Illinois Mathematics Assessment Framework PSAE Grade 11

State Assessments Beginning Spring 2006

Illinois State Board of Education

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Introduction to the Illinois Mathematics Assessment Framework PSAE Grade 11

The Illinois Mathematics Assessment Framework for PSAE Grade 11 is designed to assist educators, test developers, policy makers, and the public by clearly defining those elements of the Illinois Learning Standards that are suitable for state testing. It is not designed to replace a local mathematics curriculum and should not be considered a state mathematics curriculum. The Framework defines the mathematics content that will be assessed in the Prairie State Achievement Examination (PSAE) beginning with the 2005-06 school year.

Assessment Objectives

The Framework contains assessment objectives, clear and concise statements of testable material at grade 11. Due to practical limitations, each year's assessment will measure a sample of the content in the Framework, as well as a different subset of the content, and there will be sufficient overlap from year to year to allow annual comparisons. The assessment objectives listed may be measured in any given year. One should not presume that every objective will be measured every year. The assessment may contain multiple items that measure one or more assessment objectives. The Framework states only the range of objectives that may be assessed. The assessment objectives reflect a combination of the ACT Assessment[®] Mathematics Test and the WorkKeys[®] *Applied Mathematics* Assessment. They do not represent either the ACT Assessment Mathematics Test or the WorkKeys *Applied Mathematics* Assessment in isolation.

Content Emphasis and Reports

While the precise content on each year's assessment will vary somewhat from year to year, the relative emphasis on the State Goals and Illinois Learning Standards will not. The percents in the Mathematics Content Category Table on page 10 are typical but may not be exact for any given administration of the PSAE because the test samples the domain slightly differently each year. In addition to an overall PSAE Mathematics score, an ACT Assessment Mathematics Test score, and a WorkKeys *Applied Mathematics* Assessment score, as much information as possible will be reported.

Mathematics Assessment Structure

The mathematics portion of the PSAE is comprised of two components: the ACT Assessment Mathematics Test on Day 1 and the WorkKeys *Applied Mathematics* Assessment on Day 2. Each component contributes 50% to the total PSAE Mathematics scale score.

Framework Structure

This document employs a general organizational structure designed for ease of use. Each State Goal for mathematics is the main organizer, followed by the Illinois Learning Standards for Mathematics within each of these State Goals. Each assessment objective has a unique identifier with three components.

Example: 6.11.01

6	11	01
State Goal	Grade Level	Objective Number

The first component, "6," indicates the numbered State Goal as defined in the Illinois Learning Standards. The second component, "11," indicates the grade level. The third component, "01," indicates that this is the first assessment objective for this State Goal at this grade level.

Cognitive Complexity

Cognitive complexity refers to the level of reasoning called for by an assessment objective. The ACT Assessment Mathematics Test distributes its items among four cognitive levels:

Knowledge and Skills (KS) items test the direct use of one or more facts, definitions, formulas, or procedures.

Direct Application (**DA**) items are basically the same as KS, except that these problems are set in real-world situations. Most familiar types of word problems would fit into this category, such as problems involving percent, rate and distance, and the Pythagorean theorem.

Understanding Concepts (UC) items test depth of understanding of one or more major concepts. These items need not be difficult or complex. UC problems involve reasoning from a particular concept, perhaps drawing a conclusion or making an inference. Unlike KS and DA items, these items should appear novel to most examinees and should elicit a thoughtful, rather than a practiced, response. If guessing is discounted, an examinee would not get problems in this category correct without a significant level of understanding.

Integrated Conceptual Understanding (IC) items test understanding, as UC items do, but focus on the integration of more than one major concept. It is not sufficient for an item to test conceptual understanding of two major concepts. The item must test the relationship between the two concepts.

The distinction between KS and DA is in the setting–purely mathematical versus in-context. Setting does not distinguish UC from IC; rather, the necessity for integrating major concepts is the distinction.

Approximately 50% of the items on the ACT Assessment Mathematics Test involve Knowledge and Skills. Approximately 30% involve Direct Application, and approximately 20% involve Understanding Concepts/Integrated Conceptual Understanding. Students must apply mathematics to solve unfamiliar problems. Student must solve problems in real-world contexts and interpret the results within the context. Students must solve problems that involve connecting one area of mathematics with another.

