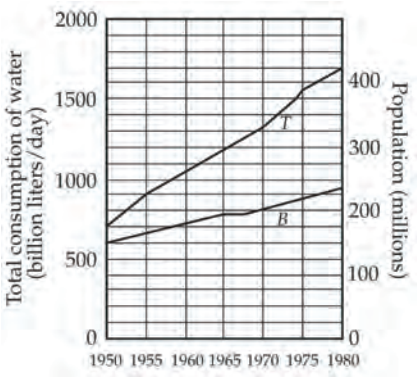

Figure 17

(a) Complete the table below with data from the graphs.

Year	1950	1955	1960	1965	1970	1975	1980
T (billion liters per day)	700				1340		
B (millions)	145				200		

(b) Use the data from the table to complete the graph below.

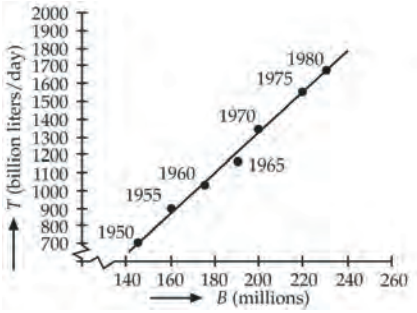

Figure 18

(c) Draw a straight line that you think best describes the pattern in your scatter plot.

(d) Find the equation of this line. Explain how you determined your equation.

(e) In 1980, researchers predicted that every five years the total consumption of water would increase by 110 to 200 billion liters per day.

(i) If that prediction is correct, what will be the minimum total consumption of water in the year 2000? Explain your answer.

(ii) What will be the maximum total consumption of water in the year 2000? Show how you got your answer.

(f) Use the experts' prediction for water-consumption increase stated in Question (e) to write equations for predicting the minimum and maximum total water consumption per day in any given year.

(g) Researchers don't expect that the increase in water consumption will continue forever. They predict that 5000 billion liters per day will be the maximum amount of water available. Between which years will people consume 5000 billion liters of water per day, according to researchers? Explain.

29. **Core Skill 5; Core Skill 6**.

The data in the accompanying table shows the annual median earnings for female and male workers in the United States from 1984 to 2004.

Year	Women's median earnings (in dollars)	Men's median (in dollars)
1984	8675	17026
1985	9328	17779
1986	10016	18782
1987	10619	19818
1988	11096	20612
1989	11736	21376
1990	12250	21522
1991	12884	21857
1992	13527	21903
1993	13896	22443
1994	14323	23656
1995	15322	25018
1996	16028	25785
1997	16716	26843
1998	17716	28755
1999	18440	30079
2000	20267	30951
2001	20851	31364
2002	21429	31647
2003	22004	32048
2004	22256	32483

(a) Create two scatter plots, one for women's median earnings over time and one for men's median earnings over time. Describe two things you notice about the scatter plots.

(b) Terry and Tomás are trying to decide what type of model will most accurately represent the data. Terry thinks that a linear model might be most appropriate for each scatter plot. Help Terry find reasonable linear function rules for each scatter plot. Explain how you found these.

(c) Using the linear models, will women's annual median earnings ever equal those of men? Why or why not?

(d) Tomás thinks that an exponential model might be most appropriate for each scatter plot. Help Tomás find reasonable exponential function rules for each scatter plot. Explain how you found these.

(e) Using the exponential models, will women's annual median earnings ever equal those of men? Why or why not?

(f) If you answered yes to either Question (c) or question (e), use that model to determine the first year women will have higher median earnings than men. Explain how you found your answer.

(g) For each year listed in the table, find the ratio of women's to men's annual median earnings expressed as a percentage. Use the data to create a scatter plot of percentage versus year. Based on this graph, do you think women's annual median earnings will ever equal those of men? Why or why not? Considering the results of the scatter plot in Question (g) above, do you think the linear model or exponential model makes more sense? Why?

30. **Core Skill 5; Core Skill 7**.

Karnataka is a state in southwest India. The accompanying table is agricultural data on fertilizer use and grain crop yield in Karnataka. Fertilizer is measured in 100,000 tons. Crop yield is measured in 10 kilograms per hectare.

Year	Fertilizer ($\times 10^5$ tons)	Yield ($\times$ 10 kg/ha)
1956	0.06	52.3
1966	0.38	45.8
1970	1.32	19.1
1974	1.95	88.9
1975	2.32	82.9
1976	2.03	93.9
1977	2.08	79.2
1978	2.9	97.9
1979	3.72	101.7
1980	3.59	102.9
1981	3.54	91.9
1982	3.87	102.2
1983	3.83	85.8
1984	4.87	102.1
1985	5.9	95.4
1986	5.55	81.19
1987	5.66	97.03
1988	5.58	83.57
1989	5.75	93.5

Throughout the years over which these data were gathered, the amount of land in cultivation remained fairly constant.

(a) Find a mathematical function that you think does a good job of modeling the relationship between fertilizer use and grain crop yield. Explain how you did it.
(b) Use the function you have chosen to predict the yield if fertilizer use is 500,000 tons.
(c) How comfortable are you with the prediction you made in Question (b)? Explain.
(d) What advice can you offer the government of Karnataka about fertilizer use? Explain.

31. **Core Skill 6**.

"My sixty-year old mother, who lives in New York, gets frightened by newspapers. One day she is afraid

of being a victim of crime, the next she is frightened of being killed in a road accident, then it's terrorists, and so on."

(a) Using reliable websites with national statistics...

(i) Estimate the chances of my mother being a victim of the above events during the next year.

(ii) Compare the likelihood of these events with the probability that women of her age will die during the coming year.

(b) Why do you suppose people fear such unrealistic dangers?

32. **Core Skill 6**.

In the general population, about 1 baby in 8,000 dies in an unexplained "crib death." The cause or causes are at present unknown. Three babies in one familiy have died. The mother is on trial, and you are on the jury. An expert witness says: "One crib death is a family tragedy; two is deeply suspicious; three is murder. The odds of even two deaths in one family are 64 million to 1."

Think about the reasoning underlying this testimony. Check the expert's math, of course, but also explain what assumptions are being made. Would the expert's testimony lead you to decide that a murder had taken place?

Sources

1. Howe, Roger
2. Adapted from Focus in High School Mathematics: Reasoning and Sense Making, copyright 2009 by the National Council of Teachers of Mathematics. All rights reserved.
3. communicated by Phil Daro; Shannon, Ann; Zimba, Jason
4. Shell Centre for Mathematical Education/Mathematics Assessment Resource Service (MARS); Roger Howe; Stephen Hawking (1994), *Black Holes and Baby Universes And Other Essays*, ISBN-13 9780553374117, Bantam Books, p. 1; Jason Zimba
5. communicated by Phil Daro
6. Reproduced with the permission of New Standards, University of Pittsburgh, and the National Center on Education and the Economy
7. communicated by Phil Daro
8. Adapted from Shell Centre for Mathematical Education/Mathematics Assessment Resource Service (MARS)
9. communicated by Phil Daro
10. COMAP
11. COMAP
12. Georgia Department of Education
13. COMAP
14. Adapted from Washington Office of the Superintendent of Public Instruction
15. COMAP
16. Adapted from Washington Office of the Superintendent of Public Instruction
17. COMAP
18. COMAP
19. Charles A. Dana Center and Achieve, Inc. (2007). Secondary Assessments and Tasks. *Mathematics Benchmarks, Grades K–12.* Austin: Authors. Online at `www.utdanacenter.org/k12mathbenchmarks`. Copyright Charles A. Dana Center/Agile Mind

20. Howe, Roger
21. Adapted from Washington Office of the Superintendent of Public Instruction
22. COMAP
23. COMAP
24. Charles A. Dana Center and Achieve, Inc. (2007). Secondary Assessments and Tasks. *Mathematics Benchmarks, Grades K–12.* Austin: Authors. Online at `www.utdanacenter.org/k12mathbenchmarks`.
25. Adapted from Washington Office of the Superintendent of Public Instruction
26. Charles A. Dana Center and Achieve, Inc. (2007). Secondary Assessments and Tasks. *Mathematics Benchmarks, Grades K–12.* Austin: Authors. Online at `www.utdanacenter.org/k12mathbenchmarks`.
27. Burkhardt, Hugh
28. COMAP
29. Charles A. Dana Center and Achieve, Inc. (2007). Secondary Assessments and Tasks. *Mathematics Benchmarks, Grades K–12.* Austin: Authors. Online at `www.utdanacenter.org/k12mathbenchmarks`.
30. COMAP
31. Adapted from Shell Centre for Mathematical Education/Mathematics Assessment Resource Service (MARS)
32. Adapted from Shell Centre for Mathematical Education/Mathematics Assessment Resource Service (MARS)

Statistics

Core Concepts

A Statistical methods take variability into account to support making informed decisions based on quantitative studies designed to answer specific questions.

