The State of Illinois

RACE TO THE TOP

APPLICATION FOR PHASE 2 FUNDING

Amendment to Section (B)(1) of the Application

Adoption of the Common Core State Standards

Office of Governor Pat Quinn
Illinois State Board of Education
July 27, 2010

CFDA Number: 84.395A

Illinois State Board of Education

100 North First Street • Springfield, Illinois 62777-0001
www.isbe.net

Jesse H. Ruiz
Chairman

Christopher A. Koch, Ed.D.
State Superintendent of Education

July 26, 2010

Mr. Arne Duncan
Secretary of Education
U.S. Department of Education
400 Maryland Avenue, SW
Washington, D.C. 20202

Re: The State of Illinois, Race to the Top Application for Phase 2 Funding, Amendment for Adoption of Common Core State Standards

Dear Secretary Duncan:

On June 24, 2010, the State of Illinois adopted the Common Core State Standards in mathematics and English language arts. As a result, we are pleased to submit for your consideration an amended Section (B)(1) to the State of Illinois' Race to the Top Application for Phase 2 Funding, which provides for the State's adoption of the Common Core State Standards. The attached amended Section (B)(1) and Appendices B-1 through B1-5 are intended to replace the Section (B)(1) and Appendices B1-1 through B1-4 included in the State's Application for Phase 2 funding submitted on June 1, 2010. The amended Section (B)(1) and Appendices provide evidence of the adoption of the Common Core State Standards and the context for such evidence to fully address criterion (B)(1)(ii).

Thank you again for your serious consideration of our State's application for Phase 2 Funding. We are hopeful that this application, as amended, will result in an opportunity to partner with you and the Department on its implementation.

Sincerely,

Christopher Koch

Christopher A. Koch, Ed.D.
State Superintendent of Education

III. RACE TO THE TOP APPLICATION ASSURANCES
(CFDA No. 84.395A)

<table>
<tr><td colspan="2">Legal Name of Applicant (Office of the Governor):
Office of the Governor Pat Quinn</td><td>Applicant's Mailing Address:
Office of the Governor Pat Quinn
207 State House
Springfield, Illinois 62706</td></tr>
<tr><td colspan="2">Employer Identification Number:
05-0527061</td><td>Organizational DUNS:
806812558</td></tr>
<tr><td colspan="2">State Race to the Top Contact Name:
(Single point of contact for communication)
Christopher A. Koch, Ed.D.</td><td>Contact Position and Office:
State Superintendent, Illinois State Board of Education</td></tr>
<tr><td colspan="2">Contact Telephone:
(217) 785-1288</td><td>Contact E-mail Address:
Chris.Koch@isbe.net</td></tr>
<tr><td colspan="3">Required Applicant Signatures:

To the best of my knowledge and belief, all of the information and data in this application are true and correct.

I further certify that I have read the application, am fully committed to it, and will support its implementation:</td></tr>
<tr><td colspan="2">Governor or Authorized Representative of the Governor (Printed Name):
Pat Quinn, Governor</td><td>Telephone:
(217) 782-0244</td></tr>
<tr><td colspan="2">Signature of Governor or Authorized Representative of the Governor:
Pat Quinn</td><td>Date:
7/13/10</td></tr>
<tr><td colspan="2">Chief State School Officer (Printed Name):
Christopher A. Koch, Ed.D.</td><td>Telephone:
(217) 782-2223</td></tr>
<tr><td colspan="2">Signature of the Chief State School Officer:
Christopher Koch</td><td>Date:
7-14-10</td></tr>
<tr><td colspan="2">President of the State Board of Education (Printed Name):
Jesse H. Ruiz, Chairman</td><td>Telephone:
(217) 782-2223</td></tr>
<tr><td colspan="2">Signature of the President of the State Board of Education:
Jesse H. Ruiz</td><td>Date:
7-19-10</td></tr>
</table>

State Attorney General Certification

I certify that the State's description of, and statements and conclusions concerning, State law, statute, and regulation in its application are complete, accurate, and constitute a reasonable interpretation of State law, statute, and regulation.
(See especially Eligibility Requirement (b), Selection Criteria (B)(1), (D)(1), (E)(1), (F)(2), (F)(3).)

I certify that the State does not have any legal, statutory, or regulatory barriers at the State level to linking data on student achievement (as defined in this notice) or student growth (as defined in this notice) to teachers and principals for the purpose of teacher and principal evaluation.

State Attorney General or Authorized Representative (Printed Name): Lisa Madigan, Illinois Attorney General	Telephone: (312) 814-3000
Signature of the State Attorney General or Authorized Representative: Lisa Madigan	Date: 7/13/10

IV. ACCOUNTABILITY, TRANSPARENCY, REPORTING AND OTHER ASSURANCES AND CERTIFICATIONS

Accountability, Transparency and Reporting Assurances

The Governor or his/her authorized representative assures that the State will comply with all of the accountability, transparency, and reporting requirements that apply to the Race to the Top program, including the following:

- For each year of the program, the State will submit a report to the Secretary, at such time and in such manner as the Secretary may require, that describes:
    - the uses of funds within the State;
    - how the State distributed the funds it received;
    - the number of jobs that the Governor estimates were saved or created with the funds;
    - the State's progress in reducing inequities in the distribution of highly qualified teachers, implementing a State longitudinal data system, and developing and implementing valid and reliable assessments for limited English proficient students and students with disabilities; and
    - if applicable, a description of each modernization, renovation, or repair project approved in the State application and funded, including the amounts awarded and project costs (ARRA Division A, Section 14008)

- The State will cooperate with any U.S. Comptroller General evaluation of the uses of funds and the impact of funding on the progress made toward closing achievement gaps (ARRA Division A, Section 14009)

- If the State uses funds for any infrastructure investment, the State will certify that the investment received the full review and vetting required by law and that the chief executive accepts responsibility that the investment is an appropriate use of taxpayer funds. This certification will include a description of the investment, the estimated total cost, and the amount of covered funds to be used. The certification will be posted on the State's website and linked to www.Recovery.gov. A State or local agency may not use funds under the ARRA for infrastructure investment funding unless this certification is made and posted. (ARRA Division A, Section 1511)

- The State will submit reports, within 10 days after the end of each calendar quarter, that contain the information required under section 1512(c) of the ARRA in accordance with any guidance issued by the Office of Management and Budget or the Department. (ARRA Division A, Section 1512(c))

- The State will cooperate with any appropriate Federal Inspector General's examination of records under the program. (ARRA Division A, Section 1515)

Other Assurances and Certifications

The Governor or his/her authorized representative assures or certifies the following:

- The State will comply with all applicable assurances in OMB Standard Forms 424B (Assurances for Non-Construction Programs) and to the extent consistent with the State's application, OMB Standard Form 424D (Assurances for Construction Programs), including the assurances relating to the legal authority to apply for assistance; access to records; conflict of interest; merit systems; nondiscrimination; Hatch Act provisions; labor standards; flood hazards; historic preservation; protection of human subjects; animal welfare; lead-based paint; Single Audit Act; and the general agreement to comply with all applicable Federal laws, executive orders and regulations.

- With respect to the certification regarding lobbying in Department Form 80-0013, no Federal appropriated funds have been paid or will be paid to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with the making or renewal of Federal grants under this program; the State will complete and submit Standard Form-LLL, "Disclosure Form to Report Lobbying," when required (34 C.F.R. Part 82, Appendix B); and the State will require the full certification, as set forth in 34 C.F.R. Part 82, Appendix A, in the award documents for all subawards at all tiers.

- The State will comply with all of the operational and administrative provisions in Title XV and XIV of the ARRA, including Buy American Requirements (ARRA Division A, Section 1605), Wage Rate Requirements (section 1606), and any applicable environmental impact requirements of the National Environmental Policy Act of 1970 (NEPA), as amended, (42 U.S.C. 4371 et seq.) (ARRA Division A, Section 1609). In using ARRA funds for infrastructure investment, recipients will comply with the requirement regarding Preferences for Quick Start Activities (ARRA Division A, Section 1602).

- Any local educational agency (LEA) receiving funding under this program will have on file with the State a set of assurances that meets the requirements of section 442 of the General Education Provisions Act (GEPA) (20 U.S.C. 1232e).

- Any LEA receiving funding under this program will have on file with the State (through either its Stabilization Fiscal Stabilization Fund application or another U.S. Department of Education Federal grant) a description of how the LEA will comply with the requirements of section 427 of GEPA (20 U.S.C. 1228a). The description must include information on the steps the LEA proposes to take to permit students, teachers, and other program beneficiaries to overcome barriers (including barriers based on gender, race, color, national origin, disability, and age) that impede access to, or participation in, the program.

- The State and other entities will comply with the Education Department General Administrative Regulations (EDGAR), including the following provisions as applicable: 34 CFR Part 74–Administration of Grants and Agreements with Institutions of Higher Education, Hospitals, and Other Non-Profit Organizations; 34 CFR Part 75–Direct Grant Programs; 34 CFR Part 77– Definitions that Apply to Department Regulations; 34 CFR Part

80– Uniform Administrative Requirements for Grants and Cooperative Agreements to State and Local Governments, including the procurement provisions; 34 CFR Part 81– General Education Provisions Act–Enforcement; 34 CFR Part 82– New Restrictions on Lobbying; 34 CFR Part 84–Governmentwide Requirements for Drug-Free Workplace (Financial Assistance); 34 CFR Part 85–Governmentwide Debarment and Suspension (Nonprocurement).

SIGNATURE BLOCK FOR CERTIFYING OFFICIAL

Governor or Authorized Representative of the Governor (Printed Name): Pat Quinn, Governor	
Signature of Governor or Authorized Representative of the Governor: Pat Quinn	Date: July 13, 2010

The State of Illinois Race to the Top
Application for Phase 2 Funding

Amendment to Section (B)(1) of the Application

Adoption of the Common Case State Standards

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B. <u>Standards and Assessments</u>

RTTT APPLICATION REQUIREMENTS

(B)(1) <u>Developing and adopting common standards</u> *(40 points)*

The extent to which the State has demonstrated its commitment to adopting a common set of high-quality standards, evidenced by (as set forth in Appendix B)—

(i) The State's participation in a consortium of States that— *(20 points)*

- (a) Is working toward jointly developing and adopting a common set of K-12 standards (as defined in this notice) that are supported by evidence that they are internationally benchmarked and build toward college and career readiness by the time of high school graduation; and
- (b) Includes a significant number of States; and

(ii) — *(20 points)*

(a) For Phase 1 applications, the State's high-quality plan demonstrating its commitment to and progress toward adopting a common set of K-12 standards (as defined in this notice) by August 2, 2010, or, at a minimum, by a later date in 2010 specified by the State, and to implementing the standards thereafter in a well-planned way; or

(b) For Phase 2 applications, the State's adoption of a common set of K-12 standards (as defined in this notice) by August 2, 2010, or, at a minimum, by a later date in 2010 specified by the State in a high-quality plan toward which the State has made significant progress, and its commitment to implementing the standards thereafter in a well-planned way.*

* Phase 2 applicants addressing selection criterion (B)(1)(ii) may amend their June 1, 2010 application submission through August 2, 2010 by submitting evidence of adopting common standards after June 1, 2010.

(B)(1) <u>Illinois Reform Conditions</u>
Developing and Adopting Common Standards

[Note: an amended Section (B)(1) is set forth in its entirety to provide context for the evidence of adoption of the Common Core State Standards.]

On June 24, 2010, the State of Illinois adopted the Common Core State Standards in mathematics and English language arts. The Common Core State Standards replaced the standards that were in place as of such date, and thus now constitute 100% of the standards, in those subject matter areas. Illinois is part of the Common Core State Standards Initiative involving 48 states, 2 territories, and the District of Columbia, and executed the Memorandum of Agreement among the initiative's participants. *(See **Appendix B1-1** for list of participants. See **Appendix B1-2** for copy of Memorandum of Agreement.)* The participating states and territories in this initiative collectively developed and have or will adopt a core set of internationally-benchmarked academic standards in mathematics and English language arts. The final Common

Core State Standards were released on June 2, 2010. *(See* ***Appendix B1-3*** *for a copy of the final standards. See* ***Appendix B1-4*** *for documentation of international benchmarking.)* The Common Core State Standards Initiative is also creating economies of scale around areas such as curriculum development and common assessments. The Common Core State Standards Initiative is jointly led by the National Governors Association (NGA), Center for Best Practices and the Council of Chief State School Officers in partnership with Achieve, ACT, and the College Board.

Illinois' membership in the Common Core State Standards Initiative builds off of the State's participation in the American Diploma Project (ADP). In October 2008, ISBE, in partnership with the Illinois Board of Higher Education, Illinois Community College Board, Office of the Governor, and the Illinois Business Roundtable, joined 33 states in the American Diploma Project. This effort has involved both an external and internal review of the Illinois Learning Standards in English/Language Arts and Math. Teams of secondary and postsecondary educators compared the Illinois Learning Standards to the ADP exemplary standards in order to clarify what it means to best prepare to succeed in college. This process has helped build support and awareness for the State's revision of the Learning Standards through the Common Core State Standards Initiative.

The Illinois Learning Standards are incorporated into the State Goals for Learning set forth in Title 23, Part 1, of the Illinois Administrative Code, 23 Ill. Adm. Code 1, Appendix D. At its June 24, 2010 meeting, the State Board of Education adopted a common set of K-12 standards, as defined in the RTTT notice, by amending the Illinois Administrative Code to adopt:

- the entire Common Core State Standards for English Language Arts and Literacy in History/Social Studies, Science, and Technical Subjects published by the Common Core State Standards Initiative; and
- the entire Common Core State Standards for Mathematics published by the Common Core State Standards Initiative.

(A copy of the adopted administrative rules amending the Illinois Administrative Code, as well as a certificate of emergency amendment signed by the State Superintendent and stamped by the Secretary of State, is included as ***Appendix B1-5.***[1]) The State Board of Education's promulgation of administrative rules that incorporate the Common Core State Standards

[1] Illinois law permits the incorporation of published material by reference. See 5 ILCS 100/5-75.

complied with the rulemaking process set forth in the Illinois Administrative Procedure Act ("IAPA"), 5 ILCS 100/1-1 *et seq*. Pursuant to the IAPA, the Common Core State Standards became effective immediately upon ISBE's June 24, 2010 filing of notice with the Secretary of State.[2]

At its June 24th meeting, the State Board of Education also authorized the solicitation of public comment on the Common Core State Standards through a concurrent regular rulemaking undertaken in accordance with the IAPA. The public comment received relative to the regular rulemaking may lead to the State supplementing the Common Core State Standards to address State-specific objectives. However, this process does not undermine the effectiveness of the Common Core State Standards in Illinois, nor will it lead to the inclusion of additional standards exceeding 15% of the State's total standards for each of the Common Core content areas (as agreed to by the State in the Common Core State Standards Initiative's Memorandum of Understanding).

Evidence for (B)(1)(i):

- A copy of the Memorandum of Agreement, executed by the State, showing that it is part of a standards consortium.
  - **APPENDIX B1-2: COMMON CORE STATE STANDARDS INITIATIVE MEMORANDUM OF AGREEMENT**
- A copy of the final standards or, if the standards are not yet final, a copy of the draft standards and anticipated date for completing the standards.
  - **APPENDIX B1-3: FINAL COMMON CORE STATE STANDARDS**
- Documentation that the standards are or will be internationally benchmarked and that, when well-implemented, will help to ensure that students are prepared for college and careers.
  - **APPENDIX B1-4: OUTLINE OF INTERNATIONAL BENCHMARKING OF COMMON CORE STATE STANDARDS AND REPORT ENTITLED, "ENSURING U.S. STUDENTS RECEIVE A WORLD-CLASS EDUCATION"**
- The number of States participating in the standards consortium and the list of these States.
  - **APPENDIX B1-1: COMMON CORE STATE STANDARDS INITIATIVE CONSORTIUM PARTICIPANTS**

Evidence for (B)(1)(ii):

The State has adopted the Common Core State Standards.
See APPENDIX B1-5:

- **CERTIFICATE OF EMERGENCY AMENDMENT**
- **STATE OF ILLINOIS ADMINISTRATIVE RULES ADOPTING THE COMMON CORE STATE STANDARDS**

[2] Section 5-45(b) of the IAPA (5 ILCS 100/5-45(b)).

AMENDED APPENDIX VOLUME II

SECTION B1

*****The Final Common Core State Standards include internal page numbering and such numbering does not correspond to the page numbers of this Appendix

Appendix B1-1

Common Core State Standards Initiative Consortium Participants

1. Alabama
2. Arizona
3. Arkansas
4. California
5. Colorado
6. Connecticut
7. Delaware
8. District of Columbia
9. Florida
10. Georgia
11. Hawaii
12. Idaho
13. Illinois
14. Indiana
15. Iowa
16. Kansas
17. Kentucky
18. Louisiana
19. Maine
20. Maryland
21. Massachusetts
22. Michigan
23. Minnesota
24. Mississippi
25. Missouri
26. Montana
27. Nebraska
28. Nevada
29. New Hampshire
30. New Jersey
31. New Mexico
32. New York
33. North Carolina
34. North Dakota
35. Ohio
36. Oklahoma
37. Oregon
38. Pennsylvania
39. Puerto Rico
40. Rhode Island
41. South Carolina
42. South Dakota
43. Tennessee
44. Utah
45. Vermont
46. Virgin Islands
47. Virginia
48. Washington
49. West Virginia
50. Wisconsin
51. Wyoming

Appendix B1-2

Memorandum of Agreement

The Council of Chief State School Officers and
The National Governors Association Center for Best Practices

Common Core Standards
Memorandum of Agreement

Purpose. This document commits states to a state-led process that will draw on evidence and lead to development and adoption of a common core of state standards (common core) in English language arts and mathematics for grades K-12. These standards will be aligned with college and work expectations, include rigorous content and skills, and be internationally benchmarked. The intent is that these standards will be aligned to state assessment and classroom practice. The second phase of this initiative will be the development of common assessments aligned to the core standards developed through this process.