The WorkKeys *Applied Mathematics* Assessment measures skill in applying mathematical reasoning to workrelated problems. There are five skill levels, from Level 3 to Level 7. As one moves from the lower levels to the higher levels, the mathematical concepts and calculations become more complex.

Level 3: Individuals with Level 3 skills can set up and solve problems with single-step mathematical operations (addition, subtraction, multiplication, or division) on whole numbers, fractions, decimals, or percentages.

Level 4: Individuals with Level 4 skills can set up and solve problems with two or more different mathematical operations (addition, subtraction, multiplication, or division) on whole numbers, fractions, decimals, or percentages.

Level 5: Individuals with Level 5 skills can set up and solve problems with multiple-step calculations on a mixture of whole numbers, fractions, decimals, or percentages, when the information is presented in a logical order.

Level 6: Individuals with Level 6 skills can set up and solve problems containing unnecessary information or information presented out of logical order and involving multiple-step calculations on a mixture of whole numbers, fractions, decimals, or percentages.

Level 7: Individuals with Level 7 skills can set up and solve problems requiring extensive calculations and several conversions between systems of measurement. They can calculate percent change, set up and manipulate complex ratios and proportions, find multiple areas or volumes of two- and three-dimensional shapes, find the best economic value of several alternatives, and locate errors in multiple-step calculations.

Calculator Use

The consideration of calculator usage on large–scale assessments is a topic that has received considerable attention over the past three decades. Many discussions about the appropriate use of such technology in a mathematics assessment context generally center on the topic of what constitutes "basic mathematical skills" and what students need to be able to do without the aid of technology. At the same time, students are being educated to function in a world that is becoming increasing reliant on technology. The balance of focusing on new processes while still holding some historical expectations is difficult, both for parents and teachers in such an era of flux and transition.

Research on student use of calculators in mathematics classes shows that students who have appropriate instruction on when and how to use calculators coupled with practice using them in learning mathematics have better attitudes toward mathematics and feel empowered. Using a calculator allows students to shift from allocating a lot of time and energy to calculations during working to solve problems. The use of calculators also allows teachers to present the students with more realistic problems than would otherwise be possible. Further, students who know when and how to use mental mathematics, estimation, paper–and–pencil algorithms, or technology achieve far higher scores than students who are relegated to programs that focus on memorization and practice of traditional skills in the absence of technology.¹ Although not required, calculators may be used for all items on the mathematics portion of the PSAE. Please see www.act.org for a current list of prohibited calculators.

Other Assessment Tools

A formula sheet is provided for the WorkKeys *Applied Mathematics* assessment on Day 2 of the PSAE. This formula sheet is available online at www.act.org/workkeys/assess/math/formulas.html.

¹ Mullis, I.V.S., Dossey, J. A., Owen, E. H., & Phillips, G. W. (1991). *The State of Mathematics Achievement: NAEP's 1990 Assessment of the Nation and the Trial Assessment of the States.* Washington, DC: National Center for Education Statistics.

Excerpt from Illinois Learning Standards²

The *Illinois Learning Standards for Mathematics* were developed by Illinois teachers for Illinois schools. These goals, standards and benchmarks are an outgrowth of the 1985 Illinois State Goals for Learning influenced by the latest thinking in school mathematics. This includes the National Council of Teachers of Mathematics; *Curriculum and Evaluation Standards for School Mathematics*; ideas underlying recent local and national curriculum projects; results of state, national, and international assessment findings; and the work and experiences of Illinois school districts and teachers.

Mathematics is a language we use to identify, describe and investigate the patterns and challenges of everyday living. It helps us to understand the events that have occurred and to predict and prepare for events to come so that we can more fully understand our world and more successfully live in it.

Mathematics encompasses arithmetic, measurement, algebra, geometry, trigonometry, statistics, probability and other fields. It deals with numbers, quantities, shapes and data, as well as numerical relationships and operations. Confronting, understanding and solving problems is at the heart of mathematics. Mathematics is much more than a collection of concepts and skills; it is a way of approaching new challenges through investigating, reasoning, visualizing and problem solving with the goal of communicating the relationships observed and problems solved to others.