B Visual displays and summary statistics condense the information in data sets into usable knowledge.

C Randomness is the foundation for using statistics to draw conclusions when testing a claim or estimating plausible values for a population characteristic.

D The design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions.

Core Skills

1. Formulate questions that can be addressed with data. Identify the relevant data, collect and organize it to respond to the question.
2. Use appropriate displays and summary statistics for data.
3. Interpret data displays and summaries critically; draw conclusions and develop recommendations.
4. Draw statistical conclusions involving population means or proportions using sample data.
5. Evaluate reports based on data.

Example Tasks

1. **Core Concept A, Core Skill 5**.
   An American Automobile Association report includes the graph shown below.

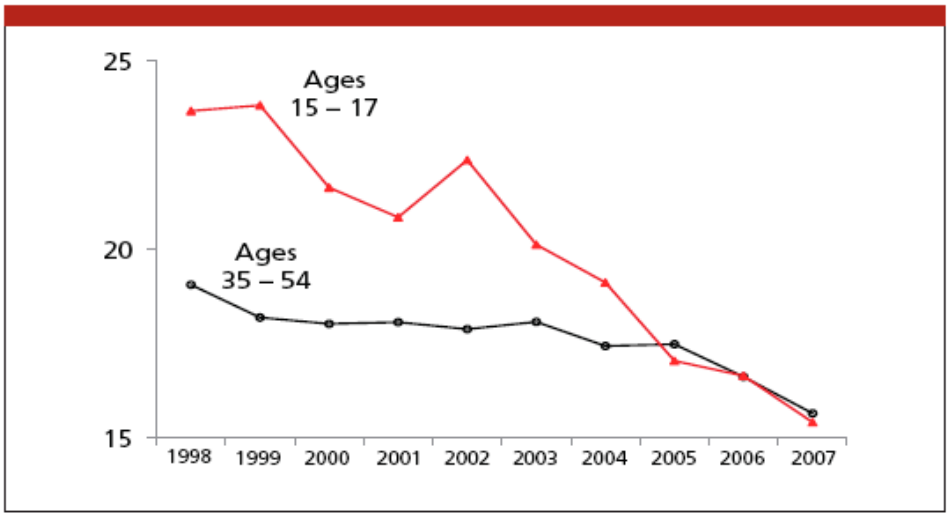

Figure 1: Number of drivers involved in fatal crashes per 100,000 population,1998–2007.

(a) Provide a written version of the information provided by this graph.
(b) The graph seems to imply that teen-aged drivers are becoming just as good as adult drivers in the 35-54 age group. Is that necessarily true? What other variables may have to be taken into account in comparing drivers for the two age groups?

(c) One of the variables that may have been listed in part 2 is the number of miles driven. How do you think rates reported in "number of drivers involved in fatal crashes per 100,000 miles driven" would differ from the rates on the graph?

2. **Core Concept B, Core Skill 2**.

Suppose the summary statistics for the number of inches of rainfall in Northampton, MA for the past 117 years, beginning in 1877, are given below.

N	Mean	Median	Min	Max	Q1	Q3
117	42.8	41.4	22.5	68.0	37.3	48.4

(a) Sketch a boxplot for these data and describe the key features of the data distribution.
(b) The news media reported that in a particular year, there were only 38 inches of rainfall. Is it appropriate for the news media to use the word only in the statement? In other words, is 38 inches an unusually small amount of rainfall for Northampton? Explain your answer.

3. **Core Concept B, Core Skill 2**.

Students in Ms. Garth's Algebra II class wanted to see if there are correlations between test scores and height and between test scores and time spent watching television. Before the students began collecting data, Ms. Garth asked them to predict what the data would reveal. Answer the following questions that Ms. Garth asked her class.

- Do you think students' heights will be correlated to their test grades? If you think a correlation will be found, will it be a positive or negative correlation? Will it be a strong or weak correlation?
- Do you think the average number of hours students watch television per week will be correlated to their test grades? If you think a correlation will be found, will it be a positive or negative correlation? Will it be a strong or weak correlation?

The students then created a table in which they recorded each student's height, average number of hours per week spent watching television (measured over a four-week period), and scores on two tests. Use the actual data collected by the students in Ms. Garth's class, as shown in the table below, to answer the following questions.

Student	Height (in inches)	TV hrs/week (average)	Test1	Test2
1	60	30	60	70
2	65	12	80	85
3	51	30	65	75
4	76	20	85	85
5	66	10	100	100
6	72	20	78	88
7	59	15	75	85
8	58	12	95	90
9	70	15	75	90
10	67	11	90	90
11	65	16	90	95
12	71	20	80	85
13	58	19	75	85

(a) Which pairs of variables seem to have a positive correlation? Explain.
(b) Which pairs of variables seem to have a negative correlation? Explain.
(c) Which pairs of variables seem to have no correlation? Explain.
(d) For each pair of variables listed below, create a scatter plot with the first variable shown on the y-axis and the second variable on the x-axis. Are the two variables correlated positively, correlated negatively, or not correlated? Determine whether each scatter plot suggests a linear trend. Do the plots confirm your decisions in parts (a)-(c) above?
- Score on test 1 versus hours watching television
- Height versus hours watching television
- Score on test 1 versus score on test 2
- Hours watching television versus score on test 2
- Height versus score on test 1

(e) Using the statistical functions of your graphing calculator, determine a line of best fit for each scatter plot that suggests a linear trend.

4. **Core Concept B, Core Skill 2, Core Skill 3**.

The table below[1] shows the reported high school graduation rates (percent of 9th grade cohorts who actually graduate), the per-pupil expenditure, PPE (in hundreds of dollars), and the percent of total taxable resources the state spends on education, PTR, for a sample of 15 states.

State	Graduation Rate	PTR	PPE
Alabama	59.0	3.4	79.24
Alaska	65.1	3.5	85.62
California	70.7	3.3	70.81
Colorado	74.6	3.0	79.39
Florida	60.5	3.0	75.39
Kansas	74.4	3.8	88.62
Kentucky	70.0	3.4	79.78
Louisiana	61.4	2.9	85.82
Michigan	69.1	4.5	91.97
Montana	76.2	3.7	89.51
Nevada	54.0	2.8	71.41
New Hampshire	76.0	3.9	93.23
New Mexico	60.1	3.7	84.31
North Dakota	79.4	3.1	91.81
Wisconsin	77.3	4.1	101.99

(a) Use technology to plot Graduation Rate versus PPE on a scatter plot that allows prediction of Graduation Rate from PPE and describe the shape of the distribution.
(b) If a linear shape is apparent, use statistical software or calculator capability to fit a least squares line to summarize the nature of that trend.

[1] Source: EPE Research Center, Education Week

(c) Interpret the slope of the least squares line found in part 2 in the context of the data.
(d) Is the relationship between Graduation Rate and PPE stronger than the one between Graduation Rate and PTR? Explain.