Background. Our state education leaders are committed to ensuring all students graduate from high school ready for college, work, and success in the global economy and society. State standards provide a key foundation to drive this reform. Today, however, state standards differ significantly in terms of the incremental content and skills expected of students.

Over the last several years, many individual states have made great strides in developing high-quality standards and assessments. These efforts provide a strong foundation for further action. For example, a majority of states (35) have joined the American Diploma Project (ADP) and have worked individually to align their state standards with college and work expectations. Of the 15 states that have completed this work, studies show significant similarities in core standards across the states. States also have made progress through initiatives to upgrade standards and assessments, for example, the New England Common Assessment Program.

Benefits to States. The time is right for a state-led, nation-wide effort to establish a common core of standards that raises the bar for all students. This initiative presents a significant opportunity to accelerate and drive education reform toward the goal of ensuring that all children graduate from high school ready for college, work, and competing in the global economy and society. With the adoption of this common core, participating states will be able to:

- Articulate to parents, teachers, and the general public expectations for students;
- Align textbooks, digital media, and curricula to the internationally benchmarked standards;
- Ensure professional development to educators is based on identified need and best practices;
- Develop and implement an assessment system to measure student performance against the common core; and
- Evaluate policy changes needed to help students and educators meet the common core standards and "end-of-high-school" expectations.

An important tenet of this work will be to increase the rigor and relevance of state standards across all participating states; therefore, no state will see a decrease in the level of student expectations that exist in their current state standards.

Process and Structure

- **Common Core State-Based Leadership.** The Council of Chief State School Officers (CCSSO) and the National Governors Association Center for Best Practices (NGA Center) shall assume responsibility for coordinating the process that will lead to state adoption of a common core set of standards. These organizations represent governors and state commissioners of education who are charged with defining K-12 expectations at the state level. As such, these organizations will facilitate a state-led process to develop a set of common core standards in English language arts and math that are:

- Fewer, clearer, and higher, to best drive effective policy and practice;
- Aligned with college and work expectations, so that all students are prepared for success upon graduating from high school;
- Inclusive of rigorous content and application of knowledge through high-order skills, so that all students are prepared for the 21st century;
- Internationally benchmarked, so that all students are prepared for succeeding in our global economy and society; and
- Research and evidence-based.

- **National Validation Committee.** CCSSO and the NGA Center will create an expert validation group that will serve several purposes, including validating end-of-course expectations, providing leadership for the development of K-12 standards, and certifying state adoption of the common core. The group will be comprised of national and international experts on standards. Participating states will have the opportunity to nominate individuals to the group. The national validation committee shall provide an independent review of the common core. The national validation committee will review the common core as it is developed and offer comments, suggestions, and validation of the process and products developed by the standards development group. The group will use evidence as the driving factor in validating the common core.

- **Develop End-of-High-School Expectations.** CCSSO and the NGA Center will convene Achieve, ACT and the College Board in an open, inclusive, and efficient process to develop a set of end-of–high-school expectations in English language arts and mathematics based on evidence. We will ask all participating states to review and provide input on these expectations. This work will be completed by July 2009.

- **Develop K-12 Standards in English Language Arts and Math.** CCSSO and the NGA Center will convene Achieve, ACT, and the College Board in an open, inclusive, and efficient process to develop K-12 standards that are grounded in empirical research and draw on best practices in standards development. We will ask participating states to provide input into the drafting of the common core and work as partners in the common core standards development process. This work will be completed by December 2009.

- **Adoption.** The goal of this effort is to develop a true common core of state standards that are internationally benchmarked. Each state adopting the common core either directly or by fully aligning its state standards may do so in accordance with current state timelines for standards adoption not to exceed three (3) years.

  This effort is voluntary for states, and it is fully intended that states adopting the common core may choose to include additional state standards beyond the common core. States that choose to align their standards to the common core standards agree to ensure that the common core represents at least 85 percent of the state's standards in English language arts and mathematics.

  Further, the goal is to establish an ongoing development process that can support continuous improvement of this first version of the common core based on research and evidence-based learning and can support the development of assessments that are aligned to the common core across the states, for accountability and other appropriate purposes.

- **National Policy Forum.** CCSSO and the NGA Center will convene a National Policy Forum (Forum) comprised of signatory national organizations (e.g., the Alliance for Excellent Education, Business Roundtable, National School Boards Association, Council of Great City Schools, Hunt Institute, National Association of State Boards of Education, National Education Association, and others) to share ideas, gather input, and inform the common core initiative. The forum is intended as a place for refining our shared understanding of the scope and elements of a common core; sharing and coordinating the various forms of implementation of a common core; providing a means to develop common messaging between and among participating organizations; and building public will and support.

- **Federal Role.** The parties support a state-led effort and not a federal effort to develop a common core of state standards; there is, however, an appropriate federal role in supporting this state-led effort. In particular, the federal government can provide key financial support for this effort in developing a common core of state standards and in moving toward common assessments, such as through the Race to the Top Fund authorized in the American Recovery and Reinvestment Act of 2009. Further, the federal government can incentivize this effort through a range of tiered incentives, such as providing states with greater flexibility in the use of existing federal funds, supporting a revised state accountability structure, and offering financial support for states to effectively implement the standards. Additionally, the federal government can provide additional long-term financial support for the development of common assessments, teacher and principal professional development, other related common core standards supports, and a research agenda that can help continually improve the common core over time. Finally, the federal government can revise and align existing federal education laws with the lessons learned from states' international benchmarking efforts and from federal research.

Agreement. The undersigned state leaders agree to the process and structure as described above and attest accordingly by our signature(s) below.

Signatures
Governor: Pat Quinn
Chief State School Officer: [signature]

Appendix B1-3

Final Common Core Standards

COMMON CORE STATE STANDARDS FOR

English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects

COMMON CORE STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

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Introduction

The Common Core State Standards for English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects ("the Standards") are the culmination of an extended, broad-based effort to fulfill the charge issued by the states to create the next generation of K–12 standards in order to help ensure that all students are college and career ready in literacy no later than the end of high school.

The present work, led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), builds on the foundation laid by states in their decades-long work on crafting high-quality education standards. The Standards also draw on the most important international models as well as research and input from numerous sources, including state departments of education, scholars, assessment developers, professional organizations, educators from kindergarten through college, and parents, students, and other members of the public. In their design and content, refined through successive drafts and numerous rounds of feedback, the Standards represent a synthesis of the best elements of standards-related work to date and an important advance over that previous work.

As specified by CCSSO and NGA, the Standards are (1) research and evidence based, (2) aligned with college and work expectations, (3) rigorous, and (4) internationally benchmarked. A particular standard was included in the document only when the best available evidence indicated that its mastery was essential for college and career readiness in a twenty-first-century, globally competitive society. The Standards are intended to be a living work: as new and better evidence emerges, the Standards will be revised accordingly.

The Standards are an extension of a prior initiative led by CCSSO and NGA to develop College and Career Readiness (CCR) standards in reading, writing, speaking, listening, and language as well as in mathematics. The CCR Reading, Writing, and Speaking and Listening Standards, released in draft form in September 2009, serve, in revised form, as the backbone for the present document. Grade-specific K–12 standards in reading, writing, speaking, listening, and language translate the broad (and, for the earliest grades, seemingly distant) aims of the CCR standards into age- and attainment-appropriate terms.

The Standards set requirements not only for English language arts (ELA) but also for literacy in history/social studies, science, and technical subjects. Just as students must learn to read, write, speak, listen, and use language effectively in a variety of content areas, so too must the Standards specify the literacy skills and understandings required for college and career readiness in multiple disciplines. Literacy standards for grade 6 and above are predicated on teachers of ELA, history/social studies, science, and technical subjects using their content area expertise to help students meet the particular challenges of reading, writing, speaking, listening, and language in their respective fields. It is important to note that the 6–12 literacy standards in history/social studies, science, and technical subjects are not meant to replace content standards in those areas but rather to supplement them. States may incorporate these standards into their standards for those subjects or adopt them as content area literacy standards.

As a natural outgrowth of meeting the charge to define college and career readiness, the Standards also lay out a vision of what it means to be a literate person in the twenty-first century. Indeed, the skills and understandings students are expected to demonstrate have wide applicability outside the classroom or workplace. Students who meet the Standards readily undertake the close, attentive reading that is at the heart of understanding and enjoying complex works of literature. They habitually perform the critical reading necessary to pick carefully through the staggering amount of information available today in print and digitally. They actively seek the wide, deep, and thoughtful engagement with high-quality literary and informational texts that builds knowledge, enlarges experience, and broadens worldviews. They reflexively demonstrate the cogent reasoning and use of evidence that is essential to both private deliberation and responsible citizenship in a democratic republic. In short, students who meet the Standards develop the skills in reading, writing, speaking, and listening that are the foundation for any creative and purposeful expression in language.

June 2, 2010

Key Design Considerations

CCR and grade-specific standards

The CCR standards anchor the document and define general, cross-disciplinary literacy expectations that must be met for students to be prepared to enter college and workforce training programs ready to succeed. The K–12 grade-specific standards define end-of-year expectations and a cumulative progression designed to enable students to meet college and career readiness expectations no later than the end of high school. The CCR and high school (grades 9–12) standards work in tandem to define the college and career readiness line—the former providing broad standards, the latter providing additional specificity. Hence, both should be considered when developing college and career readiness assessments.

Students advancing through the grades are expected to meet each year's grade-specific standards, retain or further develop skills and understandings mastered in preceding grades, and work steadily toward meeting the more general expectations described by the CCR standards.

Grade levels for K–8; grade bands for 9–10 and 11–12

The Standards use individual grade levels in kindergarten through grade 8 to provide useful specificity; the Standards use two-year bands in grades 9–12 to allow schools, districts, and states flexibility in high school course design.

A focus on results rather than means

By emphasizing required achievements, the Standards leave room for teachers, curriculum developers, and states to determine how those goals should be reached and what additional topics should be addressed. Thus, the Standards do not mandate such things as a particular writing process or the full range of metacognitive strategies that students may need to monitor and direct their thinking and learning. Teachers are thus free to provide students with whatever tools and knowledge their professional judgment and experience identify as most helpful for meeting the goals set out in the Standards.

An integrated model of literacy

Although the Standards are divided into Reading, Writing, Speaking and Listening, and Language strands for conceptual clarity, the processes of communication are closely connected, as reflected throughout this document. For example, Writing standard 9 requires that students be able to write about what they read. Likewise, Speaking and Listening standard 4 sets the expectation that students will share findings from their research.

Research and media skills blended into the Standards as a whole

To be ready for college, workforce training, and life in a technological society, students need the ability to gather, comprehend, evaluate, synthesize, and report on information and ideas, to conduct original research in order to answer questions or solve problems, and to analyze and create a high volume and extensive range of print and nonprint texts in media forms old and new. The need to conduct research and to produce and consume media is embedded into every aspect of today's curriculum. In like fashion, research and media skills and understandings are embedded throughout the Standards rather than treated in a separate section.

Shared responsibility for students' literacy development

The Standards insist that instruction in reading, writing, speaking, listening, and language be a shared responsibility within the school. The K–5 standards include expectations for reading, writing, speaking, listening, and language applicable to a range of subjects, including but not limited to ELA. The grades 6–12 standards are divided into two sections, one for ELA and the other for history/social studies, science, and technical subjects. This division reflects the unique, time-honored place of ELA teachers in developing students' literacy skills while at the same time recognizing that teachers in other areas must have a role in this development as well.

Part of the motivation behind the interdisciplinary approach to literacy promulgated by the Standards is extensive research establishing the need for college and career ready students to be proficient in reading complex informational text independently in a variety of content areas. Most of the required reading in college and workforce training programs is informational in structure and challenging in content; postsecondary education programs typically provide students with both a higher volume of such reading than is generally required in K–12 schools and comparatively little scaffolding.

The Standards are not alone in calling for a special emphasis on informational text. The 2009 reading framework of the National Assessment of Educational Progress (NAEP) requires a high and increasing proportion of informational text on its assessment as students advance through the grades.

Distribution of Literary and Informational Passages by Grade in the 2009 NAEP Reading Framework

Grade	Literary	Informational
4	50%	50%
8	45%	55%
12	30%	70%

Source: National Assessment Governing Board. (2008). *Reading framework for the 2009 National Assessment of Educational Progress.* Washington, DC: U.S. Government Printing Office.

The Standards aim to align instruction with this framework so that many more students than at present can meet the requirements of college and career readiness. In K–5, the Standards follow NAEP's lead in balancing the reading of literature with the reading of informational texts, including texts in history/social studies, science, and technical subjects. In accord with NAEP's growing emphasis on informational texts in the higher grades, the Standards demand that a significant amount of reading of informational texts take place in and outside the ELA classroom. Fulfilling the Standards for 6–12 ELA requires much greater attention to a specific category of informational text—literary nonfiction—than has been traditional. Because the ELA classroom must focus on literature (stories, drama, and poetry) as well as literary nonfiction, a great deal of informational reading in grades 6–12 must take place in other classes if the NAEP assessment framework is to be matched instructionally.[1] To measure students' growth toward college and career readiness, assessments aligned with the Standards should adhere to the distribution of texts across grades cited in the NAEP framework.

NAEP likewise outlines a distribution across the grades of the core purposes and types of student writing. The 2011 NAEP framework, like the Standards, cultivates the development of three mutually reinforcing writing capacities: writing to persuade, to explain, and to convey real or imagined experience. Evidence concerning the demands of college and career readiness gathered during development of the Standards concurs with NAEP's shifting emphases: standards for grades 9–12 describe writing in all three forms, but, consistent with NAEP, the overwhelming focus of writing throughout high school should be on arguments and informative/explanatory texts.[2]

Distribution of Communicative Purposes by Grade in the 2011 NAEP Writing Framework

Grade	To Persuade	To Explain	To Convey Experience
4	30%	35%	35%
8	35%	35%	30%
12	40%	40%	20%

Source: National Assessment Governing Board. (2007). *Writing framework for the 2011 National Assessment of Educational Progress, pre-publication edition.* Iowa City, IA: ACT, Inc.

It follows that writing assessments aligned with the Standards should adhere to the distribution of writing purposes across grades outlined by NAEP.

Focus and coherence in instruction and assessment

While the Standards delineate specific expectations in reading, writing, speaking, listening, and language, each standard need not be a separate focus for instruction and assessment. Often, several standards can be addressed by a single rich task. For example, when editing writing, students address Writing standard 5 ("Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach") as well as Language standards 1–3 (which deal with conventions of standard English and knowledge of language). When drawing evidence from literary and informational texts per Writing standard 9, students are also demonstrating their comprehension skill in relation to specific standards in Reading. When discussing something they have read or written, students are also demonstrating their speaking and listening skills. The CCR anchor standards themselves provide another source of focus and coherence.

The same ten CCR anchor standards for Reading apply to both literary and informational texts, including texts in history/social studies, science, and technical subjects. The ten CCR anchor standards for Writing cover numerous text types and subject areas. This means that students can develop mutually reinforcing skills and exhibit mastery of standards for reading and writing across a range of texts and classrooms.

[1]The percentages on the table reflect the sum of student reading, not just reading in ELA settings. Teachers of senior English classes, for example, are not required to devote 70 percent of reading to informational texts. Rather, 70 percent of student reading across the grade should be informational.
[2]As with reading, the percentages in the table reflect the sum of student writing, not just writing in ELA settings.

What is Not Covered by the Standards

The Standards should be recognized for what they are not as well as what they are. The most important intentional design limitations are as follows:

1. The Standards define what all students are expected to know and be able to do, not how teachers should teach. For instance, the use of play with young children is not specified by the Standards, but it is welcome as a valuable activity in its own right and as a way to help students meet the expectations in this document. Furthermore, while the Standards make references to some particular forms of content, including mythology, foundational U.S. documents, and Shakespeare, they do not—indeed, cannot—enumerate all or even most of the content that students should learn. The Standards must therefore be complemented by a well-developed, content-rich curriculum consistent with the expectations laid out in this document.

2. While the Standards focus on what is most essential, they do not describe all that can or should be taught. A great deal is left to the discretion of teachers and curriculum developers. The aim of the Standards is to articulate the fundamentals, not to set out an exhaustive list or a set of restrictions that limits what can be taught beyond what is specified herein.

3. The Standards do not define the nature of advanced work for students who meet the Standards prior to the end of high school. For those students, advanced work in such areas as literature, composition, language, and journalism should be available. This work should provide the next logical step up from the college and career readiness baseline established here.

4. The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

5. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post–high school lives.

   Each grade will include students who are still acquiring English. For those students, it is possible to meet the standards in reading, writing, speaking, and listening without displaying native-like control of conventions and vocabulary.