All students in Illinois schools need to have the opportunity to engage in learning experiences that foster mastery of these goals and standards. Knowledge of mathematics and the ability to apply math skills to solve problems can be an empowering force for all students—both while in school and later in their lives. Students reaching these goals and standards will have an understanding of how numbers are used and represented. They will be able to use basic operations (addition, subtraction, multiplication, division) to both solve everyday problems and confront more involved calculations in algebraic and statistical settings. They will be able to read, write, visualize and talk about ways in which mathematical problems can be solved in both theoretical and practical situations. They will be able to communicate relationships in geometric and statistical settings through drawings and graphs. These skills will provide all Illinois students with a solid foundation for success in the workplace, a basis for continued learning about mathematics, and a foundation for confronting problem situations arising throughout their lives.

APPLICATIONS OF LEARNING

Through Applications of Learning, students demonstrate and deepen their understanding of basic knowledge and skills. These applied learning skills cross academic disciplines and reinforce the important learning of the disciplines. The ability to use these skills will greatly influence students' success in school, in the workplace and in the community.

SOLVING PROBLEMS

Recognize and investigate problems; formulate and propose solutions supported by reason and evidence. The solving of problems is at the heart of "doing mathematics." When people are called on to apply their knowledge of numbers, symbols, operations, measurement, algebraic approaches, geometric concepts and relationships, and data analysis, mathematics' power emerges. Sometimes problems appear well structured, almost like textbook exercises, and simply require the application of an algorithm or the interpretation of a relationship. Other times, particularly in occupational settings, the problems are non–routine and require some imagination and careful reasoning to solve. Students must have experience with a wide variety of problem–solving methods and opportunities for solving a wide range of problems. The ability to link the problem–solving methods learned in mathematics with a knowledge of objects and concepts from other academic areas is a fundamental survival skill for life.

² Illinois State Board of Education (1997). Illinois Learning Standards

COMMUNICATING

Express and interpret information and ideas.

Everyone must be able to read and write technical material to be competitive in the modern workplace. Mathematics provides students with opportunities to grow in the ability to read, write and talk about situations involving numbers, variables, equations, figures and graphs. The ability to shift between verbal, graphical, numerical and symbolic modes of representing a problem helps people formulate, understand, solve and communicate technical information. Students must have opportunities in mathematics classes to confront problems requiring them to translate between representations, both within mathematics and between mathematics and other areas; to communicate findings both orally and in writing; and to develop displays illustrating the relationships they have observed or constructed.

USING TECHNOLOGY

Use appropriate instruments, electronic equipment, computers and networks to access information, process ideas and communicate results.

Technology provides a means to carry out operations with speed and accuracy; to display, store and retrieve information and results; and to explore and extend knowledge. The technology of paper and pencil is appropriate in many mathematical situations. In many other situations, calculators or computers are required to find answers or create images. Specialized technology may be required to make measurements, determine results or create images. Students must be able to use the technology of calculators and computers including spreadsheets, dynamical geometry systems, computer algebra systems, and data analysis and graphing software to represent information, form conjectures, solve problems and communicate results.

WORKING ON TEAMS

Learn and contribute productively as individuals and as members of groups.

The use of mathematics outside the classroom requires sharing expertise as well as applying individual knowledge and skills. Working in teams allows students to share ideas, to develop and coordinate group approaches to problems, and to share and learn from each other in communicating findings. Students must have opportunities to develop the skills and processes provided by team problem–solving experiences to be prepared to function as members of society and productive participants in the workforce.

MAKING CONNECTIONS

Recognize and apply connections of important information and ideas within and among learning areas. Mathematics is used extensively in business; the life, natural and physical sciences; the social sciences; and in the fine arts. Medicine, architecture, engineering, the industrial arts and a multitude of occupations are also dependent on mathematics. Mathematics offers necessary tools and ways of thinking to unite the concepts, relationships and procedures common to these areas. Mathematics provides a language for expressing ideas across disciplines, while, at the same time, providing connections linking number and operation, measurement, geometry, data and algebra within mathematics itself. Students must have experiences which require them to make such connections among mathematics and other disciplines. They will then see the power and utility that mathematics brings to expressing, understanding and solving problems in diverse settings beyond the classroom. **STATE GOAL 6: Demonstrate and apply a knowledge and sense of numbers, including numeration and operations (addition, subtraction, multiplication, division), patterns, ratios and proportions.**