5. **Core Concept B, Core Skill 3**.

How did housing prices change during the turbulent economic times around 2008? One way to answer the question is to compare median prices for a variety of locations over a short time span. The data display below shows box plots of the median sales price (in thousands) of existing single-family houses for a sample of 25 metropolitan areas around the United States at mid-year 2005 and 2008.

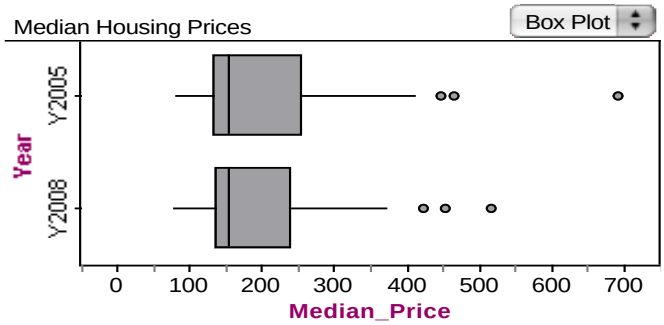

Figure 2

(a) Describe the key features of the shift in prices from 2005 to 2008.
(b) The medians of the distributions appear to differ by only a very small amount. Do you think the means would differ by a larger amount? Explain your reasoning.
(c) Why do you think the median is often used for housing prices rather than the mean?
(d) The accompanying scatter plot shows the median housing prices for 25 metropolitan areas in 2005 paired with the prices in 2008.
  (i) Describe the shape, trend and strength of the relationship between these two variables.
  (ii) Sketch the line y=x on this plot. What does the plot show with regard to the nature of the change in median prices?

Figure 3

6. **Core Concept B, Core Skill 3**.
The histogram below shows the mean annual temperatures in Pasadena for the years 1951 to 2000.

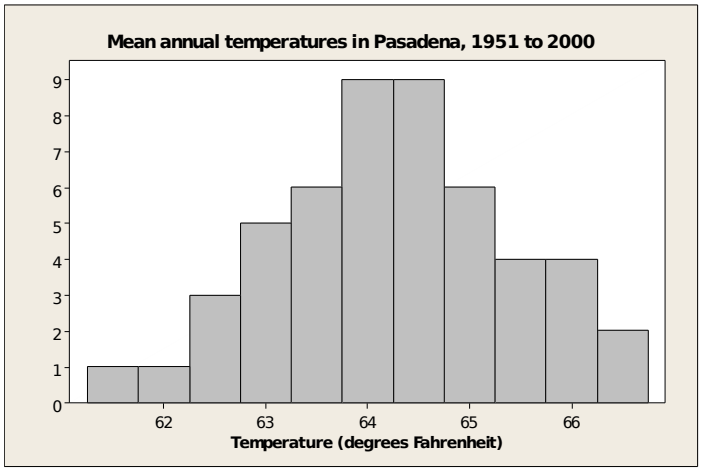

Figure 4

(a) Describe the distribution of mean annual temperatures at Pasadena.
(b) The scatter plot below shows the mean annual temperatures in Pasadena for the years 1951 to 2000. What do we learn from the scatter plot that is not obvious from the histogram.

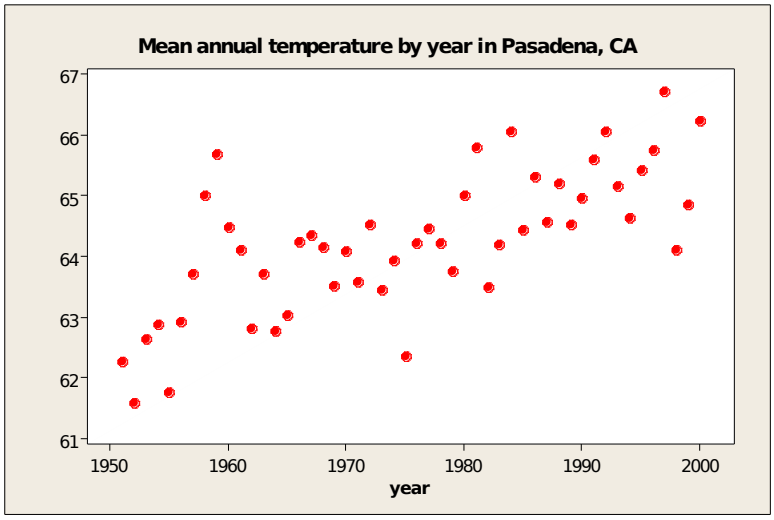

Figure 5

(c) What do we learn from the histogram that is not obvious from the scatter plot.

7. **Core Concept B, Core Skill 3**.

To determine the amount of sugar in a typical serving of breakfast cereal, a student randomly selected 60 boxes of different types of cereal from the shelves of a large grocery store. The student noticed that the side panels of some of the cereal boxes showed sugar content based on one-cup servings, while others showed sugar content based on three-quarter-cup servings. Many of the cereal boxes with side panels that showed three-quarter-cup servings were ones that appealed to young children, and the student wondered whether there might be some difference in the sugar content of the cereals that showed different-size servings on their side panels.

To investigate the question, the data were separated into two groups. One group consisted of 29 cereals that showed one-cup serving sizes; the other group consisted of 31 cereals that showed three-quarter-cup serving sizes. The boxplots shown below display sugar content (in grams) per serving of the cereals for each of the two serving sizes.

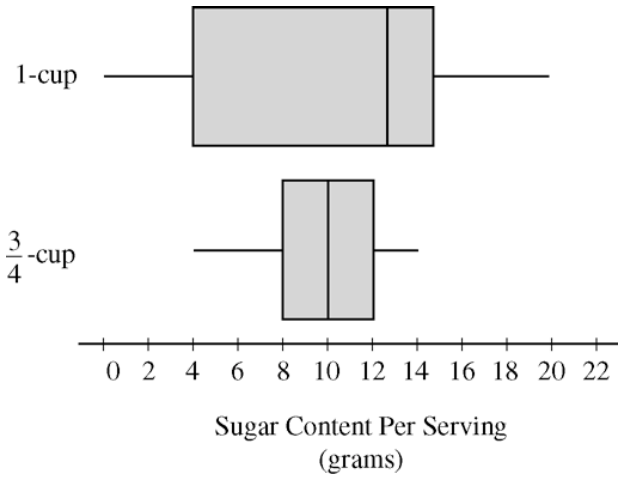

Figure 6

(a) Compare the distributions of sugar content per serving for the two serving sizes of cereals.

After analyzing the boxplots above, the student decided that instead of a comparison of sugar content per recommended serving, it might be more appropriate to compare sugar content for equal-size servings. To compare the amount of sugar in serving sizes of one cup each, the amount of sugar in each of the cereals showing three-quarter-cup servings on their side panels was multiplied by $4/3$. The bottom boxplot shown below displays sugar content (in grams) per cup for those cereals that showed a serving size of three-quarter-cup on their side panels.

(b) What new information about sugar content do the boxplots below provide?

(c) Based on the boxplots below, how would you expect the mean amounts of sugar per cup to compare for the different recommended serving sizes? Explain your reasoning.

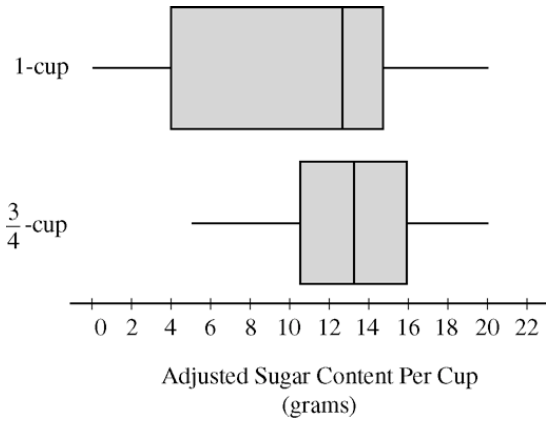

Figure 7

8. **Core Concept B, Core Skill 3**.