   The Standards should also be read as allowing for the widest possible range of students to participate fully from the outset and as permitting appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities *reading* should allow for the use of Braille, screen-reader technology, or other assistive devices, while *writing* should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, *speaking* and *listening* should be interpreted broadly to include sign language.

6. While the ELA and content area literacy components described herein are critical to college and career readiness, they do not define the whole of such readiness. Students require a wide-ranging, rigorous academic preparation and, particularly in the early grades, attention to such matters as social, emotional, and physical development and approaches to learning. Similarly, the Standards define literacy expectations in history/social studies, science, and technical subjects, but literacy standards in other areas, such as mathematics and health education, modeled on those in this document are strongly encouraged to facilitate a comprehensive, schoolwide literacy program.

Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, and Language

The descriptions that follow are not standards themselves but instead offer a portrait of students who meet the standards set out in this document. As students advance through the grades and master the standards in reading, writing, speaking, listening, and language, they are able to exhibit with increasing fullness and regularity these capacities of the literate individual.

They demonstrate independence.

Students can, without significant scaffolding, comprehend and evaluate complex texts across a range of types and disciplines, and they can construct effective arguments and convey intricate or multifaceted information. Likewise, students are able independently to discern a speaker's key points, request clarification, and ask relevant questions. They build on others' ideas, articulate their own ideas, and confirm they have been understood. Without prompting, they demonstrate command of standard English and acquire and use a wide-ranging vocabulary. More broadly, they become self-directed learners, effectively seeking out and using resources to assist them, including teachers, peers, and print and digital reference materials.

They build strong content knowledge.

Students establish a base of knowledge across a wide range of subject matter by engaging with works of quality and substance. They become proficient in new areas through research and study. They read purposefully and listen attentively to gain both general knowledge and discipline-specific expertise. They refine and share their knowledge through writing and speaking.

They respond to the varying demands of audience, task, purpose, and discipline.

Students adapt their communication in relation to audience, task, purpose, and discipline. They set and adjust purpose for reading, writing, speaking, listening, and language use as warranted by the task. They appreciate nuances, such as how the composition of an audience should affect tone when speaking and how the connotations of words affect meaning. They also know that different disciplines call for different types of evidence (e.g., documentary evidence in history, experimental evidence in science).

They comprehend as well as critique.

Students are engaged and open-minded—but discerning—readers and listeners. They work diligently to understand precisely what an author or speaker is saying, but they also question an author's or speaker's assumptions and premises and assess the veracity of claims and the soundness of reasoning.

They value evidence.

Students cite specific evidence when offering an oral or written interpretation of a text. They use relevant evidence when supporting their own points in writing and speaking, making their reasoning clear to the reader or listener, and they constructively evaluate others' use of evidence.

They use technology and digital media strategically and capably.

Students employ technology thoughtfully to enhance their reading, writing, speaking, listening, and language use. They tailor their searches online to acquire useful information efficiently, and they integrate what they learn using technology with what they learn offline. They are familiar with the strengths and limitations of various technological tools and mediums and can select and use those best suited to their communication goals.

They come to understand other perspectives and cultures.

Students appreciate that the twenty-first-century classroom and workplace are settings in which people from often widely divergent cultures and who represent diverse experiences and perspectives must learn and work together. Students actively seek to understand other perspectives and cultures through reading and listening, and they are able to communicate effectively with people of varied backgrounds. They evaluate other points of view critically and constructively. Through reading great classic and contemporary works of literature representative of a variety of periods, cultures, and worldviews, students can vicariously inhabit worlds and have experiences much different than their own.

How to Read This Document

Overall Document Organization

The Standards comprise three main sections: a comprehensive K–5 section and two content area–specific sections for grades 6–12, one for ELA and one for history/social studies, science, and technical subjects. Three appendices accompany the main document.

Each section is divided into strands. K–5 and 6–12 ELA have Reading, Writing, Speaking and Listening, and Language strands; the 6–12 history/ social studies, science, and technical subjects section focuses on Reading and Writing. Each strand is headed by a strand-specific set of College and Career Readiness Anchor Standards that is identical across all grades and content areas.

Standards for each grade within K–8 and for grades 9–10 and 11–12 follow the CCR anchor standards in each strand. Each grade-specific standard (as these standards are collectively referred to) corresponds to the same-numbered CCR anchor standard. Put another way, each CCR anchor standard has an accompanying grade-specific standard translating the broader CCR statement into grade-appropriate end-of-year expectations.

Individual CCR anchor standards can be identified by their strand, CCR status, and number (R.CCR.6, for example). Individual grade-specific standards can be identified by their strand, grade, and number (or number and letter, where applicable), so that RI.4.3, for example, stands for Reading, Informational Text, grade 4, standard 3 and W.5.1a stands for Writing, grade 5, standard 1a. Strand designations can be found in brackets alongside the full strand title.

Who is responsible for which portion of the Standards

A single K–5 section lists standards for reading, writing, speaking, listening, and language across the curriculum, reflecting the fact that most or all of the instruction students in these grades receive comes from one teacher. Grades 6–12 are covered in two content area–specific sections, the first for the English language arts teacher and the second for teachers of history/social studies, science, and technical subjects. Each section uses the same CCR anchor standards but also includes grade-specific standards tuned to the literacy requirements of the particular discipline(s).

Key Features of the Standards

Reading: Text complexity and the growth of comprehension

The Reading standards place equal emphasis on the sophistication of what students read and the skill with which they read. Standard 10 defines a grade-by-grade “staircase” of increasing text complexity that rises from beginning reading to the college and career readiness level. Whatever they are reading, students must also show a steadily growing ability to discern more from and make fuller use of text, including making an increasing number of connections among ideas and between texts, considering a wider range of textual evidence, and becoming more sensitive to inconsistencies, ambiguities, and poor reasoning in texts.

Writing: Text types, responding to reading, and research

The Standards acknowledge the fact that whereas some writing skills, such as the ability to plan, revise, edit, and publish, are applicable to many types of writing, other skills are more properly defined in terms of specific writing types: arguments, informative/explanatory texts, and narratives. Standard 9 stresses the importance of the writing-reading connection by requiring students to draw upon and write about evidence from literary and informational texts. Because of the centrality of writing to most forms of inquiry, research standards are prominently included in this strand, though skills important to research are infused throughout the document.

Speaking and Listening: Flexible communication and collaboration

Including but not limited to skills necessary for formal presentations, the Speaking and Listening standards require students to develop a range of broadly useful oral communication and interpersonal skills. Students must learn to work together, express and listen carefully to ideas, integrate information from oral, visual, quantitative, and media sources, evaluate what they hear, use media and visual displays strategically to help achieve communicative purposes, and adapt speech to context and task.

Language: Conventions, effective use, and vocabulary

The Language standards include the essential “rules” of standard written and spoken English, but they also approach language as a matter of craft and informed choice among alternatives. The vocabulary standards focus on understanding words and phrases, their relationships, and their nuances and on acquiring new vocabulary, particularly general academic and domain-specific words and phrases.

Appendices A, B, and C

Appendix A contains supplementary material on reading, writing, speaking and listening, and language as well as a glossary of key terms. Appendix B consists of text exemplars illustrating the complexity, quality, and range of reading appropriate for various grade levels with accompanying sample performance tasks. Appendix C includes annotated samples demonstrating at least adequate performance in student writing at various grade levels.

STANDARDS FOR

English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects

K–5

College and Career Readiness Anchor Standards for Reading

The K–5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Key Ideas and Details

1. Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
2. Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
3. Analyze how and why individuals, events, and ideas develop and interact over the course of a text.

Craft and Structure

4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
5. Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
6. Assess how point of view or purpose shapes the content and style of a text.

Integration of Knowledge and Ideas

7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.*
8. Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
9. Analyze how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.

Range of Reading and Level of Text Complexity

10. Read and comprehend complex literary and informational texts independently and proficiently.

*Please see “Research to Build and Present Knowledge” in Writing and “Comprehension and Collaboration” in Speaking and Listening for additional standards relevant to gathering, assessing, and applying information from print and digital sources.

Note on range and content of student reading

To build a foundation for college and career readiness, students must read widely and deeply from among a broad range of high-quality, increasingly challenging literary and informational texts. Through extensive reading of stories, dramas, poems, and myths from diverse cultures and different time periods, students gain literary and cultural knowledge as well as familiarity with various text structures and elements. By reading texts in history/social studies, science, and other disciplines, students build a foundation of knowledge in these fields that will also give them the background to be better readers in all content areas. Students can only gain this foundation when the curriculum is intentionally and coherently structured to develop rich content knowledge within and across grades. Students also acquire the habits of reading independently and closely, which are essential to their future success.

Reading Standards for Literature K–5

RL

The following standards offer a focus for instruction each year and help ensure that students gain adequate exposure to a range of texts and tasks. Rigor is also infused through the requirement that students read increasingly complex texts through the grades. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.*

Kindergartners:	Grade 1 students:	Grade 2 students:
Key Ideas and Details		
1. With prompting and support, ask and answer questions about key details in a text.	1. Ask and answer questions about key details in a text.	1. Ask and answer such questions as *who*, *what*, *where*, *when*, *why*, and *how* to demonstrate understanding of key details in a text.
2. With prompting and support, retell familiar stories, including key details.	2. Retell stories, including key details, and demonstrate understanding of their central message or lesson.	2. Recount stories, including fables and folktales from diverse cultures, and determine their central message, lesson, or moral.
3. With prompting and support, identify characters, settings, and major events in a story.	3. Describe characters, settings, and major events in a story, using key details.	3. Describe how characters in a story respond to major events and challenges.
Craft and Structure		
4. Ask and answer questions about unknown words in a text.	4. Identify words and phrases in stories or poems that suggest feelings or appeal to the senses.	4. Describe how words and phrases (e.g., regular beats, alliteration, rhymes, repeated lines) supply rhythm and meaning in a story, poem, or song.
5. Recognize common types of texts (e.g., storybooks, poems).	5. Explain major differences between books that tell stories and books that give information, drawing on a wide reading of a range of text types.	5. Describe the overall structure of a story, including describing how the beginning introduces the story and the ending concludes the action.
6. With prompting and support, name the author and illustrator of a story and define the role of each in telling the story.	6. Identify who is telling the story at various points in a text.	6. Acknowledge differences in the points of view of characters, including by speaking in a different voice for each character when reading dialogue aloud.
Integration of Knowledge and Ideas		
7. With prompting and support, describe the relationship between illustrations and the story in which they appear (e.g., what moment in a story an illustration depicts).	7. Use illustrations and details in a story to describe its characters, setting, or events.	7. Use information gained from the illustrations and words in a print or digital text to demonstrate understanding of its characters, setting, or plot.
8. (Not applicable to literature)	8. (Not applicable to literature)	8. (Not applicable to literature)
9. With prompting and support, compare and contrast the adventures and experiences of characters in familiar stories.	9. Compare and contrast the adventures and experiences of characters in stories.	9. Compare and contrast two or more versions of the same story (e.g., Cinderella stories) by different authors or from different cultures.
Range of Reading and Level of Text Complexity		
10. Actively engage in group reading activities with purpose and understanding.	10. With prompting and support, read prose and poetry of appropriate complexity for grade 1.	10. By the end of the year, read and comprehend literature, including stories and poetry, in the grades 2–3 text complexity band proficiently, with scaffolding as needed at the high end of the range.

Reading Standards for Literature K–5

RL

Grade 3 students:	Grade 4 students:	Grade 5 students:
Key Ideas and Details		
1. Ask and answer questions to demonstrate understanding of a text, referring explicitly to the text as the basis for the answers.	1. Refer to details and examples in a text when explaining what the text says explicitly and when drawing inferences from the text.	1. Quote accurately from a text when explaining what the text says explicitly and when drawing inferences from the text.
2. Recount stories, including fables, folktales, and myths from diverse cultures; determine the central message, lesson, or moral and explain how it is conveyed through key details in the text.	2. Determine a theme of a story, drama, or poem from details in the text; summarize the text.	2. Determine a theme of a story, drama, or poem from details in the text, including how characters in a story or drama respond to challenges or how the speaker in a poem reflects upon a topic; summarize the text.
3. Describe characters in a story (e.g., their traits, motivations, or feelings) and explain how their actions contribute to the sequence of events.	3. Describe in depth a character, setting, or event in a story or drama, drawing on specific details in the text (e.g., a character's thoughts, words, or actions).	3. Compare and contrast two or more characters, settings, or events in a story or drama, drawing on specific details in the text (e.g., how characters interact).
Craft and Structure		
4. Determine the meaning of words and phrases as they are used in a text, distinguishing literal from nonliteral language.	4. Determine the meaning of words and phrases as they are used in a text, including those that allude to significant characters found in mythology (e.g., Herculean).	4. Determine the meaning of words and phrases as they are used in a text, including figurative language such as metaphors and similes.
5. Refer to parts of stories, dramas, and poems when writing or speaking about a text, using terms such as chapter, scene, and stanza; describe how each successive part builds on earlier sections.	5. Explain major differences between poems, drama, and prose, and refer to the structural elements of poems (e.g., verse, rhythm, meter) and drama (e.g., casts of characters, settings, descriptions, dialogue, stage directions) when writing or speaking about a text.	5. Explain how a series of chapters, scenes, or stanzas fits together to provide the overall structure of a particular story, drama, or poem.
6. Distinguish their own point of view from that of the narrator or those of the characters.	6. Compare and contrast the point of view from which different stories are narrated, including the difference between first- and third-person narrations.	6. Describe how a narrator's or speaker's point of view influences how events are described.
Integration of Knowledge and Ideas		
7. Explain how specific aspects of a text's illustrations contribute to what is conveyed by the words in a story (e.g., create mood, emphasize aspects of a character or setting).	7. Make connections between the text of a story or drama and a visual or oral presentation of the text, identifying where each version reflects specific descriptions and directions in the text.	7. Analyze how visual and multimedia elements contribute to the meaning, tone, or beauty of a text (e.g., graphic novel, multimedia presentation of fiction, folktale, myth, poem).
8. (Not applicable to literature)	8. (Not applicable to literature)	8. (Not applicable to literature)
9. Compare and contrast the themes, settings, and plots of stories written by the same author about the same or similar characters (e.g., in books from a series).	9. Compare and contrast the treatment of similar themes and topics (e.g., opposition of good and evil) and patterns of events (e.g., the quest) in stories, myths, and traditional literature from different cultures.	9. Compare and contrast stories in the same genre (e.g., mysteries and adventure stories) on their approaches to similar themes and topics.
Range of Reading and Level of Text Complexity		
10. By the end of the year, read and comprehend literature, including stories, dramas, and poetry, at the high end of the grades 2–3 text complexity band independently and proficiently.	10. By the end of the year, read and comprehend literature, including stories, dramas, and poetry, in the grades 4–5 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literature, including stories, dramas, and poetry, at the high end of the grades 4–5 text complexity band independently and proficiently.

Reading Standards for Informational Text K–5

Kindergartners:	Grade 1 students:	Grade 2 students:
Key Ideas and Details		
1. With prompting and support, ask and answer questions about key details in a text.	1. Ask and answer questions about key details in a text.	1. Ask and answer such questions as *who, what, where, when, why,* and *how* to demonstrate understanding of key details in a text.
2. With prompting and support, identify the main topic and retell key details of a text.	2. Identify the main topic and retell key details of a text.	2. Identify the main topic of a multiparagraph text as well as the focus of specific paragraphs within the text.
3. With prompting and support, describe the connection between two individuals, events, ideas, or pieces of information in a text.	3. Describe the connection between two individuals, events, ideas, or pieces of information in a text.	3. Describe the connection between a series of historical events, scientific ideas or concepts, or steps in technical procedures in a text.
Craft and Structure		
4. With prompting and support, ask and answer questions about unknown words in a text.	4. Ask and answer questions to help determine or clarify the meaning of words and phrases in a text.	4. Determine the meaning of words and phrases in a text relevant to a *grade 2 topic or subject area*.
5. Identify the front cover, back cover, and title page of a book.	5. Know and use various text features (e.g., headings, tables of contents, glossaries, electronic menus, icons) to locate key facts or information in a text.	5. Know and use various text features (e.g., captions, bold print, subheadings, glossaries, indexes, electronic menus, icons) to locate key facts or information in a text efficiently.
6. Name the author and illustrator of a text and define the role of each in presenting the ideas or information in a text.	6. Distinguish between information provided by pictures or other illustrations and information provided by the words in a text.	6. Identify the main purpose of a text, including what the author wants to answer, explain, or describe.
Integration of Knowledge and Ideas		
7. With prompting and support, describe the relationship between illustrations and the text in which they appear (e.g., what person, place, thing, or idea in the text an illustration depicts).	7. Use the illustrations and details in a text to describe its key ideas.	7. Explain how specific images (e.g., a diagram showing how a machine works) contribute to and clarify a text.
8. With prompting and support, identify the reasons an author gives to support points in a text.	8. Identify the reasons an author gives to support points in a text.	8. Describe how reasons support specific points the author makes in a text.
9. With prompting and support, identify basic similarities in and differences between two texts on the same topic (e.g., in illustrations, descriptions, or procedures).	9. Identify basic similarities in and differences between two texts on the same topic (e.g., in illustrations, descriptions, or procedures).	9. Compare and contrast the most important points presented by two texts on the same topic.
Range of Reading and Level of Text Complexity		
10. Actively engage in group reading activities with purpose and understanding.	10. With prompting and support, read informational texts appropriately complex for grade 1.	10. By the end of year, read and comprehend informational texts, including history/social studies, science, and technical texts, in the grades 2–3 text complexity band proficiently, with scaffolding as needed at the high end of the range.