Why This Goal Is Important: Numbers and operations on numbers play fundamental roles in helping us make sense of the world around us. Operations such as addition, subtraction, multiplication and division, as well as the ability to find powers and roots, extend the notion of numbers to create tools to model situations and solve problems in our everyday lives. Discussing and solving problems related to budgets, comparing prices on merchandise, understanding the nature of interest charges, measuring fuel consumption and calculating the trajectory for space travel would all be impossible without a sense of numbers and numerical operations. All people must develop this sense of numbers and operations and be able to use it to solve problems using mental computation, paper–and–pencil algorithms, calculators and computers.

STANDARD 6A

Demonstrate knowledge and use of numbers and their representations in a broad range of theoretical and practical settings.

STANDARD 6B

Investigate, represent and solve problems using number facts, operations (addition, subtraction, multiplication, division) and their properties, algorithms and relationships.

STANDARD 6C

Compute and estimate using mental mathematics, paper–and–pencil methods, calculators and computers.

STANDARD 6D

Solve problems using comparison of quantities, ratios, proportions and percents.

STATE GOAL 7: Estimate, make and use measurements of objects, quantities and relationships and determine acceptable levels of accuracy.

Why This Goal Is Important: Measurement provides a way to answer questions about "how many," "how much" and "how far." It is an indispensable component of business, manufacturing, art, medicine and many other aspects of daily life. We describe the sizes, capacities and values of many things, from the large distances involved in space travel, to the very small quantities in computer design and microbiology, to the varying values of currencies in international monetary exchange. All people must be able to choose an appropriate level of accuracy for a measurement; to select what measuring instruments to use and to correctly determine the measures of objects, space and time. These activities require people to be able to use standard instruments including rulers, volume and capacity measures, timers and emerging measurement technologies found in the home and workplace.

STANDARD 7A

Measure and compare quantities using appropriate units, instruments and methods.

STANDARD 7B

Estimate measurements and determine acceptable levels of accuracy.

STANDARD 7C

Select and use appropriate technology, instruments and formulas to solve problems, interpret results and communicate findings.

STATE GOAL 8: Use algebraic and analytical methods to identify and describe patterns and relationships in data, solve problems and predict results.

Why This Goal Is Important: Algebra unites patterns and quantities in patterns with the means of describing change through the use of variables and functions. Its concepts and analytical methods allow people to consider general solutions to problems with common characteristics and develop related formulas. Algebra provides verbal, symbolic and graphical formats for discussing and representing settings as diverse as the pricing patterns of merchandise in a store, the behavior of a car as it accelerates or slows down, the changes in two chemicals as they react with one another, or the type of variation existing in a comparison of two factors in the economy. All people must be able to use algebraic methods to construct and examine tables of values; to interpret the relationships expressed by patterns in these tables; to relate change and variation in graphs and formulas; to reason about changes in quantities and the relationships involved in changes; and to find solutions to everyday problems using algebra's symbolic manipulation and formulas.

STANDARD 8A

Describe numerical relationships using variables and patterns.

STANDARD 8B

Interpret and describe numerical relationships using tables, graphs and symbols.

STANDARD 8C

Solve problems using systems of numbers and their properties.

STANDARD 8D

Use algebraic concepts and procedures to represent and solve problems.

STATE GOAL 9: Use geometric methods to analyze, categorize and draw conclusions about points, lines, planes and space.

Why This Goal Is Important: Geometry provides important methods for reasoning and solving problems with points, lines, planes and space. The word "geometry" comes from Greek words meaning "measurement of the Earth." While we use modern technology and employ a wider variety of mathematical tools today, we still study geometry to understand the shapes and dimensions of our world. The applications of geometry are widespread in construction, engineering, architecture, mapmaking and art. Historically, geometry is a way to develop skill in forming convincing arguments and proofs. This goal of developing a means of argument and validation remains an important part of our reasons for studying geometry today.

STANDARD 9A

Demonstrate and apply geometric concepts involving points, lines, planes and space.

STANDARD 9B

Identify, describe, classify and compare relationships using points, lines, planes and solids.

STANDARD 9C

Construct convincing arguments and proofs to solve problems.