Two pain relievers, A and B, are being compared for relief of post-surgical pain. Twenty different strengths (doses in milligrams) of each drug were tested. Eight hundred post surgical patients were randomly divided into 40 different groups. Twenty groups were given drug A. Each group was given a different strength. Similarly, the other twenty groups were given different strengths of drug B. Drug strengths used ranged from 210 to 400 milligrams. Thirty minutes after receiving the drug, each patient was asked to describe his or her pain relief on a scale of 0 (no decrease in pain) to 100 (pain totally gone).

The strength of the drug, given in milligrams, and average pain rating for each group are shown in the scatter plot below. Drug A is indicated with A's and drug B with B's.

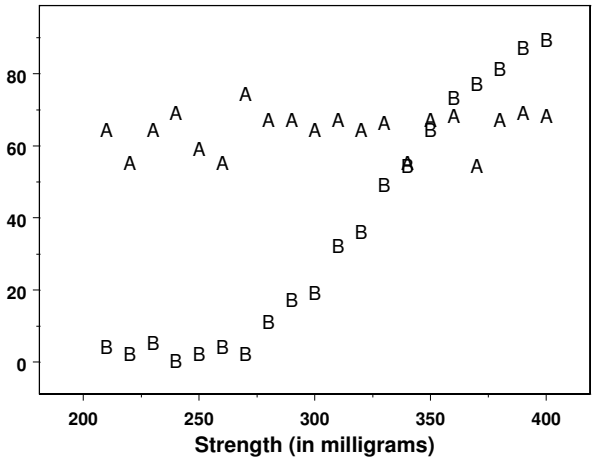

Figure 8

(a) Based on the scatter plot, carefully describe the effect of drug A and how it is related to strength in milligrams.

(b) Based on the scatter plot, carefully describe the effect of drug B and how it is related to strength in milligrams.

(c) Which drug would you give and at what strength, if the goal is to get pain relief of at least 50 at the lowest possible strength? Justify your answer based on the scatter plot.

9. **Core Concept B, Core Skill 3**.

SAT mathematics scores for a particular year are approximately normally distributed with a mean of 510 and a standard deviation of 100.

(a) What is the probability that a randomly selected score is greater than 610? Greater than 710? Between 410 and 710?

(b) If a student is known to score 750, what is the student's percentile score (the proportion of scores below 750)?

10. **Core Concept C, Core Skill 3, Core Skill 4**.

There is little doubt that caffeine stimulates bodily activity, but how much does it take to produce a significant effect? This is a question that involves measuring the effect of two or more interventions (generally called treatments) and deciding if the different interventions have differing effects. To obtain a partial answer to the question on caffeine, it was decided to compare two levels of caffeine with a control of no caffeine on a response to a finger tapping exercise.

Thirty male students were randomly assigned to one of three treatment groups of 10 students each. Each group was given one of three doses of caffeine (0, 100, and 200 milligrams) and two hours later the students were given a finger tapping exercise. The response is the number of taps per minute, as shown in the table below.[2]

Finger taps per minute from a caffeine experiment

0 mg caffeine	100 mg caffeine	200 mg caffeine
242	248	246
245	246	248
244	245	250
248	247	252
247	248	248
248	250	250
242	247	246
244	246	248
246	243	245
242	244	250
Mean:244.8	246.4	248.3

The accompanying plot shows that the data sets tend to be somewhat symmetric and have no extreme data points (outliers) that would have undue influence on the analysis. The sample mean, then, is a suitable measure of center, and will be used as the statistic for comparing treatments.

[2]Source: Draper and Smith (1981) Applied Regression Analysis, John Wiley and Sons

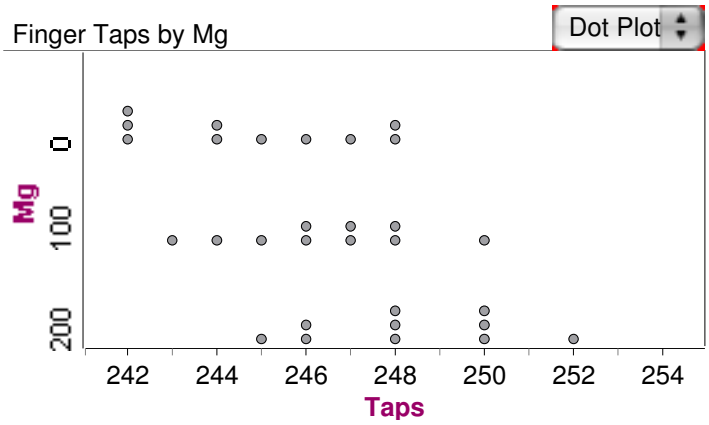

Figure 9

(a) The mean for the 100 mg data is 1.6 taps larger than that for the 0 mg data. In light of the variation in the data, is that enough to be confident that the 100 mg treatment truly results in more tapping activity than the 0 mg treatment? In other words, could this difference of 1.6 taps be explained simply by the randomization (the luck of the draw, so to speak) rather than any real difference in the treatments?

Assume for the moment that the 100 mg treatment has no effect on the tapping, and the difference in the means is simply due to the randomization. Would a difference of 1.6 units be likely to occur under these conditions? An empirical answer to this question can be found by re-randomizing the first two groups many times and studying the distribution of differences in sample means. The re-randomizing is accomplished by combining the data in the first two columns, randomly splitting them into two groups of ten each representing 0 and 100 mg, and then calculating the difference in the sample means. (This can be expedited with the use of technology.)

The plot below shows the differences produced in 200 re-randomizations of the data for 100 and 0 mg. The observed difference of 1.6 taps is equaled or exceeded 18 out of 200 times, 9% of the runs. Do these data provide strong evidence to reject the claim that the control and the 100 mg treatment do not differ with respect to their mean finger tapping counts? Explain your reasoning and the nature of the error that you may have made.

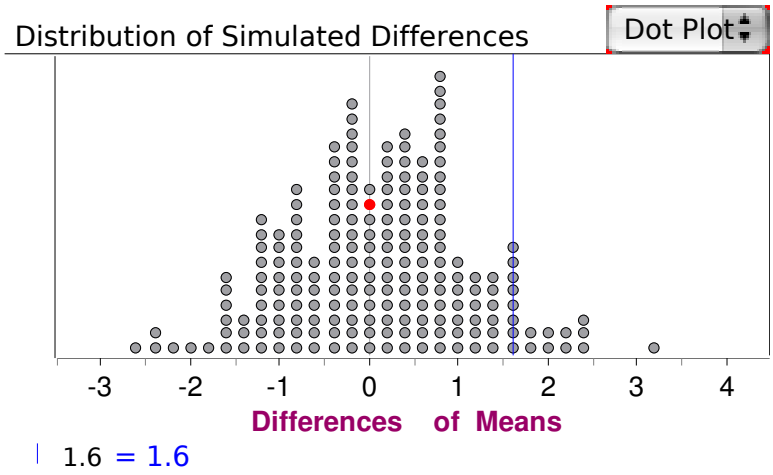

Figure 10

(b) What about the 200 mg treatment as compared to 0 mg? Construct a similar re-randomization process to decide if the difference between the mean of the 200 mg treatment and that of the 0 mg treatment is to large to be explained by chance alone. Explain your decision process and the nature of the error you may have made.

11. **Core Concept C, Core Skill 4**.

A random number generator is said to generate even or odd digits independently and each with probability 0.5. Consider the following scenarios and questions:
I. The generator generates three odd digits in a row, with probability $(.5)^3 = 0.125$ under the model given above. Does this suggest that the model is questionable?
II. The generator generates four odd digits in a row, with probability $(.5)^4 = 0.0625$ under the model given above. Does this suggest that the model is questionable?
III. The generator generates five odd digits in a row, with probability $(.5)^5 = 0.03125$ under the model given above. Does this suggest that the model is questionable?