Reading Standards for Informational Text K–5

RI

Grade 3 students:	Grade 4 students:	Grade 5 students:
Key Ideas and Details		
1. Ask and answer questions to demonstrate understanding of a text, referring explicitly to the text as the basis for the answers.	1. Refer to details and examples in a text when explaining what the text says explicitly and when drawing inferences from the text.	1. Quote accurately from a text when explaining what the text says explicitly and when drawing inferences from the text.
2. Determine the main idea of a text; recount the key details and explain how they support the main idea.	2. Determine the main idea of a text and explain how it is supported by key details; summarize the text.	2. Determine two or more main ideas of a text and explain how they are supported by key details; summarize the text.
3. Describe the relationship between a series of historical events, scientific ideas or concepts, or steps in technical procedures in a text, using language that pertains to time, sequence, and cause/effect.	3. Explain events, procedures, ideas, or concepts in a historical, scientific, or technical text, including what happened and why, based on specific information in the text.	3. Explain the relationships or interactions between two or more individuals, events, ideas, or concepts in a historical, scientific, or technical text based on specific information in the text.
Craft and Structure		
4. Determine the meaning of general academic and domain-specific words and phrases in a text relevant to a *grade 3 topic or subject area*.	4. Determine the meaning of general academic and domain-specific words or phrases in a text relevant to a *grade 4 topic or subject area*.	4. Determine the meaning of general academic and domain-specific words and phrases in a text relevant to a *grade 5 topic or subject area*.
5. Use text features and search tools (e.g., key words, sidebars, hyperlinks) to locate information relevant to a given topic efficiently.	5. Describe the overall structure (e.g., chronology, comparison, cause/effect, problem/solution) of events, ideas, concepts, or information in a text or part of a text.	5. Compare and contrast the overall structure (e.g., chronology, comparison, cause/effect, problem/solution) of events, ideas, concepts, or information in two or more texts.
6. Distinguish their own point of view from that of the author of a text.	6. Compare and contrast a firsthand and secondhand account of the same event or topic; describe the differences in focus and the information provided.	6. Analyze multiple accounts of the same event or topic, noting important similarities and differences in the point of view they represent.
Integration of Knowledge and Ideas		
7. Use information gained from illustrations (e.g., maps, photographs) and the words in a text to demonstrate understanding of the text (e.g., where, when, why, and how key events occur).	7. Interpret information presented visually, orally, or quantitatively (e.g., in charts, graphs, diagrams, time lines, animations, or interactive elements on Web pages) and explain how the information contributes to an understanding of the text in which it appears.	7. Draw on information from multiple print or digital sources, demonstrating the ability to locate an answer to a question quickly or to solve a problem efficiently.
8. Describe the logical connection between particular sentences and paragraphs in a text (e.g., comparison, cause/effect, first/second/third in a sequence).	8. Explain how an author uses reasons and evidence to support particular points in a text.	8. Explain how an author uses reasons and evidence to support particular points in a text, identifying which reasons and evidence support which point(s).
9. Compare and contrast the most important points and key details presented in two texts on the same topic.	9. Integrate information from two texts on the same topic in order to write or speak about the subject knowledgeably.	9. Integrate information from several texts on the same topic in order to write or speak about the subject knowledgeably.
Range of Reading and Level of Text Complexity		
10. By the end of the year, read and comprehend informational texts, including history/social studies, science, and technical texts, at the high end of the grades 2–3 text complexity band independently and proficiently.	10. By the end of year, read and comprehend informational texts, including history/social studies, science, and technical texts, in the grades 4–5 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend informational texts, including history/social studies, science, and technical texts, at the high end of the grades 4–5 text complexity band independently and proficiently.

Reading Standards: Foundational Skills (K–5)

RF

These standards are directed toward fostering students' understanding and working knowledge of concepts of print, the alphabetic principle, and other basic conventions of the English writing system. These foundational skills are not an end in and of themselves; rather, they are necessary and important components of an effective, comprehensive reading program designed to develop proficient readers with the capacity to comprehend texts across a range of types and disciplines. Instruction should be differentiated: good readers will need much less practice with these concepts than struggling readers will. The point is to teach students what they need to learn and not what they already know—to discern when particular children or activities warrant more or less attention.

Note: In kindergarten, children are expected to demonstrate increasing awareness and competence in the areas that follow.

Kindergartners:	Grade 1 students:
Print Concepts	
1. Demonstrate understanding of the organization and basic features of print. a. Follow words from left to right, top to bottom, and page by page. b. Recognize that spoken words are represented in written language by specific sequences of letters. c. Understand that words are separated by spaces in print. d. Recognize and name all upper- and lowercase letters of the alphabet.	1. Demonstrate understanding of the organization and basic features of print. a. Recognize the distinguishing features of a sentence (e.g., first word, capitalization, ending punctuation).
Phonological Awareness	
2. Demonstrate understanding of spoken words, syllables, and sounds (phonemes). a. Recognize and produce rhyming words. b. Count, pronounce, blend, and segment syllables in spoken words. c. Blend and segment onsets and rimes of single-syllable spoken words. d. Isolate and pronounce the initial, medial vowel, and final sounds (phonemes) in three-phoneme (consonent-vowel-consonent, or CVC) words.* (This does not include CVCs ending with /l/, /r/, or /x/.) e. Add or substitute individual sounds (phonemes) in simple, one-syllable words to make new words.	2. Demonstrate understanding of spoken words, syllables, and sounds (phonemes). a. Distinguish long from short vowel sounds in spoken single-syllable words. b. Orally produce single-syllable words by blending sounds (phonemes), including consonant blends. c. Isolate and pronounce initial, medial vowel, and final sounds (phonemes) in spoken single-syllable words. d. Segment spoken single-syllable words into their complete sequence of individual sounds (phonemes).

*Words, syllables, or phonemes written in /slashes/refer to their pronunciation or phonology.
Thus, /CVC/ is a word with three phonemes regardless of the number of letters in the spelling of the word.

Reading Standards: Foundational Skills (K–5)

RF

Note: In kindergarten children are expected to demonstrate increasing awareness and competence in the areas that follow.

Kindergartners:	Grade 1 students:	Grade 2 students:
Phonics and Word Recognition		
3. Know and apply grade-level phonics and word analysis skills in decoding words. a. Demonstrate basic knowledge of one-to-one letter-sound correspondences by producing the primary or many of the most frequent sound for each consonant. b. Associate the long and short sounds with common spellings (graphemes) for the five major vowels. c. Read common high-frequency words by sight (e.g., *the*, *of*, *to*, *you*, *she*, *my*, *is*, *are*, *do*, *does*). d. Distinguish between similarly spelled words by identifying the sounds of the letters that differ.	3. Know and apply grade-level phonics and word analysis skills in decoding words. a. Know the spelling-sound correspondences for common consonant digraphs. b. Decode regularly spelled one-syllable words. c. Know final -e and common vowel team conventions for representing long vowel sounds. d. Use knowledge that every syllable must have a vowel sound to determine the number of syllables in a printed word. e. Decode two-syllable words following basic patterns by breaking the words into syllables. f. Read words with inflectional endings. g. Recognize and read grade-appropriate irregularly spelled words.	3. Know and apply grade-level phonics and word analysis skills in decoding words. a. Distinguish long and short vowels when reading regularly spelled one-syllable words. b. Know spelling-sound correspondences for additional common vowel teams. c. Decode regularly spelled two-syllable words with long vowels. d. Decode words with common prefixes and suffixes. e. Identify words with inconsistent but common spelling-sound correspondences. f. Recognize and read grade-appropriate irregularly spelled words.
Fluency		
4. Read emergent-reader texts with purpose and understanding.	4. Read with sufficient accuracy and fluency to support comprehension. a. Read on-level text with purpose and understanding. b. Read on-level text orally with accuracy, appropriate rate, and expression on successive readings. c. Use context to confirm or self-correct word recognition and understanding, rereading as necessary.	4. Read with sufficient accuracy and fluency to support comprehension. a. Read on-level text with purpose and understanding. b. Read on-level text orally with accuracy, appropriate rate, and expression on successive readings. c. Use context to confirm or self-correct word recognition and understanding, rereading as necessary.

Reading Standards: Foundational Skills (K–5)

RF

Grade 3 students:	Grade 4 students:	Grade 5 students:
Phonics and Word Recognition		
3. Know and apply grade-level phonics and word analysis skills in decoding words. a. Identify and know the meaning of the most common prefixes and derivational suffixes. b. Decode words with common Latin suffixes. c. Decode multisyllable words. d. Read grade-appropriate irregularly spelled words.	3. Know and apply grade-level phonics and word analysis skills in decoding words. a. Use combined knowledge of all letter-sound correspondences, syllabication patterns, and morphology (e.g., roots and affixes) to read accurately unfamiliar multisyllabic words in context and out of context.	3. Know and apply grade-level phonics and word analysis skills in decoding words. a. Use combined knowledge of all letter-sound correspondences, syllabication patterns, and morphology (e.g., roots and affixes) to read accurately unfamiliar multisyllabic words in context and out of context.
Fluency		
4. Read with sufficient accuracy and fluency to support comprehension. a. Read on-level text with purpose and understanding. b. Read on-level prose and poetry orally with accuracy, appropriate rate, and expression on successive readings c. Use context to confirm or self-correct word recognition and understanding, rereading as necessary.	4. Read with sufficient accuracy and fluency to support comprehension. a. Read on-level text with purpose and understanding. b. Read on-level prose and poetry orally with accuracy, appropriate rate, and expression on successive readings. c. Use context to confirm or self-correct word recognition and understanding, rereading as necessary.	4. Read with sufficient accuracy and fluency to support comprehension. a. Read on-level text with purpose and understanding. b. Read on-level prose and poetry orally with accuracy, appropriate rate, and expression on successive readings. c. Use context to confirm or self-correct word recognition and understanding, rereading as necessary.

College and Career Readiness Anchor Standards for Writing

The K–5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Text Types and Purposes*

1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences.

Production and Distribution of Writing

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

Research to Build and Present Knowledge

7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

Range of Writing

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

*These broad types of writing include many subgenres. See Appendix A for definitions of key writing types.

Note on range and content of student writing

To build a foundation for college and career readiness, students need to learn to use writing as a way of offering and supporting opinions, demonstrating understanding of the subjects they are studying, and conveying real and imagined experiences and events. They learn to appreciate that a key purpose of writing is to communicate clearly to an external, sometimes unfamiliar audience, and they begin to adapt the form and content of their writing to accomplish a particular task and purpose. They develop the capacity to build knowledge on a subject through research projects and to respond analytically to literary and informational sources. To meet these goals, students must devote significant time and effort to writing, producing numerous pieces over short and extended time frames throughout the year.

Writing Standards K–5

W

The following standards for K–5 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. Each year in their writing, students should demonstrate increasing sophistication in all aspects of language use, from vocabulary and syntax to the development and organization of ideas, and they should address increasingly demanding content and sources. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.* The expected growth in student writing ability is reflected both in the standards themselves and in the collection of annotated student writing samples in Appendix C.

Kindergartners:	Grade 1 students:	Grade 2 students:
Text Types and Purposes		
1. Use a combination of drawing, dictating, and writing to compose opinion pieces in which they tell a reader the topic or the name of the book they are writing about and state an opinion or preference about the topic or book (e.g., *My favorite book is . . .*).	1. Write opinion pieces in which they introduce the topic or name the book they are writing about, state an opinion, supply a reason for the opinion, and provide some sense of closure.	1. Write opinion pieces in which they introduce the topic or book they are writing about, state an opinion, supply reasons that support the opinion, use linking words (e.g., *because*, *and*, *also*) to connect opinion and reasons, and provide a concluding statement or section.
2. Use a combination of drawing, dictating, and writing to compose informative/explanatory texts in which they name what they are writing about and supply some information about the topic.	2. Write informative/explanatory texts in which they name a topic, supply some facts about the topic, and provide some sense of closure.	2. Write informative/explanatory texts in which they introduce a topic, use facts and definitions to develop points, and provide a concluding statement or section.
3. Use a combination of drawing, dictating, and writing to narrate a single event or several loosely linked events, tell about the events in the order in which they occurred, and provide a reaction to what happened.	3. Write narratives in which they recount two or more appropriately sequenced events, include some details regarding what happened, use temporal words to signal event order, and provide some sense of closure.	3. Write narratives in which they recount a well-elaborated event or short sequence of events, include details to describe actions, thoughts, and feelings, use temporal words to signal event order, and provide a sense of closure.
Production and Distribution of Writing		
4. (Begins in grade 3)	4. (Begins in grade 3)	4. (Begins in grade 3)
5. With guidance and support from adults, respond to questions and suggestions from peers and add details to strengthen writing as needed.	5. With guidance and support from adults, focus on a topic, respond to questions and suggestions from peers, and add details to strengthen writing as needed.	5. With guidance and support from adults and peers, focus on a topic and strengthen writing as needed by revising and editing.
6. With guidance and support from adults, explore a variety of digital tools to produce and publish writing, including in collaboration with peers.	6. With guidance and support from adults, use a variety of digital tools to produce and publish writing, including in collaboration with peers.	6. With guidance and support from adults, use a variety of digital tools to produce and publish writing, including in collaboration with peers.
Research to Build and Present Knowledge		
7. Participate in shared research and writing projects (e.g., explore a number of books by a favorite author and express opinions about them).	7. Participate in shared research and writing projects (e.g., explore a number of "how-to" books on a given topic and use them to write a sequence of instructions).	7. Participate in shared research and writing projects (e.g., read a number of books on a single topic to produce a report; record science observations).
8. With guidance and support from adults, recall information from experiences or gather information from provided sources to answer a question.	8. With guidance and support from adults, recall information from experiences or gather information from provided sources to answer a question.	8. Recall information from experiences or gather information from provided sources to answer a question.
9. (Begins in grade 4)	9. (Begins in grade 4)	9. (Begins in grade 4)
Range of Writing		
10. (Begins in grade 3)	10. (Begins in grade 3)	10. (Begins in grade 3)

Writing Standards K–5

W

Grade 3 students:	Grade 4 students:	Grade 5 students:
Text Types and Purposes		
1. Write opinion pieces on topics or texts, supporting a point of view with reasons. a. Introduce the topic or text they are writing about, state an opinion, and create an organizational structure that lists reasons. b. Provide reasons that support the opinion. c. Use linking words and phrases (e.g., *because*, *therefore*, *since*, *for example*) to connect opinion and reasons. d. Provide a concluding statement or section.	1. Write opinion pieces on topics or texts, supporting a point of view with reasons and information. a. Introduce a topic or text clearly, state an opinion, and create an organizational structure in which related ideas are grouped to support the writer's purpose. b. Provide reasons that are supported by facts and details. c. Link opinion and reasons using words and phrases (e.g., *for instance*, *in order to*, *in addition*). d. Provide a concluding statement or section related to the opinion presented.	1. Write opinion pieces on topics or texts, supporting a point of view with reasons and information. a. Introduce a topic or text clearly, state an opinion, and create an organizational structure in which ideas are logically grouped to support the writer's purpose. b. Provide logically ordered reasons that are supported by facts and details. c. Link opinion and reasons using words, phrases, and clauses (e.g., *consequently*, *specifically*). d. Provide a concluding statement or section related to the opinion presented.
2. Write informative/explanatory texts to examine a topic and convey ideas and information clearly. a. Introduce a topic and group related information together; include illustrations when useful to aiding comprehension. b. Develop the topic with facts, definitions, and details. c. Use linking words and phrases (e.g., *also*, *another*, *and*, *more*, *but*) to connect ideas within categories of information. d. Provide a concluding statement or section.	2. Write informative/explanatory texts to examine a topic and convey ideas and information clearly. a. Introduce a topic clearly and group related information in paragraphs and sections; include formatting (e.g., headings), illustrations, and multimedia when useful to aiding comprehension. b. Develop the topic with facts, definitions, concrete details, quotations, or other information and examples related to the topic. c. Link ideas within categories of information using words and phrases (e.g., *another*, *for example*, *also*, *because*). d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Provide a concluding statement or section related to the information or explanation presented.	2. Write informative/explanatory texts to examine a topic and convey ideas and information clearly. a. Introduce a topic clearly, provide a general observation and focus, and group related information logically; include formatting (e.g., headings), illustrations, and multimedia when useful to aiding comprehension. b. Develop the topic with facts, definitions, concrete details, quotations, or other information and examples related to the topic. c. Link ideas within and across categories of information using words, phrases, and clauses (e.g., *in contrast*, *especially*). d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Provide a concluding statement or section related to the information or explanation presented.
3. Write narratives to develop real or imagined experiences or events using effective technique, descriptive details, and clear event sequences. a. Establish a situation and introduce a narrator and/or characters; organize an event sequence that unfolds naturally. b. Use dialogue and descriptions of actions, thoughts, and feelings to develop experiences and events or show the response of characters to situations. c. Use temporal words and phrases to signal event order. d. Provide a sense of closure.	3. Write narratives to develop real or imagined experiences or events using effective technique, descriptive details, and clear event sequences. a. Orient the reader by establishing a situationand introducing a narrator and/or characters; organize an event sequence that unfolds naturally. b. Use dialogue and description to develop experiences and events or show the responses of characters to situations. c. Use a variety of transitional words and phrases to manage the sequence of events. d. Use concrete words and phrases and sensory details to convey experiences and events precisely. e. Provide a conclusion that follows from the narrated experiences or events.	3. Write narratives to develop real or imagined experiences or events using effective technique, descriptive details, and clear event sequences. a. Orient the reader by establishing a situation and introducing a narrator and/or characters; organize an event sequence that unfolds naturally. b. Use narrative techniques, such as dialogue, description, and pacing, to develop experiences and events or show the responses of characters to situations. c. Use a variety of transitional words, phrases, and clauses to manage the sequence of events. d. Use concrete words and phrases and sensory details to convey experiences and events precisely. e. Provide a conclusion that follows from the narrated experiences or events.