STANDARD 9D

Use trigonometric ratios and circular functions to solve problems.

STATE GOAL 10: Collect, organize and analyze data using statistical methods; predict results; and interpret uncertainty using concepts of probability.

Why This Goal Is Important: The ability to understand and interpret data (e.g., opinion polls, stock prices, tax rates, crime statistics, scientific studies, weather reports) grows more important each day. Students must be able to organize data, make sense of variables and patterns, and judge the logical reasonableness of any claims and interpretations made. Even very young students can count objects and communicate their findings with charts and graphs. Students of all ages can collect, display and interpret data to answer specific questions. They also must construct and analyze arguments that involve data and its interpretation. All students need to understand and apply the role probability plays in data collection and decision making. Data analysis and use are important abilities necessary for all careers.

STANDARD 10A

Organize, describe and make predictions from existing data.

STANDARD 10B

Formulate questions, design data collection methods, gather and analyze data and communicate findings.

STANDARD 10C

Determine, describe and apply the probabilities of events.

Mathematics Content Category Table PSAE Grade 11

The following percents are typical but may not be exact for any given administration of the PSAE. Many items measure more than one assessment objective, either within a state goal or across more than one state goal. The interconnection of the content within these state goals is crucial in instruction and curriculum development.

	PSAE Percent (93 items)	ACT* Assessment Component (60 items)	WorkKeys* Assessment Component (33 items)
State Goal 6 – Number Sense	29%	18%	63%
Standard 6A Representations and Ordering	2%	2%	3%
Standards 6B, 6C Computation, Operations, Estimation, and Properties	18%	11%	42%
Standard 6D Ratios, Proportions, and Percents	9%	5%	18%
State Goal 7 – Measurement	18%	11%	37%
Standards 7A, 7B, 7C Units, Tools, Estimation, and Applications	18%	11%	37%
State Goal 8 – Algebra	32%	40%	0%
Standard 8A Representations, Patterns, and Expressions	16%	20%	0%
Standard 8B Connections Using Tables, Graphs, and Symbols	6%	7%	0%
Standards 8C, 8D Writing, Interpreting, and Solving Equations	10%	13%	0%
State Goal 9 – Geometry	19%	24%	0%
Standard 9A Properties of Single Figures and Coordinate Geometry	14%	17%	0%
Standard 9B Relationships Between and Among Multiple Figures	1%	1%	0%
Standard 9C Justifications of Conjectures and Conclusions	1%	1%	0%
Standard 9D Trigonometry	3%	5%	0%
State Goal 10 – Data Analysis, Statistics, and Probability	4%	7%	0%
Standards 10A, 10B Data Analysis and Statistics	3%	5%	0%
Standard 10C Probability	1%	2%	0%

Note: The mathematics portion of the PSAE is a combination of the ACT Assessment Mathematics component and the WorkKeys *Applied Mathematics* Assessment component. **Note Added on 3/18/08:** A new scoring methodology was announced that affects only reading and mathematics. For more information, please see http://www.isbe.net/assessment/listserv/2008/mar14.htm.

*These percents are typical, based on an analysis of forms from multiple years. These percents in this chart were derived by considering only an item's primary classification.

Mathematics – State Goal 6: Number Sense

Grade 11

Standard 6A – Representations and Ordering

6.11.01 Recognize, represent, order, compare real numbers, and locate real numbers on a number line (e.g., π , $\sqrt{2}$, $\sqrt{5}$, 2/3, -1.6).

6.11.02 Represent numbers in equivalent forms (e.g., fraction/decimal/percent, exponential/logarithmic, radical/rational exponents, absolute value, scientific notation).

6.11.03 Use matrices to organize data.

Mathematics – State Goal 6: Number Sense

Grade 11

Standards 6B, 6C – Computation, Operations, Estimation, and Properties

6.11.04 Apply the rules of order of operations to real-number expressions.

6.11.05 Simplify or test expressions by applying field properties (commutative, associative, distributive), order properties (transitive, reflexive, symmetric), and properties of equality for the set of real numbers.

6.11.06 Apply number theory concepts to the solution of problems (e.g., prime and composite numbers, prime factorization, greatest common factor, least common multiple, divisibility rules).