(a) At which point in the sequence of odd digits would you begin to question the truth of the model? (If five odds in a row are not enough, continue the sequence until doubt sets in.)
(b) Write a short description explaining the interactive roles of probability and data in the decision-making process.

12. **Core Concept C, Core Skill 4**.

A random sample of 100 students from a specific high school resulted in 45% of them favoring a plan to implement block scheduling. Is it plausible that a majority of the students in the school actually favor the block schedule? Simulation can help answer the questions.

The accompanying plot shows a simulated distribution of sample proportions for samples of size 100 from a population in which 50% of the students favor the plan, and another distribution from a population in which 60% of the students favor the plan. (Each simulation contains 200 runs.) What do you conclude about the plausibility of a population proportion of 0.50 when the sample proportion is only 0.45? What about the plausibility of 0.60 for the population proportion?

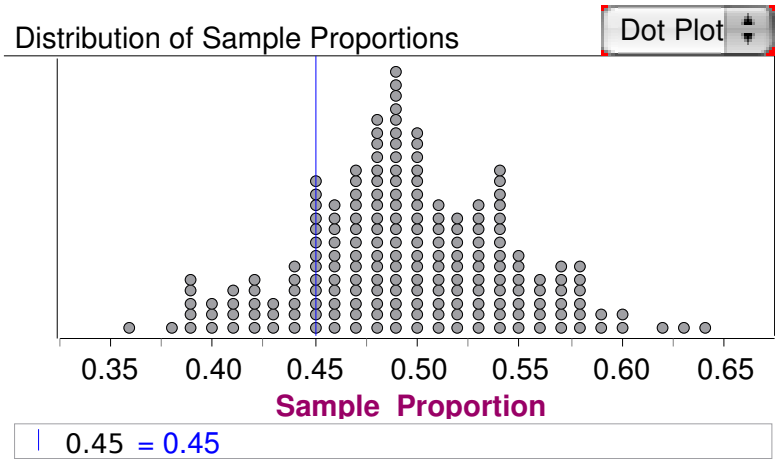

Figure 11

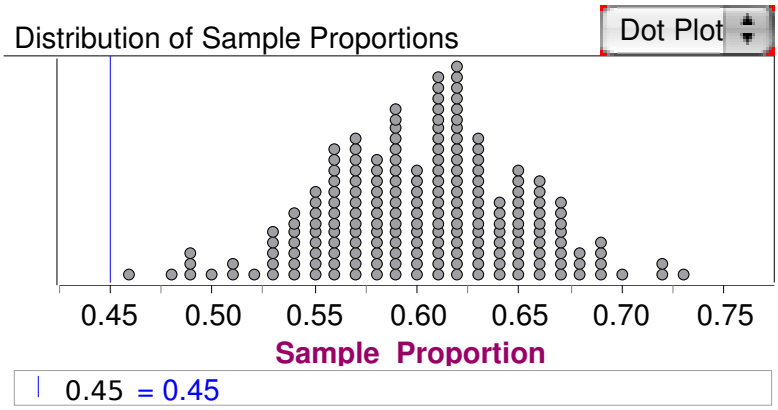

Figure 12

13. **Core Concept C, Core Skill 4**.

A tremendous amount of bottled water is being consumed these days. Do the consumers really think bottled water tastes better than city water or well water? Or, is this just a fad? To test tastes preferences in one school, $n = 50$ students were given samples of bottled water, well water from typical area homes having wells, and city water. Each student tasted all three types of water in random order. The results: 26 preferred bottled water, 16 preferred city water and 8 preferred well water.[3]

(a) Suppose the students running the experiment decided to check the claim that students had no preference for bottled water over non-bottled water. Under this claim and the randomization of the order of tasting, the preference for bottled over non-bottle water should have probability of about $p = 0.5$. The probabilities for X, the number of students choosing bottled water when $n = 50$ and $p = 0.5$, is shown below, with a vertical line at $X = 26$.

[3] Source: http://library.thinkquest.org/04apr/00222/text/survey.htm

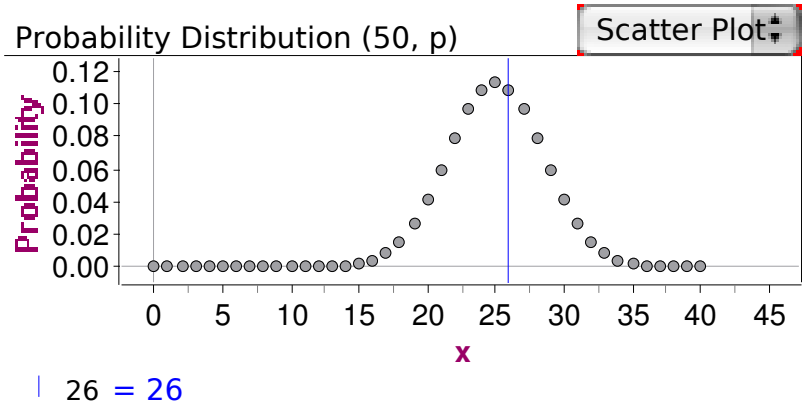

Figure 13

Does this provide strong evidence to reject the claim that there is no preference for bottled water? Explain your reasoning.

(b) Some students suggest that making this into a two-choice problem is incorrect, because there are really three choices for each taster. So, the correct claim to test is that students have no preference among bottled water, well water and city water. Under this claim the preference for bottled water should have probability of about $p = 0.33$. The probabilities for X, the number of students choosing bottled water with $n = 50$ and $p = 0.33$, is shown below, with a vertical line at $X = 26$.

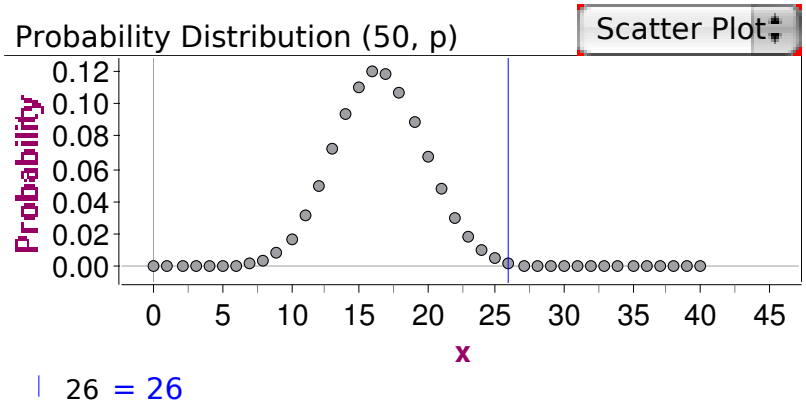

Figure 14

Does this provide strong evidence to reject the claim of no preference for bottled water among the three choices? Explain your reasoning.

(c) Which of the two claims outlined above seems more appropriate for this experiment?

(d) In the decisions made above, explain the error that could have been made.

14. **Core Concept D**.

From a class containing 12 girls and 10 boys, three students are to be selected to serve on a school disciplinary panel. Here are four different methods of making the selection.

I. Select the first three names on the class roll.
II. Select the first three students who volunteer.
III. Place the names of the 22 students in a hat, mix them thoroughly, and select three names from the mix.
IV. Select the first three students who show up for class tomorrow.

Which is the best sampling method, among these four, if you want the school panel to represent an unbiased view of the opinions of your class.

15. **Core Concept D, Core Skill 1**.

Biology students are to conduct a study of the effect of different durations of light and dark on the growth of radish seedlings. To pare the number of possible durations down to a manageable level, the students decided to focus the question on three different durations: 24 hours of light, 12 hours of light and 12 hours of darkness, and 24 hours of darkness. Plastic bags are to be used as growth chambers; there are 120 radish seeds available for the study.