Writing Standards K–5

W

Grade 3 students:	Grade 4 students:	Grade 5 students:
Production and Distribution of Writing		
4. With guidance and support from adults, produce writing in which the development and organization are appropriate to task and purpose. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	4. Produce clear and coherent writing in which the development and organization are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	4. Produce clear and coherent writing in which the development and organization are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
5. With guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, and editing. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 3 on pages 28 and 29.)	5. With guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, and editing. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 4 on pages 28 and 29.)	5. With guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 5 on pages 28 and 29.)
6. With guidance and support from adults, use technology to produce and publish writing (using keyboarding skills) as well as to interact and collaborate with others.	6. With some guidance and support from adults, use technology, including the Internet, to produce and publish writing as well as to interact and collaborate with others; demonstrate sufficient command of keyboarding skills to type a minimum of one page in a single sitting.	6. With some guidance and support from adults, use technology, including the Internet, to produce and publish writing as well as to interact and collaborate with others; demonstrate sufficient command of keyboarding skills to type a minimum of two pages in a single sitting.
Research to Build and Present Knowledge		
7. Conduct short research projects that build knowledge about a topic.	7. Conduct short research projects that build knowledge through investigation of different aspects of a topic.	7. Conduct short research projects that use several sources to build knowledge through investigation of different aspects of a topic.
8. Recall information from experiences or gather information from print and digital sources; take brief notes on sources and sort evidence into provided categories.	8. Recall relevant information from experiences or gather relevant information from print and digital sources; take notes and categorize information, and provide a list of sources.	8. Recall relevant information from experiences or gather relevant information from print and digital sources; summarize or paraphrase information in notes and finished work, and provide a list of sources.
9. (Begins in grade 4)	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grade 4 Reading standards* to literature (e.g., "Describe in depth a character, setting, or event in a story or drama, drawing on specific details in the text [e.g., a character's thoughts, words, or actions]."). b. Apply *grade 4 Reading standards* to informational texts (e.g., "Explain how an author uses reasons and evidence to support particular points in a text").	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grade 5 Reading standards* to literature (e.g., "Compare and contrast two or more characters, settings, or events in a story or a drama, drawing on specific details in the text [e.g., how characters interact]"). b. Apply *grade 5 Reading standards* to informational texts (e.g., "Explain how an author uses reasons and evidence to support particular points in a text, identifying which reasons and evidence support which point[s]").
Range of Writing		
10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

College and Career Readiness Anchor Standards for Speaking and Listening

The K–5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Comprehension and Collaboration

1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
2. Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.
3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

Presentation of Knowledge and Ideas

4. Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
5. Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.
6. Adapt speech to a variety of contexts and communicative tasks, demonstrating command of formal English when indicated or appropriate.

Note on range and content of student speaking and listening

To build a foundation for college and career readiness, students must have ample opportunities to take part in a variety of rich, structured conversations—as part of a whole class, in small groups, and with a partner. Being productive members of these conversations requires that students contribute accurate, relevant information; respond to and develop what others have said; make comparisons and contrasts; and analyze and synthesize a multitude of ideas in various domains.

New technologies have broadened and expanded the role that speaking and listening play in acquiring and sharing knowledge and have tightened their link to other forms of communication. Digital texts confront students with the potential for continually updated content and dynamically changing combinations of words, graphics, images, hyperlinks, and embedded video and audio.

Speaking and Listening Standards K–5

SL

The following standards for K–5 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.*

Kindergartners:	Grade 1 students:	Grade 2 students:
Comprehension and Collaboration		
1. Participate in collaborative conversations with diverse partners about *kindergarten topics and texts* with peers and adults in small and larger groups. a. Follow agreed-upon rules for discussions (e.g., listening to others and taking turns speaking about the topics and texts under discussion). b. Continue a conversation through multiple exchanges.	1. Participate in collaborative conversations with diverse partners about *grade 1 topics and texts* with peers and adults in small and larger groups. a. Follow agreed-upon rules for discussions (e.g., listening to others with care, speaking one at a time about the topics and texts under discussion). b. Build on others' talk in conversations by responding to the comments of others through multiple exchanges. c. Ask questions to clear up any confusion about the topics and texts under discussion.	1. Participate in collaborative conversations with diverse partners about *grade 2 topics and texts* with peers and adults in small and larger groups. a. Follow agreed-upon rules for discussions (e.g., gaining the floor in respectful ways, listening to others with care, speaking one at a time about the topics and texts under discussion). b. Build on others' talk in conversations by linking their comments to the remarks of others. c. Ask for clarification and further explanation as needed about the topics and texts under discussion.
2. Confirm understanding of a text read aloud or information presented orally or through other media by asking and answering questions about key details and requesting clarification if something is not understood.	2. Ask and answer questions about key details in a text read aloud or information presented orally or through other media.	2. Recount or describe key ideas or details from a text read aloud or information presented orally or through other media.
3. Ask and answer questions in order to seek help, get information, or clarify something that is not understood.	3. Ask and answer questions about what a speaker says in order to gather additional information or clarify something that is not understood.	3. Ask and answer questions about what a speaker says in order to clarify comprehension, gather additional information, or deepen understanding of a topic or issue.
Presentation of Knowledge and Ideas		
4. Describe familiar people, places, things, and events and, with prompting and support, provide additional detail.	4. Describe people, places, things, and events with relevant details, expressing ideas and feelings clearly.	4. Tell a story or recount an experience with appropriate facts and relevant, descriptive details, speaking audibly in coherent sentences.
5. Add drawings or other visual displays to descriptions as desired to provide additional detail.	5. Add drawings or other visual displays to descriptions when appropriate to clarify ideas, thoughts, and feelings.	5. Create audio recordings of stories or poems; add drawings or other visual displays to stories or recounts of experiences when appropriate to clarify ideas, thoughts, and feelings.
6. Speak audibly and express thoughts, feelings, and ideas clearly.	6. Produce complete sentences when appropriate to task and situation. (See grade 1 Language standards 1 and 3 on page 26 for specific expectations.)	6. Produce complete sentences when appropriate to task and situation in order to provide requested detail or clarification. (See grade 2 Language standards 1 and 3 on pages 26 and 27 for specific expectations.)

Speaking and Listening Standards K–5

SL

Grade 3 students:	Grade 4 students:	Grade 5 students:
Comprehension and Collaboration		
1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 3 topics and texts*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion. b. Follow agreed-upon rules for discussions (e.g., gaining the floor in respectful ways, listening to others with care, speaking one at a time about the topics and texts under discussion). c. Ask questions to check understanding of information presented, stay on topic, and link their comments to the remarks of others. d. Explain their own ideas and understanding in light of the discussion.	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 4 topics and texts*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion. b. Follow agreed-upon rules for discussions and carry out assigned roles. c. Pose and respond to specific questions to clarify or follow up on information, and make comments that contribute to the discussion and link to the remarks of others. d. Review the key ideas expressed and explain their own ideas and understanding in light of the discussion.	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 5 topics and texts*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion. b. Follow agreed-upon rules for discussions and carry out assigned roles. c. Pose and respond to specific questions by making comments that contribute to the discussion and elaborate on the remarks of others. d. Review the key ideas expressed and draw conclusions in light of information and knowledge gained from the discussions.
2. Determine the main ideas and supporting details of a text read aloud or information presented in diverse media and formats, including visually, quantitatively, and orally.	2. Paraphrase portions of a text read aloud or information presented in diverse media and formats, including visually, quantitatively, and orally.	2. Summarize a written text read aloud or information presented in diverse media and formats, including visually, quantitatively, and orally.
3. Ask and answer questions about information from a speaker, offering appropriate elaboration and detail.	3. Identify the reasons and evidence a speaker provides to support particular points.	3. Summarize the points a speaker makes and explain how each claim is supported by reasons and evidence.
Presentation of Knowledge and Ideas		
4. Report on a topic or text, tell a story, or recount an experience with appropriate facts and relevant, descriptive details, speaking clearly at an understandable pace.	4. Report on a topic or text, tell a story, or recount an experience in an organized manner, using appropriate facts and relevant, descriptive details to support main ideas or themes; speak clearly at an understandable pace.	4. Report on a topic or text or present an opinion, sequencing ideas logically and using appropriate facts and relevant, descriptive details to support main ideas or themes; speak clearly at an understandable pace.
5. Create engaging audio recordings of stories or poems that demonstrate fluid reading at an understandable pace; add visual displays when appropriate to emphasize or enhance certain facts or details.	5. Add audio recordings and visual displays to presentations when appropriate to enhance the development of main ideas or themes.	5. Include multimedia components (e.g., graphics, sound) and visual displays in presentations when appropriate to enhance the development of main ideas or themes.
6. Speak in complete sentences when appropriate to task and situation in order to provide requested detail or clarification. (See grade 3 Language standards 1 and 3 on pages 28 and 29 for specific expectations.)	6. Differentiate between contexts that call for formal English (e.g., presenting ideas) and situations where informal discourse is appropriate (e.g., small-group discussion); use formal English when appropriate to task and situation. (See grade 4 Language standards 1 on pages 28 and 29 for specific expectations.)	6. Adapt speech to a variety of contexts and tasks, using formal English when appropriate to task and situation. (See grade 5 Language standards 1 and 3 on pages 28 and 29 for specific expectations.)

College and Career Readiness Anchor Standards for Language

The K–5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Conventions of Standard English

1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.

Knowledge of Language

3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.

Vocabulary Acquisition and Use

4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings.
6. Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.

Note on range and content of student language use

To build a foundation for college and career readiness in language, students must gain control over many conventions of standard English grammar, usage, and mechanics as well as learn other ways to use language to convey meaning effectively. They must also be able to determine or clarify the meaning of grade-appropriate words encountered through listening, reading, and media use; come to appreciate that words have nonliteral meanings, shadings of meaning, and relationships to other words; and expand their vocabulary in the course of studying content. The inclusion of Language standards in their own strand should not be taken as an indication that skills related to conventions, effective language use, and vocabulary are unimportant to reading, writing, speaking, and listening; indeed, they are inseparable from such contexts.

Language Standards K–5

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The following standards for grades K–5 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.* Beginning in grade 3, skills and understandings that are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking are marked with an asterisk (*). See the table on page 30 for a complete list and Appendix A for an example of how these skills develop in sophistication.

Kindergartners:	Grade 1 students:	Grade 2 students:
Conventions of Standard English		
1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Print many upper- and lowercase letters. b. Use frequently occurring nouns and verbs. c. Form regular plural nouns orally by adding /s/ or /es/ (e.g., *dog, dogs*; *wish, wishes*). d. Understand and use question words (interrogatives) (e.g., *who, what, where, when, why, how*). e. Use the most frequently occurring prepositions (e.g., *to, from, in, out, on, off, for, of, by, with*). f. Produce and expand complete sentences in shared language activities.	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Print all upper- and lowercase letters. b. Use common, proper, and possessive nouns. c. Use singular and plural nouns with matching verbs in basic sentences (e.g., *He hops*; *We hop*). d. Use personal, possessive, and indefinite pronouns (e.g., *I, me, my; they, them, their; anyone, everything*). e. Use verbs to convey a sense of past, present, and future (e.g., *Yesterday I walked home; Today I walk home; Tomorrow I will walk home*). f. Use frequently occurring adjectives. g. Use frequently occurring conjunctions (e.g., *and, but, or, so, because*). h. Use determiners (e.g., articles, demonstratives). i. Use frequently occurring prepositions (e.g., *during, beyond, toward*). j. Produce and expand complete simple and compound declarative, interrogative, imperative, and exclamatory sentences in response to prompts.	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Use collective nouns (e.g., *group*). b. Form and use frequently occurring irregular plural nouns (e.g., *feet, children, teeth, mice, fish*). c. Use reflexive pronouns (e.g., *myself, ourselves*). d. Form and use the past tense of frequently occurring irregular verbs (e.g., *sat, hid, told*). e. Use adjectives and adverbs, and choose between them depending on what is to be modified. f. Produce, expand, and rearrange complete simple and compound sentences (e.g., *The boy watched the movie; The little boy watched the movie; The action movie was watched by the little boy*).
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Capitalize the first word in a sentence and the pronoun *I*. b. Recognize and name end punctuation. c. Write a letter or letters for most consonant and short-vowel sounds (phonemes). d. Spell simple words phonetically, drawing on knowledge of sound-letter relationships.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Capitalize dates and names of people. b. Use end punctuation for sentences. c. Use commas in dates and to separate single words in a series. d. Use conventional spelling for words with common spelling patterns and for frequently occurring irregular words. e. Spell untaught words phonetically, drawing on phonemic awareness and spelling conventions.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Capitalize holidays, product names, and geographic names. b. Use commas in greetings and closings of letters. c. Use an apostrophe to form contractions and frequently occurring possessives. d. Generalize learned spelling patterns when writing words (e.g., cage → badge; boy → boil). e. Consult reference materials, including beginning dictionaries, as needed to check and correct spellings.

Language Standards K–5

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Kindergartners:	Grade 1 students:	Grade 2 students:
Knowledge of Language		
3. (Begins in grade 2)	3. (Begins in grade 2)	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Compare formal and informal uses of English.
Vocabulary Acquisition and Use		
4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *kindergarten reading and content.* a. Identify new meanings for familiar words and apply them accurately (e.g., knowing *duck* is a bird and learning the verb *to duck*). b. Use the most frequently occurring inflections and affixes (e.g., *-ed*, *-s*, *re-*, *un-*, *pre-*, *-ful*, *-less*) as a clue to the meaning of an unknown word.	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 1 reading and content*, choosing flexibly from an array of strategies. a. Use sentence-level context as a clue to the meaning of a word or phrase. b. Use frequently occurring affixes as a clue to the meaning of a word. c. Identify frequently occurring root words (e.g., *look*) and their inflectional forms (e.g., *looks*, *looked*, *looking*).	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 2 reading and content*, choosing flexibly from an array of strategies. a. Use sentence-level context as a clue to the meaning of a word or phrase. b. Determine the meaning of the new word formed when a known prefix is added to a known word (e.g., *happy/unhappy*, *tell/retell*). c. Use a known root word as a clue to the meaning of an unknown word with the same root (e.g., *addition*, *additional*). d. Use knowledge of the meaning of individual words to predict the meaning of compound words (e.g., *birdhouse, lighthouse, housefly; bookshelf, notebook, bookmark*). e. Use glossaries and beginning dictionaries, both print and digital, to determine or clarify the meaning of words and phrases.
5. With guidance and support from adults, explore word relationships and nuances in word meanings. a. Sort common objects into categories (e.g., shapes, foods) to gain a sense of the concepts the categories represent. b. Demonstrate understanding of frequently occurring verbs and adjectives by relating them to their opposites (antonyms). c. Identify real-life connections between words and their use (e.g., note places at school that are *colorful*). d. Distinguish shades of meaning among verbs describing the same general action (e.g., *walk, march, strut, prance*) by acting out the meanings.	5. With guidance and support from adults, demonstrate understanding of word relationships and nuances in word meanings. a. Sort words into categories (e.g., colors, clothing) to gain a sense of the concepts the categories represent. b. Define words by category and by one or more key attributes (e.g., a *duck* is a bird that swims; a *tiger* is a large cat with stripes). c. Identify real-life connections between words and their use (e.g., note places at home that are *cozy*). d. Distinguish shades of meaning among verbs differing in manner (e.g., *look, peek, glance, stare, glare, scowl*) and adjectives differing in intensity (e.g., *large, gigantic*) by defining or choosing them or by acting out the meanings.	5. Demonstrate understanding of word relationships and nuances in word meanings. a. Identify real-life connections between words and their use (e.g., describe foods that are *spicy* or *juicy*). b. Distinguish shades of meaning among closely related verbs (e.g., *toss, throw, hurl*) and closely related adjectives (e.g., *thin, slender, skinny, scrawny*).
6. Use words and phrases acquired through conversations, reading and being read to, and responding to texts.	6. Use words and phrases acquired through conversations, reading and being read to, and responding to texts, including using frequently occurring conjunctions to signal simple relationships (e.g., *because*).	6. Use words and phrases acquired through conversations, reading and being read to, and responding to texts, including using adjectives and adverbs to describe (e.g., *When other kids are happy that makes me happy*).