6.11.07 Determine the effects of operations on the magnitudes of quantities (e.g., multiplication, division, powers, roots).

6.11.08 Determine the appropriate solution, including rounding, from a context (e.g., rounding up, down, to the nearest integer).

6.11.09 Solve problems involving estimates or data (e.g., use averages to estimate the cost of a job that includes labor and materials).

6.11.10 Perform numerical computations with real numbers.

6.11.11 Perform numerical computations with non-real complex numbers.

6.11.12 Solve problems using simple matrix operations (addition, subtraction, multiplication, scalar multiplication).

6.11.13 Set up, evaluate, or solve single- and multi-step number sentences and word problems with rational numbers using the four basic operations.

6.11.14 Determine the most cost effective option using single- and multi-step calculations and then comparing results.

6.11.15 Judge the reasonableness of solutions, and find mistakes in calculation, logic, and formula application.

6.11.16 Simplify numerical problems involving absolute value.

Mathematics – State Goal 6: Number Sense

Grade 11

Standard 6D – Ratios, Proportions, and Percents

6.11.17 Set up, evaluate, or solve number sentences or word problems involving ratios and proportions with rational numbers (e.g., scale drawing, unit rate, scale factor, rate of change).

6.11.18 Set up, evaluate, or solve common problems involving percent (e.g., sales tax, tip, interest, discount, markup, commission, compound interest).

6.11.19 Set up, evaluate, or solve problems stated in terms of direct and inverse variation of simple quantities.

Mathematics – State Goal 7: Measurement

Grade 11

Standards 7A, 7B, 7C – Units, Tools, Estimation, and Applications

7.11.01 Change from one unit to another within the same system of measurement, including calculations with mixed units (e.g., 3½ hours plus 4 hours and 20 minutes; 2½ feet minus 16 inches).

7.11.02 Change from one unit in one system of measurement to a unit in another system of measurement, given a conversion factor.

7.11.03 Determine and calculate to an indicated precision the length, width, height, perimeter/circumference, area, volume, surface area, angle measures, or sums of angle measures of common geometric figures or combinations of common geometric figures.

7.11.04 Describe the general trends of how the change in one measure affects other measures in the same figure (e.g., length, area, volume).

7.11.05 Determine the linear measure, perimeter, area, surface area, and volume of similar figures.

7.11.06 Determine the ratio of perimeters, areas, and volumes of figures.

7.11.07 Use measures expressed as rates (e.g., speed, density), measures expressed as products (e.g., person-days), and dimensional analysis (e.g., converting ft/sec to yards/min) to solve problems.

Mathematics - State Goal 8: Algebra

Grade 11

Standard 8A – Representations, Patterns, and Expressions

8.11.01 Simplify or identify equivalent algebraic expressions (e.g., exponential, rational, logarithmic, factored, polynomial).

8.11.02 Represent mathematical relationships using symbolic algebra.

8.11.03 Identify essential quantitative relationships in a situation, and determine the class or classes of functions (e.g., linear, quadratic, exponential) that model the relationships.

8.11.04 Determine a specific term, a finite sum, or a rule that generates terms of a pattern.

8.11.05 Model and describe slope as a constant rate of change.

8.11.06 Evaluate variable expressions and functions.

Standard 8B - Connections Using Tables, Graphs, and Symbols

8.11.07 Identify an equation of a line or an equation of a line of best fit from given information (e.g., from a set of ordered pairs, graphs, tables).

8.11.08 Recognize and describe the general shape and properties of functions from graphs, tables, or equations (e.g., linear, absolute value, quadratic, exponential, logarithmic).

8.11.09 Identify slope from an equation, table of values, or graph.

8.11.10 Interpret the role of the coefficients and constants on the graphs of linear and quadratic functions, given a set of equations.

8.11.11 Analyze functions by investigating domain, range, rates of change, intercepts, and zeros.

8.11.12 Create and connect representations that are tabular, graphic, numeric, and symbolic from a set of data.

8.11.13 Represent quantitative relationships graphically, and interpret the meaning of the graph or a specific part of the graph as it relates to the situation represented by the graph.