(a) Is the best design for this study a sample survey, an experiment, or an observational study? Explain your reasoning.
(b) Describe how you would design a study to compare the three duration schemes, using all 120 seeds and as many plastic bags as you need.

16. **Core Concept D, Core Skill 1**.

Students in a high school mathematics class decided that their term project would be a study of the strictness of the parents or guardians of students in the school. Their goal was to estimate the proportion of students in the school who thought of their parents or guardians as "strict". They do not have time to interview all 1000 students in the school, so they plan to obtain data from a sample of students.

(a) Is the best design for this study a sample survey, an experiment, or an observational study? Explain your reasoning.
(b) The students quickly realized that, as there is no definition of "strict", they could not simply ask a student, "Are your parents or guardians strict?" Write three questions that could provide objective data related to strictness.
(c) Describe an appropriate method for obtaining a sample of 100 students, based on your answer in part (a) above.

17. **Core Skill 1, Core Skill 2**.

The data in the accompanying table shows the fate of 1316 passengers on the Titanic described in terms of two categorical variables, class of travel and survival. Use these data to answer the questions posed below.

	Class of Travel			
Survived	First	Second	Third	Total
Yes	203	118	178	499
No	122	167	528	817
Total	325	285	706	1316

(a) Do these data come from a sample survey, an experiment or an observational study? Explain your reasoning.
(b) What proportion of the passengers survived? What proportion of the passengers traveled in first class?
(c) Construct a table of (conditional) relative frequencies by column. Interpret the results in context and use them to discuss the association between class of travel and survival.
(d) Construct a table of (conditional) relative frequencies by row. Do these result in a similar interpretation with regard to association as that in part (c)?

18. **Core Skill 1, Core Skill 2, Core Skill 3**.

A simple random sample of 100 high school seniors was selected from a large school district. The gender of each student was recorded, and each student was asked the following questions.

- Have you ever had a part-time job?
- If you answered yes to the previous question, was your part-time job in the summer only?

The responses are summarized in the table below.

	Gender		
Job Experience	Male	Female	Total
Never had a part-time job	21	31	52
Had a part-time job during summer only	15	13	28
Had a part-time job but not only during summer	12	8	20
Total	48	52	100

(a) Construct a graphical display that shows the relationship between gender and job experience for the students in the sample.
(b) Write a few sentences summarizing what the display in part (a) reveals about the relationship between gender and job experience for the students in the sample.
(c) Is it appropriate to use these data to construct an estimate of the proportion of seniors in the school district who never had a part-time job? Why or why not?

19. **Core Skill 1, Core Skill 3**.

The 54 students in a middle school class were asked two questions about musical preferences: "Do you like rock?" "Do you like rap?" The responses are summarized in the table below.

(a) Is this a sample survey, an experiment, or an observational study?
(b) What percentage of the students in the class like rock?
(c) Does there appear to be a positive association between liking rock and liking rap for this class (i.e., do the students who like rock also tend to like rap)? Justify your answer by pointing out a feature of the table that supports it.
(d) Do you think the results for this class would generalize to the entire middle school? To a high school class in a nearby school? Explain your reasoning.

	Like Rap		
Like Rock	Yes	No	Row Totals
Yes	27	6	33
No	4	17	21
Column Totals	31	23	54

20. **Core Skill 1, Core Skill 5**.

What are the consequences of delayed defibrillation for those who have cardiac arrest? In a study of 6789 patients, at 369 hospitals, who had experienced cardiac arrest, 2045 had to wait longer than the recommended two minute maximum for defibrillation. Among those receiving delayed defibrillation, 22.2% survived to hospital discharge. Among those whose defibrillation was not delayed, 39.3% survived to discharge. The study concluded that, "Delayed defibrillation is common and is associated with lower rates of survival after in-hospital cardiac arrest."

(a) Is this study a sample survey, an experiment or an observational study?

(b) Note the use of the phrase "is associated with" in the study conclusion. Would it be correct to replace this phrase with "causes"? Why or why not?

21. **Core Skill 2, Core Skill 3**.

Two gasoline additives were tested to check their claims that their use will increase gasoline mileage in cars. Thirty cars were randomly selected and each was filled with gasoline. The cars were run until the gas tanks were empty. The distance traveled was recorded for each car.

Additive A was randomly assigned to 15 of the cars and the remaining 15 cars got additive B. The cars were again run under the same driving conditions until the tanks were empty and the miles driven was recorded. For each car the difference in miles driven was calculated as miles with additive minus miles without the additive. If that number was greater than zero it meant that the car went a greater distance with the additive than without it. Negative differences meant that the car went a greater distance without the additive than with it.

The following table summarizes the calculated differences.

Additive	Values Below Q1	Q1	Median	Q3	Values Above Q3
A	-10, -8, -2	1	3	4	5, 7, 9
B	-5, -3, -3	-2	1	25	35, 37, 40

(a) On the grid below, display parallel boxplots (showing outliers, if any) of the differences for the two additives. Make sure that you label the plots so that we can tell which one is A and which is B.

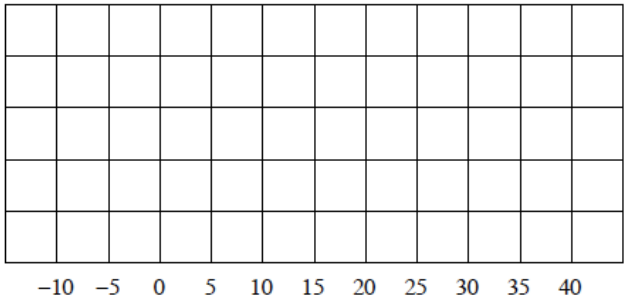

Figure 15

(b) Two ways that the effectiveness of a gasoline additive can be evaluated are by looking at

- The proportion of cars that have increased gas mileage when the additive is used in those cars
- The average mean increase in gas mileage when the additive is used in those cars.

(i) Which additive, A or B, would you recommend if the goal is to increase gas mileage in the highest proportion of cars? Explain your answer.

(ii) Which additive, A or B, would you recommend if the goal is to have the highest average (mean) increase in gas mileage? Explain your answer.

22. **Core Skill 3**.

The accompanying graph shows the relationship between scores on Exam 1 and scores on Exam 2 for all the students studying statistics from the same teacher in a recent year. The darker line (with slope 1) is the $y = x$ line. The lighter line (slope 0.6) is the least squares line of best fit.

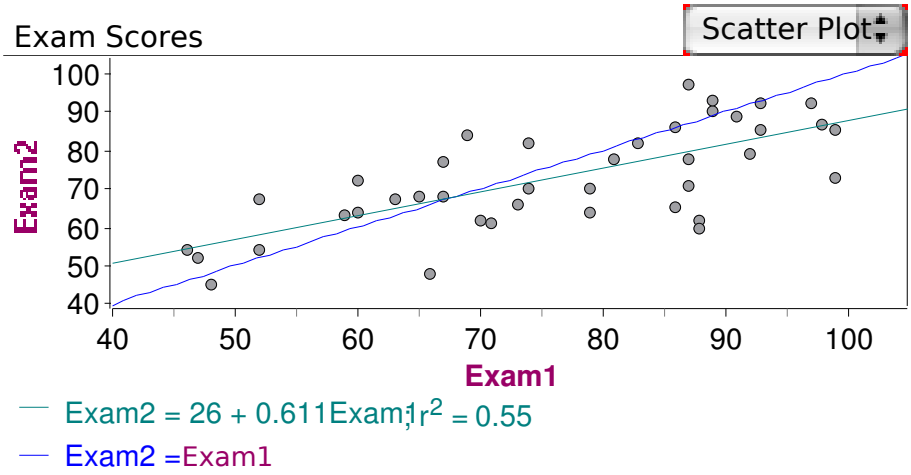

Figure 16

(a) What does the pattern tell you about the students who scored 60 or below on the first exam? What does the pattern tell you about the students who scored 80 or above on the first exam?
(b) Did more students improve their score or lower their score as they moved from exam 1 to exam 2? What feature of the plot leads to you answer of this question?
(c) Make a concise statement as to why the slope of the least squares line is less than 1.