Language Standards K–5

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Grade 3 students:	Grade 4 students:	Grade 5 students:
Conventions of Standard English		
1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Explain the function of nouns, pronouns, verbs, adjectives, and adverbs in general and their functions in particular sentences. b. Form and use regular and irregular plural nouns. c. Use abstract nouns (e.g., *childhood*). d. Form and use regular and irregular verbs. e. Form and use the simple (e.g., *I walked; I walk; I will walk*) verb tenses. f. Ensure subject-verb and pronoun-antecedent agreement.* g. Form and use comparative and superlative adjectives and adverbs, and choose between them depending on what is to be modified. h. Use coordinating and subordinating conjunctions. i. Produce simple, compound, and complex sentences.	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Use relative pronouns (*who, whose, whom, which, that*) and relative adverbs (*where, when, why*). b. Form and use the progressive (e.g., *I was walking; I am walking; I will be walking*) verb tenses. c. Use modal auxiliaries (e.g., *can, may, must*) to convey various conditions. d. Order adjectives within sentences according to conventional patterns (e.g., *a small red bag* rather than *a red small bag*). e. Form and use prepositional phrases. f. Produce complete sentences, recognizing and correcting inappropriate fragments and run-ons.* g. Correctly use frequently confused words (e.g., *to, too, two; there, their*).*	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Explain the function of conjunctions, prepositions, and interjections in general and their function in particular sentences. b. Form and use the perfect (e.g., *I had walked; I have walked; I will have walked*) verb tenses. c. Use verb tense to convey various times, sequences, states, and conditions. d. Recognize and correct inappropriate shifts in verb tense.* e. Use correlative conjunctions (e.g., *either/or, neither/nor*).
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Capitalize appropriate words in titles. b. Use commas in addresses. c. Use commas and quotation marks in dialogue. d. Form and use possessives. e. Use conventional spelling for high-frequency and other studied words and for adding suffixes to base words (e.g., *sitting, smiled, cries, happiness*). f. Use spelling patterns and generalizations (e.g., word families, position-based spellings, syllable patterns, ending rules, meaningful word parts) in writing words. g. Consult reference materials, including beginning dictionaries, as needed to check and correct spellings.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use correct capitalization. b. Use commas and quotation marks to mark direct speech and quotations from a text. c. Use a comma before a coordinating conjunction in a compound sentence. d. Spell grade-appropriate words correctly, consulting references as needed.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use punctuation to separate items in a series.* b. Use a comma to separate an introductory element from the rest of the sentence. c. Use a comma to set off the words *yes* and *no* (e.g., *Yes, thank you*), to set off a tag question from the rest of the sentence (e.g., *It's true, isn't it?*), and to indicate direct address (e.g., *Is that you, Steve?*). d. Use underlining, quotation marks, or italics to indicate titles of works. e. Spell grade-appropriate words correctly, consulting references as needed.

Language Standards K–5

Grade 3 students:	Grade 4 students:	Grade 5 students:
Knowledge of Language		
3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Choose words and phrases for effect.* b. Recognize and observe differences between the conventions of spoken and written standard English.	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Choose words and phrases to convey ideas precisely.* b. Choose punctuation for effect.* c. Differentiate between contexts that call for formal English (e.g., presenting ideas) and situations where informal discourse is appropriate (e.g., small-group discussion).	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Expand, combine, and reduce sentences for meaning, reader/listener interest, and style. b. Compare and contrast the varieties of English (e.g., dialects, registers) used in stories, dramas, or poems.
Vocabulary Acquisition and Use		
4. Determine or clarify the meaning of unknown and multiple-meaning word and phrases based on *grade 3 reading and content,* choosing flexibly from a range of strategies. a. Use sentence-level context as a clue to the meaning of a word or phrase. b. Determine the meaning of the new word formed when a known affix is added to a known word (e.g., *agreeable/disagreeable, comfortable/uncomfortable, care/careless, heat/preheat*). c. Use a known root word as a clue to the meaning of an unknown word with the same root (e.g., *company, companion*). d. Use glossaries or beginning dictionaries, both print and digital, to determine or clarify the precise meaning of key words and phrases.	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 4 reading and content,* choosing flexibly from a range of strategies. a. Use context (e.g., definitions, examples, or restatements in text) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek and Latin affixes and roots as clues to the meaning of a word (e.g., *telegraph, photograph, autograph*). c. Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation and determine or clarify the precise meaning of key words and phrases.	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 5 reading and content,* choosing flexibly from a range of strategies. a. Use context (e.g., cause/effect relationships and comparisons in text) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek and Latin affixes and roots as clues to the meaning of a word (e.g., *photograph, photosynthesis*). c. Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation and determine or clarify the precise meaning of key words and phrases.
5. Demonstrate understanding of word relationships and nuances in word meanings. a. Distinguish the literal and nonliteral meanings of words and phrases in context (e.g., *take steps*). b. Identify real-life connections between words and their use (e.g., describe people who are *friendly* or *helpful*). c. Distinguish shades of meaning among related words that describe states of mind or degrees of certainty (e.g., *knew, believed, suspected, heard, wondered*).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Explain the meaning of simple similes and metaphors (e.g., *as pretty as a picture*) in context. b. Recognize and explain the meaning of common idioms, adages, and proverbs. c. Demonstrate understanding of words by relating them to their opposites (antonyms) and to words with similar but not identical meanings (synonyms).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figurative language, including similes and metaphors, in context. b. Recognize and explain the meaning of common idioms, adages, and proverbs. c. Use the relationship between particular words (e.g., synonyms, antonyms, homographs) to better understand each of the words.
6. Acquire and use accurately grade-appropriate conversational, general academic, and domain-specific words and phrases, including those that signal spatial and temporal relationships (e.g., *After dinner that night we went looking for them*).	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases, including those that signal precise actions, emotions, or states of being (e.g., *quizzed, whined, stammered*) and that are basic to a particular topic (e.g., *wildlife, conservation,* and *endangered* when discussing animal preservation).	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases, including those that signal contrast, addition, and other logical relationships (e.g., *however, although, nevertheless, similarly, moreover, in addition*).

Language Progressive Skills, by Grade

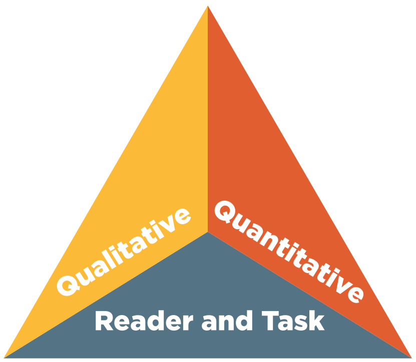
The following skills, marked with an asterisk (*) in Language standards 1–3, are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking.

Standard	Grade(s)							
	3	4	5	6	7	8	9–10	11–12
L.3.1f. Ensure subject-verb and pronoun-antecedent agreement.	■	■	■	■	■	■	■	■
L.3.3a. Choose words and phrases for effect.	■	■	■	■	■	■	■	■
L.4.1f. Produce complete sentences, recognizing and correcting inappropriate fragments and run-ons.		■	■	■	■	■	■	■
L.4.1g. Correctly use frequently confused words (e.g., *to/too/two; there/their*).		■	■	■	■	■	■	■
L.4.3a. Choose words and phrases to convey ideas precisely.*		■	■	■				
L.4.3b. Choose punctuation for effect.		■	■	■	■	■	■	■
L.5.1d. Recognize and correct inappropriate shifts in verb tense.			■	■	■	■	■	■
L.5.2a. Use punctuation to separate items in a series.†			■	■	■	■		
L.6.1c. Recognize and correct inappropriate shifts in pronoun number and person.				■	■	■	■	■
L.6.1d. Recognize and correct vague pronouns (i.e., ones with unclear or ambiguous antecedents).				■	■	■	■	■
L.6.1e. Recognize variations from standard English in their own and others' writing and speaking, and identify and use strategies to improve expression in conventional language.				■	■	■	■	■
L.6.2a. Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/parenthetical elements.				■	■	■	■	■
L.6.3a. Vary sentence patterns for meaning, reader/listener interest, and style.‡				■	■	■	■	
L.6.3b. Maintain consistency in style and tone.				■	■	■	■	■
L.7.1c. Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.					■	■	■	■
L.7.3a. Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.					■	■	■	■
L.8.1d. Recognize and correct inappropriate shifts in verb voice and mood.						■	■	■
L.9–10.1a. Use parallel structure.							■	■

*Subsumed by L.7.3a
†Subsumed by L.9–10.1a
‡Subsumed by L.11–12.3a

Standard 10: Range, Quality, and Complexity of Student Reading K–5

Measuring Text Complexity: Three Factors



Qualitative evaluation of the text: Levels of meaning, structure, language conventionality and clarity, and knowledge demands

Quantitative evaluation of the text: Readability measures and other scores of text complexity

Matching reader to text and task: Reader variables (such as motivation, knowledge, and experiences) and task variables (such as purpose and the complexity generated by the task assigned and the questions posed)

Note: More detailed information on text complexity and how it is measured is contained in Appendix A.

Range of Text Types for K–5

Students in K–5 apply the Reading standards to the following range of text types, with texts selected from a broad range of cultures and periods.

Literature			Informational Text
Stories	**Dramas**	**Poetry**	**Literary Nonfiction and Historical, Scientific, and Technical Texts**
Includes children's adventure stories, folktales, legends, fables, fantasy, realistic fiction, and myth	Includes staged dialogue and brief familiar scenes	Includes nursery rhymes and the subgenres of the narrative poem, limerick, and free verse poem	Includes biographies and autobiographies; books about history, social studies, science, and the arts; technical texts, including directions, forms, and information displayed in graphs, charts, or maps; and digital sources on a range of topics

* Read-aloud
** Read-along

Texts Illustrating the Complexity, Quality, and Range of Student Reading K–5

	Literature: Stories, Drama, Poetry	Informational Texts: Literary Nonfiction and Historical, Scientific, and Technical Texts
K*	▪ *Over in the Meadow* by John Langstaff (traditional) (c1800)* ▪ *A Boy, a Dog, and a Frog* by Mercer Mayer (1967) ▪ *Pancakes for Breakfast* by Tomie DePaola (1978) ▪ *A Story, A Story* by Gail E. Haley (1970)* ▪ *Kitten's First Full Moon* by Kevin Henkes (2004)*	▪ *My Five Senses* by Aliki (1962)** ▪ *Truck* by Donald Crews (1980) ▪ *I Read Signs* by Tana Hoban (1987) ▪ *What Do You Do With a Tail Like This?* by Steve Jenkins and Robin Page (2003)* ▪ *Amazing Whales!* by Sarah L. Thomson (2005)*
1*	▪ "Mix a Pancake" by Christina G. Rossetti (1893)** ▪ *Mr. Popper's Penguins* by Richard Atwater (1938)* ▪ *Little Bear* by Else Holmelund Minarik, illustrated by Maurice Sendak (1957)** ▪ *Frog and Toad Together* by Arnold Lobel (1971)** ▪ *Hi! Fly Guy* by Tedd Arnold (2006)	▪ *A Tree Is a Plant* by Clyde Robert Bulla, illustrated by Stacey Schuett (1960)** ▪ *Starfish* by Edith Thacher Hurd (1962) ▪ *Follow the Water from Brook to Ocean* by Arthur Dorros (1991)** ▪ *From Seed to Pumpkin* by Wendy Pfeffer, illustrated by James Graham Hale (2004)* ▪ *How People Learned to Fly* by Fran Hodgkins and True Kelley (2007)*
2–3	▪ "Who Has Seen the Wind?" by Christina G. Rossetti (1893) ▪ *Charlotte's Web* by E. B. White (1952)* ▪ *Sarah, Plain and Tall* by Patricia MacLachlan (1985) ▪ *Tops and Bottoms* by Janet Stevens (1995) ▪ *Poppleton in Winter* by Cynthia Rylant, illustrated by Mark Teague (2001)	▪ *A Medieval Feast* by Aliki (1983) ▪ *From Seed to Plant* by Gail Gibbons (1991) ▪ *The Story of Ruby Bridges* by Robert Coles (1995)* ▪ *A Drop of Water: A Book of Science and Wonder* by Walter Wick (1997) ▪ *Moonshot: The Flight of Apollo 11* by Brian Floca (2009)
4–5	▪ *Alice's Adventures in Wonderland* by Lewis Carroll (1865) ▪ "Casey at the Bat" by Ernest Lawrence Thayer (1888) ▪ *The Black Stallion* by Walter Farley (1941) ▪ "Zlateh the Goat" by Isaac Bashevis Singer (1984) ▪ *Where the Mountain Meets the Moon* by Grace Lin (2009)	▪ *Discovering Mars: The Amazing Story of the Red Planet* by Melvin Berger (1992) ▪ *Hurricanes: Earth's Mightiest Storms* by Patricia Lauber (1996) ▪ *A History of US* by Joy Hakim (2005) ▪ *Horses* by Seymour Simon (2006) ▪ *Quest for the Tree Kangaroo: An Expedition to the Cloud Forest of New Guinea* by Sy Montgomery (2006)

Note: Given space limitations, the illustrative texts listed above are meant only to show individual titles that are representative of a wide range of topics and genres. (See Appendix B for excerpts of these and other texts illustrative of K–5 text complexity, quality, and range.) At a curricular or instructional level, within and across grade levels, texts need to be selected around topics or themes that generate knowledge and allow students to study those topics or themes in depth. On the next page is an example of progressions of texts building knowledge across grade levels.

*Children at the kindergarten and grade 1 levels should be expected to read texts independently that have been specifically written to correlate to their reading level and their word knowledge. Many of the titles listed above are meant to supplement carefully structured independent reading with books to read along with a teacher or that are read aloud to students to build knowledge and cultivate a joy in reading.

Staying on Topic Within a Grade and Across Grades: How to Build Knowledge Systematically in English Language Arts K–5

Building knowledge systematically in English language arts is like giving children various pieces of a puzzle in each grade that, over time, will form one big picture. At a curricular or instructional level, texts—within and across grade levels—need to be selected around topics or themes that systematically develop the knowledge base of students. Within a grade level, there should be an adequate number of titles on a single topic that would allow children to study that topic for a sustained period. The knowledge children have learned about particular topics in early grade levels should then be expanded and developed in subsequent grade levels to ensure an increasingly deeper understanding of these topics. Children in the upper elementary grades will generally be expected to read these texts independently and reflect on them in writing. However, children in the early grades (particularly K–2) should participate in rich, structured conversations with an adult in response to the written texts that are read aloud, orally comparing and contrasting as well as analyzing and synthesizing, in the manner called for by the *Standards*.

Preparation for reading complex informational texts should begin at the very earliest elementary school grades. What follows is one example that uses domain-specific nonfiction titles across grade levels to illustrate how curriculum designers and classroom teachers can infuse the English language arts block with rich, age-appropriate content knowledge and vocabulary in history/social studies, science, and the arts. Having students listen to informational read-alouds in the early grades helps lay the necessary foundation for students' reading and understanding of increasingly complex texts on their own in subsequent grades.

Exemplar Texts on a Topic Across Grades	K	1	2–3	4–5
The Human Body **Students can begin learning about the human body starting in kindergarten and then review and extend their learning during each subsequent grade.**	**The five senses and associated body parts** • *My Five Senses* by Aliki (1989) • *Hearing* by Maria Rius (1985) • *Sight* by Maria Rius (1985) • *Smell* by Maria Rius (1985) • *Taste* by Maria Rius (1985) • *Touch* by Maria Rius (1985) **Taking care of your body: Overview (hygiene, diet, exercise, rest)** • *My Amazing Body: A First Look at Health & Fitness* by Pat Thomas (2001) • *Get Up and Go!* by Nancy Carlson (2008) • *Go Wash Up* by Doering Tourville (2008) • *Sleep* by Paul Showers (1997) • *Fuel the Body* by Doering Tourville (2008)	**Introduction to the systems of the human body and associated body parts** • *Under Your Skin: Your Amazing Body* by Mick Manning (2007) • *Me and My Amazing Body* by Joan Sweeney (1999) • *The Human Body* by Gallimard Jeunesse (2007) • *The Busy Body Book* by Lizzy Rockwell (2008) • *First Encyclopedia of the Human Body* by Fiona Chandler (2004) **Taking care of your body: Germs, diseases, and preventing illness** • *Germs Make Me Sick* by Marilyn Berger (1995) • *Tiny Life on Your Body* by Christine Taylor-Butler (2005) • *Germ Stories* by Arthur Kornberg (2007) • *All About Scabs* by GenichiroYagu (1998)	**Digestive and excretory systems** • *What Happens to a Hamburger* by Paul Showers (1985) • *The Digestive System* by Christine Taylor-Butler (2008) • *The Digestive System* by Rebecca L. Johnson (2006) • *The Digestive System* by Kristin Petrie (2007) **Taking care of your body: Healthy eating and nutrition** • *Good Enough to Eat* by Lizzy Rockwell (1999) • *Showdown at the Food Pyramid* by Rex Barron (2004) **Muscular, skeletal, and nervous systems** • *The Mighty Muscular and Skeletal Systems* Crabtree Publishing (2009) • *Muscles* by Seymour Simon (1998) • *Bones* by Seymour Simon (1998) • *The Astounding Nervous System* Crabtree Publishing (2009) • *The Nervous System* by Joelle Riley (2004)	**Circulatory system** • *The Heart* by Seymour Simon (2006) • *The Heart and Circulation* by Carol Ballard (2005) • *The Circulatory System* by Kristin Petrie (2007) • *The Amazing Circulatory System* by John Burstein (2009) **Respiratory system** • *The Lungs* by Seymour Simon (2007) • *The Respiratory System* by Susan Glass (2004) • *The Respiratory System* by Kristin Petrie (2007) • *The Remarkable Respiratory System* by John Burstein (2009) **Endocrine system** • *The Endocrine System* by Rebecca Olien (2006) • *The Exciting Endocrine System* by John Burstein (2009)

STANDARDS FOR

English Language Arts

6–12

College and Career Readiness Anchor Standards for Reading

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Key Ideas and Details

1. Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
2. Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
3. Analyze how and why individuals, events, and ideas develop and interact over the course of a text.