Mathematics - State Goal 8: Algebra

Grade 11

Standards 8C, 8D – Writing, Interpreting, and Solving Equations

8.11.14 Model problems using mathematical functions and relations (e.g., linear, non-linear).

8.11.15 Interpret the graph of a system of equations and inequalities, including cases where there are no solutions.

8.11.16 Solve linear equations and inequalities, including selecting and evaluating formulas.

8.11.17 Solve systems of equations and inequalities.

8.11.18 Solve quadratic equations over the complex number system, including selecting and evaluating formulas.

8.11.19 Solve problems that include nonlinear functions, including selecting and evaluating formulas (i.e., absolute value, trigonometric, logarithmic, exponential).

8.11.20 Identify, interpret, and write equations for circles and other conic sections.

8.11.21 Recognize and apply mathematical and algebraic axioms, theorems of algebra, and deductive reasoning.

8.11.22 Identify equivalent forms of equations, inequalities, and systems of equations.

Mathematics - State Goal 9: Geometry

Grade 11

Standard 9A – Properties of Single Figures and Coordinate Geometry

9.11.01 Apply the Pythagorean theorem.

9.11.02 Identify and represent transformations (rotations, reflections, translations, dilations) of an object in the plane, and describe the effects of transformations on points in words or coordinates.

9.11.03 Determine how changing the scale factor affects the size and position of a figure in the plane.

9.11.04 Classify plane figures according to their properties.

9.11.05 Identify, apply, or solve problems that require knowledge of geometric properties of plane figures (e.g., triangles, quadrilaterals, parallel lines cut by a transversal, angles, diagonals, triangle inequality).

9.11.06 Identify a three-dimensional object from different perspectives.

9.11.07 Identify the relationship between two-dimensional patterns (e.g., nets) and related three-dimensional objects (e.g., cylinders, prisms, cones).

9.11.08 Identify two- and three-dimensional figures that would match a set of given conditions.

9.11.09 Solve problems that involve calculating distance, midpoint, and slope using coordinate geometry.

9.11.10 Identify, apply, and solve problems that require knowledge of geometric relationships of circles (e.g. arcs, chords, tangents, secants, central angles, inscribed angles).

9.11.11 Graph, locate, and identify points on a coordinate system.

Standard 9B – Relationships Between and Among Multiple Figures

9.11.12 Solve problems involving similar figures.

9.11.13 Solve problems using triangle congruence.

9.11.14 Describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

9.11.15 Identify relationships between circles and other objects in the plane (e.g., inscribed circles, concentric circles, internal/external tangency).

Mathematics - State Goal 9: Geometry

Grade 11

Standard 9C – Justifications of Conjectures and Conclusions

9.11.16 Recognize and apply the conditions that assure congruence and similarity.

9.11.17 Recognize and apply mathematical and geometric axioms, fundamental theorems of geometry, and deductive reasoning.

9.11.18 Identify a counter-example to disprove a conjecture.

Standard 9D – Trigonometry

9.11.19 Determine distances and angle measures using indirect measurement (e.g., properties of right triangles, Law of Sines, Law of Cosines).

9.11.20 Solve problems using 45°-45°-90° and 30°-60°-90° triangles.

9.11.21 Identify graphs of a given trigonometric function $(\sin x, \cos x)$ using its characteristics (e.g., period, amplitude).

9.11.22 Define, identify, and evaluate trigonometric ratios.

9.11.23 Use trigonometric identities (e.g., $\sin^2 x + \cos^2 x = 1$)

Grade 11

Standards 10A, 10B – Data Analysis and Statistics

10.11.01 Read, interpret, predict, interpolate, extrapolate, and use information from a variety of graphs, charts, and tables.

10.11.02 Translate from one representation of data to another (e.g., a bar graph to a circle graph).

10.11.03 Solve problems involving Venn diagrams.

10.11.04 Find an unknown value in a dataset given information about descriptive statistics.

10.11.05 Calculate, interpret, and use measures of central tendency and dispersion.

10.11.06 Compare two or more data sets on measures of central tendency and dispersion.

Standard 10C – Probability

10.11.07 Compute the probability of an event composed of single or repeated trials with or without replacement.

10.11.08 Compute probabilities for compound events.

10.11.09 Determine geometric probability based on area.

10.11.10 Apply counting techniques (e.g., permutations, combinations, Fundamental Counting Principle).