23. **Core Skill 3**.

Gas chromatography is a technique used to detect very small amounts of a substance. To study the calibration of a gas chromatograph, five measurements were taken for each of four specimens containing different but known amounts of the substance being studied. A graph of the output readings from the gas chromatograph versus the input amounts is shown on the accompanying plot.[4]

(a) Based on just the scatter plot and the regression line, does the simple linear model appear to provide a good description of the performance of the chromatograph?
(b) On studying the residual plot, what concerns might you have about the simple linear regression line as a model for the performance of the chromatograph? (Residuals are deviations of the points from the regression line.)

[4]Source: http://lib.stat.cmu.edu/DASL/Datafiles/Chromatography.html

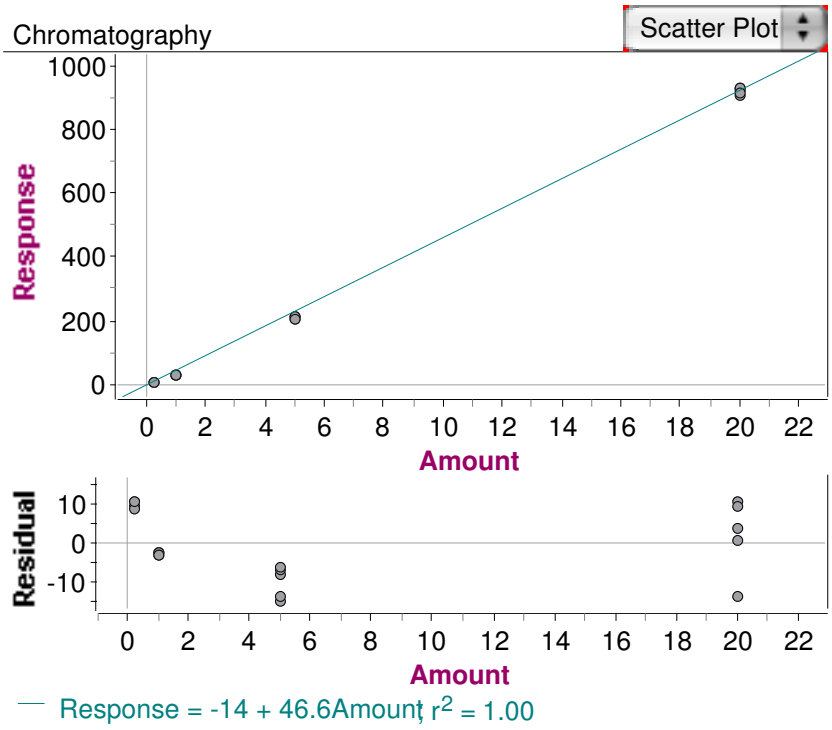

Figure 17

24. **Core Skill 3**.

Which of the following data sets are not good candidates to be modeled by a normal distribution? Explain your reasoning in each case.

(a) Salaries of employees of a large corporation.
(b) The life lengths of batteries of a specific brand used in laptop computers.
(c) Measurements of the diameters of 15-inch wheels produced by an automobile manufacturer.
(d) Prices of single-family houses in your city or county.

25. **Core Skill 3, Core Skill 4**.

Each of the simulated distributions generated above can be modeled by a normal distribution with center at the fixed population proportion, p, used to generate the distribution and standard deviation (SD) of approximately 0.05. From properties of the normal distribution it can be seen that about 95% of the possible values of a sample proportion lie within approximately 2SD of their mean (p).

(a) Letting p denote a population proportion and an observed sample proportion, argue that any population with a value of p greater than $p + 2$SD would not produce the observed within its middle 95% of possible outcomes. (In the context of Block Scheduling, and using an arbitrary but common definition of plausible, any value of p greater than $0.45+0.10 = 0.55$ would not be a plausible value for the population proportion of students favoring the block scheduling plan.)
(b) Letting p denote a population proportion and an observed sample proportion, argue that any population with a value of p less than $p - 2$SD would not produce the observed within its middle 95% of possible outcomes. (In the context of Block Scheduling, any value of p less than $0.45 - 0.10 = 0.35$ would not be a plausible value for the population proportion of students favoring the block scheduling plan.)
(c) Based on parts (a) and (b), argue that the interval

$$p \pm 2\text{SD}$$

forms an interval of plausible values for the true population proportion. That is, the interval $p \pm 2$SD contains about 95% of the possible values of the sample proportions that can be generated from a population with true proportion p. This 2SD bound is commonly referred to as the margin of error.

(d) In general, will the margin of error increase or decrease as the sample size increases? Explain your reasoning.

26. **Core Skill 5**.

Researchers have noticed that the number of golf courses and the number of divorces in the United States are strongly correlated and both have been increasing over the last several decades. Can you conclude that the increasing number of golf courses is causing the number of divorces to increase? Explain your answer.

27. **Core Skill 5**.

The following appeared in the Sunday Times of London:

> Research finds the website [Facebook] is damaging students' academic performance. . . . Facebook users . . . are more likely to perform poorly in exams, according to new research. . . . The majority of students who use Facebook every day are underachieving by as much as an entire grade compared with those who shun the site.

This story was then picked up by many other newspapers and became a popular headline across the United States and beyond.

(a) What elements of the story in the Times sound like cause-and-effect arguments?

The source of the data in this case was the report of a study at the Ohio State University entitled, "A Description of Facebook Use and Academic Performance Among Undergraduate and Graduate Students." A total of 219 participants, mostly from selected classes in the school of education whose instructors agreed to participate, constituted the sample of this pilot study. The sample included both graduate and undergraduate students, with self-reported data on both how much they studied and grade point averages. The data were reported in ranges, like 3.0 to 3.5 for grade point averages. [5]

(b) Many of the weaknesses of the study were reported by the authors. What do you think some of these weaknesses are?

(c) Was the Sunday Times correct in reporting the results in cause-and-effect style? Explain your reasoning.

28. **Core Skill 5**.

A presidential campaign advertisement for former New York City mayor Rudy Giuliani said, "I had prostate cancer, 5, 6 years ago. My chance of surviving prostate cancer-and thank God, I was cured of it-in the United States was eighty-two percent. My chance of surviving prostate cancer in England was only 44 percent under socialized medicine."

The quoted percentages were based on 5-year survival rates, not mortality rates, reported from studies in the United States and the United Kingdom, respectively. But, these 5-year survival rates have little to do with mortality rates, as can be seen from the definitions given below:

$$\text{5-year survival rate} = \frac{\text{number of patients diagnosed with cancer still alive 5 years after diagnosis}}{\text{number of patients diagnosed with cancer}}$$

$$\text{annual mortality rate} = \frac{\text{number of people who died from cancer over 1 year}}{\text{number of people in the group}}$$

(a) How could changes in medical practice increase the 5-year survival rates without changing the mortality rate for any given year?

[5] Source:http://online.wsj.com/article/SB124034974305240495.html

(b) In the US, most prostate cancer is detected by screening for prostate-specific antigens (PSA). In the UK, most prostate cancer is detected by symptoms. How might this difference in medical practice affect their respective 5-year survival rates?
(c) Was Mr. Giuliani making a fair comparison? Explain your reasoning.

29. **Core Skill 5**.

"Parents Rate Schools Much Higher Than Do Americans Overall" is the title of a report on a Gallup poll of August 24, 2009. The results of the poll show that "three in four American parents (76%) are satisfied with the education their children receive in school, compared to 45% of the general public who are satisfied with the state of schools nationwide." An explanation of how the poll was conducted includes the following.