Craft and Structure

4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
5. Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
6. Assess how point of view or purpose shapes the content and style of a text.

Integration of Knowledge and Ideas

7. Integrate and evaluate content presented in diverse formats and media, including visually and quantitatively, as well as in words.*
8. Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
9. Analyze how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.

Range of Reading and Level of Text Complexity

10. Read and comprehend complex literary and informational texts independently and proficiently.

*Please see "Research to Build Knowledge" in Writing and "Comprehension and Collaboration" in Speaking and Listening for additional standards relevant to gathering, assessing, and applying information from print and digital sources.

Note on range and content of student reading

To become college and career ready, students must grapple with works of exceptional craft and thought whose range extends across genres, cultures, and centuries. Such works offer profound insights into the human condition and serve as models for students' own thinking and writing. Along with high-quality contemporary works, these texts should be chosen from among seminal U.S. documents, the classics of American literature, and the timeless dramas of Shakespeare. Through wide and deep reading of literature and literary nonfiction of steadily increasing sophistication, students gain a reservoir of literary and cultural knowledge, references, and images; the ability to evaluate intricate arguments; and the capacity to surmount the challenges posed by complex texts.

Reading Standards for Literature 6–12

RL

The following standards offer a focus for instruction each year and help ensure that students gain adequate exposure to a range of texts and tasks. Rigor is also infused through the requirement that students read increasingly complex texts through the grades. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.*

Grade 6 students:	Grade 7 students:	Grade 8 students:
Key Ideas and Details		
1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text.
2. Determine a theme or central idea of a text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.	2. Determine a theme or central idea of a text and analyze its development over the course of the text; provide an objective summary of the text.	2. Determine a theme or central idea of a text and analyze its development over the course of the text, including its relationship to the characters, setting, and plot; provide an objective summary of the text.
3. Describe how a particular story's or drama's plot unfolds in a series of episodes as well as how the characters respond or change as the plot moves toward a resolution.	3. Analyze how particular elements of a story or drama interact (e.g., how setting shapes the characters or plot).	3. Analyze how particular lines of dialogue or incidents in a story or drama propel the action, reveal aspects of a character, or provoke a decision.
Craft and Structure		
4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of a specific word choice on meaning and tone.	4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of rhymes and other repetitions of sounds (e.g., alliteration) on a specific verse or stanza of a poem or section of a story or drama.	4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts.
5. Analyze how a particular sentence, chapter, scene, or stanza fits into the overall structure of a text and contributes to the development of the theme, setting, or plot.	5. Analyze how a drama's or poem's form or structure (e.g., soliloquy, sonnet) contributes to its meaning.	5. Compare and contrast the structure of two or more texts and analyze how the differing structure of each text contributes to its meaning and style.
6. Explain how an author develops the point of view of the narrator or speaker in a text.	6. Analyze how an author develops and contrasts the points of view of different characters or narrators in a text.	6. Analyze how differences in the points of view of the characters and the audience or reader (e.g., created through the use of dramatic irony) create such effects as suspense or humor.

Reading Standards for Literature 6–12

RL

Grade 6 students:	Grade 7 students:	Grade 8 students:
Integration of Knowledge and Ideas		
7. Compare and contrast the experience of reading a story, drama, or poem to listening to or viewing an audio, video, or live version of the text, including contrasting what they "see" and "hear" when reading the text to what they perceive when they listen or watch.	7. Compare and contrast a written story, drama, or poem to its audio, filmed, staged, or multimedia version, analyzing the effects of techniques unique to each medium (e.g., lighting, sound, color, or camera focus and angles in a film).	7. Analyze the extent to which a filmed or live production of a story or drama stays faithful to or departs from the text or script, evaluating the choices made by the director or actors.
8. (Not applicable to literature)	8. (Not applicable to literature)	8. (Not applicable to literature)
9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics.	9. Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors of fiction use or alter history.	9. Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new.
Range of Reading and Level of Text Complexity		
10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band independently and proficiently.

Reading Standards for Literature 6–12

RL

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9–10 students:	Grades 11–12 students:
Key Ideas and Details	
1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.
2. Determine a theme or central idea of a text and analyze in detail its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text.	2. Determine two or more themes or central ideas of a text and analyze their development over the course of the text, including how they interact and build on one another to produce a complex account; provide an objective summary of the text.
3. Analyze how complex characters (e.g., those with multiple or conflicting motivations) develop over the course of a text, interact with other characters, and advance the plot or develop the theme.	3. Analyze the impact of the author's choices regarding how to develop and relate elements of a story or drama (e.g., where a story is set, how the action is ordered, how the characters are introduced and developed).
Craft and Structure	
4. Determine the meaning of words and phrases as they are used in the text, including figurative and connotative meanings; analyze the cumulative impact of specific word choices on meaning and tone (e.g., how the language evokes a sense of time and place; how it sets a formal or informal tone).	4. Determine the meaning of words and phrases as they are used in the text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including words with multiple meanings or language that is particularly fresh, engaging, or beautiful. (Include Shakespeare as well as other authors.)
5. Analyze how an author's choices concerning how to structure a text, order events within it (e.g., parallel plots), and manipulate time (e.g., pacing, flashbacks) create such effects as mystery, tension, or surprise.	5. Analyze how an author's choices concerning how to structure specific parts of a text (e.g., the choice of where to begin or end a story, the choice to provide a comedic or tragic resolution) contribute to its overall structure and meaning as well as its aesthetic impact.
6. Analyze a particular point of view or cultural experience reflected in a work of literature from outside the United States, drawing on a wide reading of world literature.	6. Analyze a case in which grasping point of view requires distinguishing what is directly stated in a text from what is really meant (e.g., satire, sarcasm, irony, or understatement).
Integration of Knowledge and Ideas	
7. Analyze the representation of a subject or a key scene in two different artistic mediums, including what is emphasized or absent in each treatment (e.g., Auden's "Musée des Beaux Arts" and Breughel's *Landscape with the Fall of Icarus*).	7. Analyze multiple interpretations of a story, drama, or poem (e.g., recorded or live production of a play or recorded novel or poetry), evaluating how each version interprets the source text. (Include at least one play by Shakespeare and one play by an American dramatist.)
8. (Not applicable to literature)	8. (Not applicable to literature)
9. Analyze how an author draws on and transforms source material in a specific work (e.g., how Shakespeare treats a theme or topic from Ovid or the Bible or how a later author draws on a play by Shakespeare).	9. Demonstrate knowledge of eighteenth-, nineteenth- and early-twentieth-century foundational works of American literature, including how two or more texts from the same period treat similar themes or topics.
Range of Reading and Level of Text Complexity	
10. By the end of grade 9, read and comprehend literature, including stories, dramas, and poems, in the grades 9–10 text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 10, read and comprehend literature, including stories, dramas, and poems, at the high end of the grades 9–10 text complexity band independently and proficiently.	10. By the end of grade 11, read and comprehend literature, including stories, dramas, and poems, in the grades 11–CCR text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 12, read and comprehend literature, including stories, dramas, and poems, at the high end of the grades 11–CCR text complexity band independently and proficiently.

Reading Standards for Informational Text 6–12

RI

Grade 6 students:	Grade 7 students:	Grade 8 students:
Key Ideas and Details		
1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text.
2. Determine a central idea of a text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.	2. Determine two or more central ideas in a text and analyze their development over the course of the text; provide an objective summary of the text.	2. Determine a central idea of a text and analyze its development over the course of the text, including its relationship to supporting ideas; provide an objective summary of the text.
3. Analyze in detail how a key individual, event, or idea is introduced, illustrated, and elaborated in a text (e.g., through examples or anecdotes).	3. Analyze the interactions between individuals, events, and ideas in a text (e.g., how ideas influence individuals or events, or how individuals influence ideas or events).	3. Analyze how a text makes connections among and distinctions between individuals, ideas, or events (e.g., through comparisons, analogies, or categories).
Craft and Structure		
4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings.	4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the impact of a specific word choice on meaning and tone.	4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts.
5. Analyze how a particular sentence, paragraph, chapter, or section fits into the overall structure of a text and contributes to the development of the ideas.	5. Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to the development of the ideas.	5. Analyze in detail the structure of a specific paragraph in a text, including the role of particular sentences in developing and refining a key concept.
6. Determine an author's point of view or purpose in a text and explain how it is conveyed in the text.	6. Determine an author's point of view or purpose in a text and analyze how the author distinguishes his or her position from that of others.	6. Determine an author's point of view or purpose in a text and analyze how the author acknowledges and responds to conflicting evidence or viewpoints.
Integration of Knowledge and Ideas		
7. Integrate information presented in different media or formats (e.g., visually, quantitatively) as well as in words to develop a coherent understanding of a topic or issue.	7. Compare and contrast a text to an audio, video, or multimedia version of the text, analyzing each medium's portrayal of the subject (e.g., how the delivery of a speech affects the impact of the words).	7. Evaluate the advantages and disadvantages of using different mediums (e.g., print or digital text, video, multimedia) to present a particular topic or idea.
8. Trace and evaluate the argument and specific claims in a text, distinguishing claims that are supported by reasons and evidence from claims that are not.	8. Trace and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient to support the claims.	8. Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient; recognize when irrelevant evidence is introduced.
9. Compare and contrast one author's presentation of events with that of another (e.g., a memoir written by and a biography on the same person).	9. Analyze how two or more authors writing about the same topic shape their presentations of key information by emphasizing different evidence or advancing different interpretations of facts.	9. Analyze a case in which two or more texts provide conflicting information on the same topic and identify where the texts disagree on matters of fact or interpretation.
Range of Reading and Level of Text Complexity		
10. By the end of the year, read and comprehend literary nonfiction in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literary nonfiction in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literary nonfiction at the high end of the grades 6–8 text complexity band independently and proficiently.

Reading Standards for Informational Text 6–12

RI

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9–10 students:	Grades 11–12 students:
Key Ideas and Details	
1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.
2. Determine a central idea of a text and analyze its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text.	2. Determine two or more central ideas of a text and analyze their development over the course of the text, including how they interact and build on one another to provide a complex analysis; provide an objective summary of the text.
3. Analyze how the author unfolds an analysis or series of ideas or events, including the order in which the points are made, how they are introduced and developed, and the connections that are drawn between them.	3. Analyze a complex set of ideas or sequence of events and explain how specific individuals, ideas, or events interact and develop over the course of the text.
Craft and Structure	
4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the cumulative impact of specific word choices on meaning and tone (e.g., how the language of a court opinion differs from that of a newspaper).	4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze how an author uses and refines the meaning of a key term or terms over the course of a text (e.g., how Madison defines *faction* in *Federalist* No. 10).
5. Analyze in detail how an author's ideas or claims are developed and refined by particular sentences, paragraphs, or larger portions of a text (e.g., a section or chapter).	5. Analyze and evaluate the effectiveness of the structure an author uses in his or her exposition or argument, including whether the structure makes points clear, convincing, and engaging.
6. Determine an author's point of view or purpose in a text and analyze how an author uses rhetoric to advance that point of view or purpose.	6. Determine an author's point of view or purpose in a text in which the rhetoric is particularly effective, analyzing how style and content contribute to the power, persuasiveness, or beauty of the text.
Integration of Knowledge and Ideas	
7. Analyze various accounts of a subject told in different mediums (e.g., a person's life story in both print and multimedia), determining which details are emphasized in each account.	7. Integrate and evaluate multiple sources of information presented in different media or formats (e.g., visually, quantitatively) as well as in words in order to address a question or solve a problem.
8. Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is valid and the evidence is relevant and sufficient; identify false statements and fallacious reasoning.	8. Delineate and evaluate the reasoning in seminal U.S. texts, including the application of constitutional principles and use of legal reasoning (e.g., in U.S. Supreme Court majority opinions and dissents) and the premises, purposes, and arguments in works of public advocacy (e.g., *The Federalist,* presidential addresses).
9. Analyze seminal U.S. documents of historical and literary significance (e.g., Washington's Farewell Address, the Gettysburg Address, Roosevelt's Four Freedoms speech, King's "Letter from Birmingham Jail"), including how they address related themes and concepts.	9. Analyze seventeenth-, eighteenth-, and nineteenth-century foundational U.S. documents of historical and literary significance (including The Declaration of Independence, the Preamble to the Constitution, the Bill of Rights, and Lincoln's Second Inaugural Address) for their themes, purposes, and rhetorical features.
Range of Reading and Level of Text Complexity	
10. By the end of grade 9, read and comprehend literary nonfiction in the grades 9–10 text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 10, read and comprehend literary nonfiction at the high end of the grades 9–10 text complexity band independently and proficiently.	10. By the end of grade 11, read and comprehend literary nonfiction in the grades 11–CCR text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 12, read and comprehend literary nonfiction at the high end of the grades 11–CCR text complexity band independently and proficiently.

College and Career Readiness Anchor Standards for Writing

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Text Types and Purposes*

1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences.

Production and Distribution of Writing

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

Research to Build and Present Knowledge

7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

Range of Writing

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

*These broad types of writing include many subgenres. See Appendix A for definitions of key writing types.

Note on range and content of student writing

For students, writing is a key means of asserting and defending claims, showing what they know about a subject, and conveying what they have experienced, imagined, thought, and felt. To be college- and career-ready writers, students must take task, purpose, and audience into careful consideration, choosing words, information, structures, and formats deliberately. They need to know how to combine elements of different kinds of writing—for example, to use narrative strategies within argument and explanation within narrative—to produce complex and nuanced writing. They need to be able to use technology strategically when creating, refining, and collaborating on writing. They have to become adept at gathering information, evaluating sources, and citing material accurately, reporting findings from their research and analysis of sources in a clear and cogent manner. They must have the flexibility, concentration, and fluency to produce high-quality first-draft text under a tight deadline as well as the capacity to revisit and make improvements to a piece of writing over multiple drafts when circumstances encourage or require it.

Writing Standards 6–12

W

The following standards for grades 6–12 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. Each year in their writing, students should demonstrate increasing sophistication in all aspects of language use, from vocabulary and syntax to the development and organization of ideas, and they should address increasingly demanding content and sources. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.* The expected growth in student writing ability is reflected both in the standards themselves and in the collection of annotated student writing samples in Appendix C.

Grade 6 students:	Grade 7 students:	Grade 8 students:
Text Types and Purposes		
1. Write arguments to support claims with clear reasons and relevant evidence. a. Introduce claim(s) and organize the reasons and evidence clearly. b. Support claim(s) with clear reasons and relevant evidence, using credible sources and demonstrating an understanding of the topic or text. c. Use words, phrases, and clauses to clarify the relationships among claim(s) and reasons. d. Establish and maintain a formal style. e. Provide a concluding statement or section that follows from the argument presented.	1. Write arguments to support claims with clear reasons and relevant evidence. a. Introduce claim(s), acknowledge alternate or opposing claims, and organize the reasons and evidence logically. b. Support claim(s) with logical reasoning and relevant evidence, using accurate, credible sources and demonstrating an understanding of the topic or text. c. Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), reasons, and evidence. d. Establish and maintain a formal style. e. Provide a concluding statement or section that follows from and supports the argument presented.	1. Write arguments to support claims with clear reasons and relevant evidence. a. Introduce claim(s), acknowledge and distinguish the claim(s) from alternate or opposing claims, and organize the reasons and evidence logically. b. Support claim(s) with logical reasoning and relevant evidence, using accurate, credible sources and demonstrating an understanding of the topic or text. c. Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), counterclaims, reasons, and evidence. d. Establish and maintain a formal style. e. Provide a concluding statement or section that follows from and supports the argument presented.
2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. a. Introduce a topic; organize ideas, concepts, and information, using strategies such as definition, classification, comparison/contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate transitions to clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style. f. Provide a concluding statement or section that follows from the information or explanation presented.	2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information, using strategies such as definition, classification, comparison/contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate transitions to create cohesion and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style. f. Provide a concluding statement or section that follows from and supports the information or explanation presented.	2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style. f. Provide a concluding statement or section that follows from and supports the information or explanation presented.