> Results are based on telephone interviews with $1{,}010$ national adults, aged 18 and older, conducted Aug. 6-9, 2009. For results based on the total sample of national adults, one can say with 95% confidence that the margin of error is ± 4 percentage points. For results based on the sample of 233 parents with children in kindergarten through grade 12, the maximum margin of sampling error is ± 8 percentage points.
>
> Interviews are conducted with respondents on land-line telephones (for respondents with a land-line telephone) and cellular phones (for respondents who are cell-phone only).
>
> In addition to sampling error, question wording and practical difficulties in conducting surveys can introduce error or bias into the findings of public opinion polls.

(a) Explain the meaning of the 4% margin of error in the context of this poll.
(b) Why is the margin of error for the poll of parents with children in school larger than that for the overall poll?
(c) Why is it important to mention both cellular and land-line phones?
(d) Other than error due to the fact the data comes from a sample, describe other potential sources of error in a poll of this nature.

30. **Core Skill 5**.

In 1962, research statistics showed that the percentage of obesity in America's population was at 13%. By 1980 it has risen to 15%, by 1994 to 23%, and by the year 2000 the obesity progression in America had reached an unprecedented 31%. The U.S. Surgeon General report declared that obesity is responsible for $300{,}000$ deaths every year. These overwhelming research statistics reveal an alarming obesity trend, the need for diagnosis, and a call to action.

In the midst of an informational and research feeding frenzy on the obesity epidemic, statistics are easy to come by. The most widely disseminated CDC research statistics on American obesity tell us that 63% of adult Americans have a Body Mass Index (BMI) in excess of 25.0 and are therefore overweight; more than a quarter surpass 30.0, having been declared obese. And perhaps the most riveting statistics concern obesity in kids: research shows that childhood obesity has more than tripled over the past two decades.

Still, by focusing on the polite abstraction of Body Mass Index rather than actual bodyweight, the CDC has hindered the war on the prevention of obesity in America. Indeed, CDC research epidemiologists are faithful keepers of the public health record; but for reasons of technical precision and political propriety, they have scrupulously avoided the publication of the most crucial and powerful obesity statistics, raw bodyweight averages for the American population.

The new IHRSA/ASD Obesity/Weight Control Report has published these graphic and visceral images of a dangerously overweight population. The "real" research statistics on obesity reveal that:

- 3.8 million Americans carry over 300 pounds.
- With the average adult woman weighing in at a staggering 163!

Perhaps the most shocking statistics underscoring obesity in the United States is that $400{,}000$ Americans

(mostly men) fall into a super-massive $400+$ pound category.[6]

(a) Explain the controversy between the two measures of obesity, the BMI (which takes into account height and weight) and bodyweight averages.
(b) The percentages in the first paragraph are based on BMI. Is there any other data on the article to which these figures can be compared fairly?
(c) Does the article make a strong scientific case for using bodyweight rather than BMI to measure obesity?

Sources

1. Scheaffer, Richard
2. Adapted from AP Statistics 2001 No 1
3. Charles A. Dana Center and Achieve, Inc. (2007). Secondary Assessments and Tasks. *Mathematics Benchmarks, Grades K–12.* Austin: Authors. Online at `www.utdanacenter.org/k12mathbenchmarks`.
4. Adapted from Achieve Benchmarks revision
5. Adapted from Achieve Benchmarks revision
6. Adapted from AP Statistics 1998 No 1
7. Adapted from AP Statistics 2008 No 1
8. Adapted from AP Statistics 2000 No 1
9. Adapted from Achieve Benchmarks revision
10. Adapted from Achieve Benchmarks revision
11. Adapted from Achieve Benchmarks revision
12. Adapted from Achieve Benchmarks revision
13. Adapted from Achieve Benchmarks revision
14. Adapted from Achieve Benchmarks revision
15. Adapted from Guidelines for Assessment and Instruction in Statistics Education (GAISE)
16. Adapted by Richard Scheaffer from Mathematics Teacher
17. Adapted from Achieve Benchmarks revision
18. Adapted from AP Statistics 2009 No 1
19. Adapted from Guidelines for Assessment and Instruction in Statistics Education (GAISE)
20. Chan, P. S. et. al. (2008), Delayed Time to Defibrillation after In-Hospital Cardiac Arrest, *The New England Journal of Medicine*, 358, 1, 9-17.
21. Adapted from AP Statistics 2004 No 1
22. Scheaffer, Richard
23. Adapted from Achieve Benchmarks revision
24. Adapted from Achieve Benchmarks revision
25. Adapted from Achieve Benchmarks revision
26. Adapted from Achieve Benchmarks revision
27. Sunday Times of London
28. Gigerenzer, G, et.al. (2008), Helping Doctors and Patients Make Sense of Health Statistics, *Psychological Science in the Public Interest*, 8, 2, 53-96.

[6] Source:http://www.americansportsdata.com/obesityresearch.asp

29. Scheaffer, Richard
30. Statistics_ADP

Appendix B1-4

International Benchmarking of Common Core Standards

The following is information on international benchmarking of the Common Core Standards provided by the Council of Chief State School Officers.

International Benchmarking and the Common Core

The Common Core State Standards (CCSS) are designed to be **college- and career-ready** and **internationally benchmarked**. To that end, the development process included the review and consideration of many sources, including research studies, existing standards from the U.S and abroad, and the professional judgment of teachers, content area experts, and college faculty. This paper will briefly describe how international benchmarking was used to develop the CCSS.

What documents were used to ensure that the CCSS were internationally benchmarked?
To ensure that the standards prepare students to be globally competitive, the development team used a number of sources, including: the frameworks for PISA and TIMSS; the International Baccalaureate syllabi; the American Institutes for Research report, **Informing Grades 1-6 Mathematics Standards Development: What Can Be Learned From High-Performing Hong Kong, Korea, and Singapore** and; the A+ Composite found in **A Coherent Curriculum: The Case for Mathematics *by Bill Schmidt, Richard Houang, and Leland Cogan.***

In addition, the development team looked to the standards of a number of individual countries and provinces to inform the content, structure and language of the CCSS. In *mathematics*, twelve set of standards were selected to help guide the writing of the standards: Belgium, Canada [Alberta], China, Chinese Taipei, England, Finland, Hong Kong, India, Ireland, Japan, Korea, and Singapore.[1] In *English language arts*, the writing team looked closely at ten sets of standards from Australia (New South Wales and Victoria), Canada (Alberta, British Columbia, and Ontario), England, Finland, Hong Kong, Ireland, and Singapore.[2]

How were the international benchmarks used to inform the development of the CCSS?
The goal of the international benchmarking in the common core state standards development process was to ensure that the CCSS are as rigorous as comparable standards in the high-performing and other countries. However, the use of international benchmarks as evidence is no easy feat; it is not simply a matter of identifying the "best" source and copying it, or of aggregating all viable sources to find some set of shared expectations. Rather, international benchmarks were used to guide critical decisions in the following areas:

- *Whether particular content should be included:* One of the principal ways international standards were used in this development process was as a guide when making tough decisions about whether content should be included or excluded.
- *When content should be introduced and how that content should progress*: The progression of topics in the international mathematics standards helped the development

team make decisions about when to introduce topics in the CCSS as well as when to stop focusing on them.

- *Ensuring focus and coherence*: Standards from other countries tend to be very focused, including only what is absolutely necessary.
- *Organizing and formatting the standards*: Certain organizational aspects or characteristics of international standards that promoted clarity and ease of reading and use served as a model for the CCSS.
- *Determining emphasis on particular topics in standards*: Where emphasis on particular topics was found repeatedly in international standard, this was instructive in determining their importance for inclusion in the CCSS.

* * * * *

When the final version of the K-12 Common Core State Standards is released, it will be accompanied by a discussion of the evidence that was used in their development. In the meantime, the evidence from the September 2009 draft of the College and Career Ready Standards is available: The URL for the ELA document is http://www.corestandards.org/Files/ELAEvidence.pdf, and the URL for the mathematics document is http://www.corestandards.org/Files/MathEvidence.pdf.