Writing Standards 6–12

W

Grade 6 students:	Grade 7 students:	Grade 8 students:
Text Types and Purposes (continued)		
3. Write narratives to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences. a. Engage and orient the reader by establishing a context and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. b. Use narrative techniques, such as dialogue, pacing, and description, to develop experiences, events, and/or characters. c. Use a variety of transition words, phrases, and clauses to convey sequence and signal shifts from one time frame or setting to another. d. Use precise words and phrases, relevant descriptive details, and sensory language to convey experiences and events. e. Provide a conclusion that follows from the narrated experiences or events.	3. Write narratives to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences. a. Engage and orient the reader by establishing a context and point of view and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. b. Use narrative techniques, such as dialogue, pacing, and description, to develop experiences, events, and/or characters. c. Use a variety of transition words, phrases, and clauses to convey sequence and signal shifts from one time frame or setting to another. d. Use precise words and phrases, relevant descriptive details, and sensory language to capture the action and convey experiences and events. e. Provide a conclusion that follows from and reflects on the narrated experiences or events.	3. Write narratives to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences. a. Engage and orient the reader by establishing a context and point of view and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. b. Use narrative techniques, such as dialogue, pacing, description, and reflection, to develop experiences, events, and/or characters. c. Use a variety of transition words, phrases, and clauses to convey sequence, signal shifts from one time frame or setting to another, and show the relationships among experiences and events. d. Use precise words and phrases, relevant descriptive details, and sensory language to capture the action and convey experiences and events. e. Provide a conclusion that follows from and reflects on the narrated experiences or events.
Production and Distribution of Writing		
4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 6 on page 52.)	5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 7 on page 52.)	5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 8 on page 52.)
6. Use technology, including the Internet, to produce and publish writing as well as to interact and collaborate with others; demonstrate sufficient command of keyboarding skills to type a minimum of three pages in a single sitting.	6. Use technology, including the Internet, to produce and publish writing and link to and cite sources as well as to interact and collaborate with others, including linking to and citing sources.	6. Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas efficiently as well as to interact and collaborate with others.

Writing Standards 6–12

W

Grade 6 students:	Grade 7 students:	Grade 8 students:
Research to Build and Present Knowledge		
7. Conduct short research projects to answer a question, drawing on several sources and refocusing the inquiry when appropriate.	7. Conduct short research projects to answer a question, drawing on several sources and generating additional related, focused questions for further research and investigation.	7. Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.
8. Gather relevant information from multiple print and digital sources; assess the credibility of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and providing basic bibliographic information for sources.	8. Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and following a standard format for citation.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grade 6 Reading standards* to literature (e.g., "Compare and contrast texts in different forms or genres [e.g., stories and poems; historical novels and fantasy stories] in terms of their approaches to similar themes and topics"). b. Apply *grade 6 Reading standards* to literary nonfiction (e.g., "Trace and evaluate the argument and specific claims in a text, distinguishing claims that are supported by reasons and evidence from claims that are not").	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grade 7 Reading standards* to literature (e.g., "Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors of fiction use or alter history"). b. Apply *grade 7 Reading standards* to literary nonfiction (e.g. "Trace and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient to support the claims").	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grade 8 Reading standards* to literature (e.g., "Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new"). b. Apply *grade 8 Reading standards* to literary nonfiction (e.g., "Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient; recognize when irrelevant evidence is introduced").
Range of Writing		
10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

Writing Standards 6–12

W

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9–10 students:	Grades 11–12 students:
Text Types and Purposes	
1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence. a. Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly, supplying evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level and concerns. c. Use words, phrases, and clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from and supports the argument presented.	1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence. a. Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level, concerns, values, and possible biases. c. Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from and supports the argument presented.
2. Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. a. Introduce a topic; organize complex ideas, concepts, and information to make important connections and distinctions; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with well-chosen, relevant, and sufficient facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use appropriate and varied transitions to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts. d. Use precise language and domain-specific vocabulary to manage the complexity of the topic. e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).	2. Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. a. Introduce a topic; organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use appropriate and varied transitions and syntax to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts. d. Use precise language, domain-specific vocabulary, and techniques such as metaphor, simile, and analogy to manage the complexity of the topic. e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).

Writing Standards 6–12

W

Grades 9–10 students:	Grades 11–12 students:
Text Types and Purposes (continued)	
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences. a. Engage and orient the reader by setting out a problem, situation, or observation, establishing one or multiple point(s) of view, and introducing a narrator and/or characters; create a smooth progression of experiences or events. b. Use narrative techniques, such as dialogue, pacing, description, reflection, and multiple plot lines, to develop experiences, events, and/or characters. c. Use a variety of techniques to sequence events so that they build on one another to create a coherent whole. d. Use precise words and phrases, telling details, and sensory language to convey a vivid picture of the experiences, events, setting, and/or characters. e. Provide a conclusion that follows from and reflects on what is experienced, observed, or resolved over the course of the narrative.	3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences. a. Engage and orient the reader by setting out a problem, situation, or observation and its significance, establishing one or multiple point(s) of view, and introducing a narrator and/or characters; create a smooth progression of experiences or events. b. Use narrative techniques, such as dialogue, pacing, description, reflection, and multiple plot lines, to develop experiences, events, and/or characters. c. Use a variety of techniques to sequence events so that they build on one another to create a coherent whole and build toward a particular tone and outcome (e.g., a sense of mystery, suspense, growth, or resolution). d. Use precise words and phrases, telling details, and sensory language to convey a vivid picture of the experiences, events, setting, and/or characters. e. Provide a conclusion that follows from and reflects on what is experienced, observed, or resolved over the course of the narrative.
Production and Distribution of Writing	
4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grades 9–10 on page 54.)	5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grades 11–12 on page 54.)
6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.	6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.
Research to Build and Present Knowledge	
7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.	7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the usefulness of each source in answering the research question; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the task, purpose, and audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation.

Writing Standards 6–12

W

Grades 9–10 students:	Grades 11–12 students:
Research to Build and Present Knowledge (continued)	
9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grades 9–10 Reading standards* to literature (e.g., "Analyze how an author draws on and transforms source material in a specific work [e.g., how Shakespeare treats a theme or topic from Ovid or the Bible or how a later author draws on a play by Shakespeare]"). b. Apply *grades 9–10 Reading standards* to literary nonfiction (e.g., "Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is valid and the evidence is relevant and sufficient; identify false statements and fallacious reasoning").	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grades 11–12 Reading standards* to literature (e.g., "Demonstrate knowledge of eighteenth-, nineteenth- and early-twentieth-century foundational works of American literature, including how two or more texts from the same period treat similar themes or topics"). b. Apply *grades 11–12 Reading standards* to literary nonfiction (e.g., "Delineate and evaluate the reasoning in seminal U.S. texts, including the application of constitutional principles and use of legal reasoning [e.g., in U.S. Supreme Court Case majority opinions and dissents] and the premises, purposes, and arguments in works of public advocacy [e.g., *The Federalist*, presidential addresses]").
Range of Writing	
10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

College and Career Readiness Anchor Standards for Speaking and Listening

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Comprehension and Collaboration

1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
2. Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.
3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

Presentation of Knowledge and Ideas

4. Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
5. Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.
6. Adapt speech to a variety of contexts and communicative tasks, demonstrating command of formal English when indicated or appropriate.

Note on range and content of student speaking and listening

To become college and career ready, students must have ample opportunities to take part in a variety of rich, structured conversations—as part of a whole class, in small groups, and with a partner—built around important content in various domains. They must be able to contribute appropriately to these conversations, to make comparisons and contrasts, and to analyze and synthesize a multitude of ideas in accordance with the standards of evidence appropriate to a particular discipline. Whatever their intended major or profession, high school graduates will depend heavily on their ability to listen attentively to others so that they are able to build on others' meritorious ideas while expressing their own clearly and persuasively.

New technologies have broadened and expanded the role that speaking and listening play in acquiring and sharing knowledge and have tightened their link to other forms of communication. The Internet has accelerated the speed at which connections between speaking, listening, reading, and writing can be made, requiring that students be ready to use these modalities nearly simultaneously. Technology itself is changing quickly, creating a new urgency for students to be adaptable in response to change.

Speaking and Listening Standards 6–12

SL

The following standards for grades 6–12 offer a focus for instruction in each year to help ensure that students gain adequate mastery of a range of skills and applications. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.*

Grade 6 students:	Grade 7 students:	Grade 8 students:
Comprehension and Collaboration		
1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 6 topics, texts, and issues*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. b. Follow rules for collegial discussions, set specific goals and deadlines, and define individual roles as needed. c. Pose and respond to specific questions with elaboration and detail by making comments that contribute to the topic, text, or issue under discussion. d. Review the key ideas expressed and demonstrate understanding of multiple perspectives through reflection and paraphrasing.	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 7 topics, texts, and issues,* building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. b. Follow rules for collegial discussions, track progress toward specific goals and deadlines, and define individual roles as needed. c. Pose questions that elicit elaboration and respond to others' questions and comments with relevant observations and ideas that bring the discussion back on topic as needed. d. Acknowledge new information expressed by others and, when warranted, modify their own views.	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 8 topics, texts, and issues,* building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. b. Follow rules for collegial discussions and decision-making, track progress toward specific goals and deadlines, and define individual roles as needed. c. Pose questions that connect the ideas of several speakers and respond to others' questions and comments with relevant evidence, observations, and ideas. d. Acknowledge new information expressed by others, and, when warranted, qualify or justify their own views in light of the evidence presented.
2. Interpret information presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how it contributes to a topic, text, or issue under study.	2. Analyze the main ideas and supporting details presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how the ideas clarify a topic, text, or issue under study.	2. Analyze the purpose of information presented in diverse media and formats (e.g., visually, quantitatively, orally) and evaluate the motives (e.g., social, commercial, political) behind its presentation.
3. Delineate a speaker's argument and specific claims, distinguishing claims that are supported by reasons and evidence from claims that are not.	3. Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and the relevance and sufficiency of the evidence.	3. Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and relevance and sufficiency of the evidence and identifying when irrelevant evidence is introduced.
Presentation of Knowledge and Ideas		
4. Present claims and findings, sequencing ideas logically and using pertinent descriptions, facts, and details to accentuate main ideas or themes; use appropriate eye contact, adequate volume, and clear pronunciation.	4. Present claims and findings, emphasizing salient points in a focused, coherent manner with pertinent descriptions, facts, details, and examples; use appropriate eye contact, adequate volume, and clear pronunciation.	4. Present claims and findings, emphasizing salient points in a focused, coherent manner with relevant evidence, sound valid reasoning, and well-chosen details; use appropriate eye contact, adequate volume, and clear pronunciation.
5. Include multimedia components (e.g., graphics, images, music, sound) and visual displays in presentations to clarify information.	5. Include multimedia components and visual displays in presentations to clarify claims and findings and emphasize salient points.	5. Integrate multimedia and visual displays into presentations to clarify information, strengthen claims and evidence, and add interest.
6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. (See grade 6 Language standards 1 and 3 on page 52 for specific expectations.)	6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. (See grade 7 Language standards 1 and 3 on page 52 for specific expectations.)	6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. (See grade 8 Language standards 1 and 3 on page 52 for specific expectations.)

Speaking and Listening Standards 6–12

SL

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9–10 students:	Grades 11–12 students:
Comprehension and Collaboration	
1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grades 9–10 topics, texts, and issues,* building on others' ideas and expressing their own clearly and persuasively. a. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas. b. Work with peers to set rules for collegial discussions and decision-making (e.g., informal consensus, taking votes on key issues, presentation of alternate views), clear goals and deadlines, and individual roles as needed. c. Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions. d. Respond thoughtfully to diverse perspectives, summarize points of agreement and disagreement, and, when warranted, qualify or justify their own views and understanding and make new connections in light of the evidence and reasoning presented.	1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grades 11–12 topics, texts, and issues,* building on others' ideas and expressing their own clearly and persuasively. a. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas. b. Work with peers to promote civil, democratic discussions and decision-making, set clear goals and deadlines, and establish individual roles as needed. c. Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives. d. Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task.
2. Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, orally) evaluating the credibility and accuracy of each source.	2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, identifying any fallacious reasoning or exaggerated or distorted evidence.	3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used.
Presentation of Knowledge and Ideas	
4. Present information, findings, and supporting evidence clearly, concisely, and logically such that listeners can follow the line of reasoning and the organization, development, substance, and style are appropriate to purpose, audience, and task.	4. Present information, findings, and supporting evidence, conveying a clear and distinct perspective, such that listeners can follow the line of reasoning, alternative or opposing perspectives are addressed, and the organization, development, substance, and style are appropriate to purpose, audience, and a range of formal and informal tasks.
5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.	5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. (See grades 9–10 Language standards 1 and 3 on pages 54 for specific expectations.)	6. Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate. (See grades 11–12 Language standards 1 and 3 on page 54 for specific expectations.)

College and Career Readiness Anchor Standards for Language

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Conventions of Standard English

1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.

Knowledge of Language

3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.

Vocabulary Acquisition and Use

4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings.
6. Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.

Note on range and content of student language use

To be college and career ready in language, students must have firm control over the conventions of standard English. At the same time, they must come to appreciate that language is as at least as much a matter of craft as of rules and be able to choose words, syntax, and punctuation to express themselves and achieve particular functions and rhetorical effects. They must also have extensive vocabularies, built through reading and study, enabling them to comprehend complex texts and engage in purposeful writing about and conversations around content. They need to become skilled in determining or clarifying the meaning of words and phrases they encounter, choosing flexibly from an array of strategies to aid them. They must learn to see an individual word as part of a network of other words—words, for example, that have similar denotations but different connotations. The inclusion of Language standards in their own strand should not be taken as an indication that skills related to conventions, effective language use, and vocabulary are unimportant to reading, writing, speaking, and listening; indeed, they are inseparable from such contexts.

Language Standards 6–12

L

The following standards for grades 6–12 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.* Beginning in grade 3, skills and understandings that are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking are marked with an asterisk (*). See the table on page 56 for a complete listing and Appendix A for an example of how these skills develop in sophistication.

Grade 6 students:	Grade 7 students:	Grade 8 students:
Conventions of Standard English		
1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Ensure that pronouns are in the proper case (subjective, objective, possessive). b. Use intensive pronouns (e.g., *myself, ourselves*). c. Recognize and correct inappropriate shifts in pronoun number and person.* d. Recognize and correct vague pronouns (i.e., ones with unclear or ambiguous antecedents).* e. Recognize variations from standard English in their own and others' writing and speaking, and identify and use strategies to improve expression in conventional language.*	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Explain the function of phrases and clauses in general and their function in specific sentences. b. Choose among simple, compound, complex, and compound-complex sentences to signal differing relationships among ideas. c. Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.*	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Explain the function of verbals (gerunds, participles, infinitives) in general and their function in particular sentences. b. Form and use verbs in the active and passive voice. c. Form and use verbs in the indicative, imperative, interrogative, conditional, and subjunctive mood. d. Recognize and correct inappropriate shifts in verb voice and mood.*
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/parenthetical elements.* b. Spell correctly.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use a comma to separate coordinate adjectives (e.g., *It was a fascinating, enjoyable movie* but not *He wore an old[,] green shirt*). b. Spell correctly.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use punctuation (comma, ellipsis, dash) to indicate a pause or break. b. Use an ellipsis to indicate an omission. c. Spell correctly.
Knowledge of Language		
3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Vary sentence patterns for meaning, reader/listener interest, and style.* b. Maintain consistency in style and tone.*	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.*	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Use verbs in the active and passive voice and in the conditional and subjunctive mood to achieve particular effects (e.g., emphasizing the actor or the action; expressing uncertainty or describing a state contrary to fact).

Language Standards 6–12

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Grade 6 students:	Grade 7 students:	Grade 8 students:
Vocabulary Acquisition and Use		
4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 6 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., *audience, auditory, audible*). c. Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 7 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., *belligerent, bellicose, rebel*). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	4. Determine or clarify the meaning of unknown and multiple-meaning words or phrases based on *grade 8 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., *precede, recede, secede*). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., personification) in context. b. Use the relationship between particular words (e.g., cause/effect, part/whole, item/category) to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., *stingy, scrimping, economical, unwasteful, thrifty*).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., literary, biblical, and mythological allusions) in context. b. Use the relationship between particular words (e.g., synonym/antonym, analogy) to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., *refined, respectful, polite, diplomatic, condescending*).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g. verbal irony, puns) in context. b. Use the relationship between particular words to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., *bullheaded, willful, firm, persistent, resolute*).
6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.

Language Standards 6–12

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The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9–10 students:	Grades 11–12 students:
Conventions of Standard English	
1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Use parallel structure.* b. Use various types of phrases (noun, verb, adjectival, adverbial, participial, prepositional, absolute) and clauses (independent, dependent; noun, relative, adverbial) to convey specific meanings and add variety and interest to writing or presentations.	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Apply the understanding that usage is a matter of convention, can change over time, and is sometimes contested. b. Resolve issues of complex or contested usage, consulting references (e.g., *Merriam-Webster's Dictionary of English Usage, Garner's Modern American Usage*) as needed.
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use a semicolon (and perhaps a conjunctive adverb) to link two or more closely related independent clauses. b. Use a colon to introduce a list or quotation. c. Spell correctly.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Observe hyphenation conventions. b. Spell correctly.
Knowledge of Language	
3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening. a. Write and edit work so that it conforms to the guidelines in a style manual (e.g., *MLA Handbook*, Turabian's *Manual for Writers*) appropriate for the discipline and writing type.	3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening. a. Vary syntax for effect, consulting references (e.g., Tufte's *Artful Sentences*) for guidance as needed; apply an understanding of syntax to the study of complex texts when reading.

Language Standards 6–12

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Grades 9–10 students:	Grades 11–12 students:
Vocabulary Acquisition and Use	
4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grades 9–10 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence, paragraph, or text; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Identify and correctly use patterns of word changes that indicate different meanings or parts of speech (e.g., *analyze, analysis, analytical; advocate, advocacy*). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning, its part of speech, or its etymology. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grades 11–12 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence, paragraph, or text; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Identify and correctly use patterns of word changes that indicate different meanings or parts of speech (e.g., *conceive, conception, conceivable*). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning, its part of speech, its etymology, or its standard usage. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., euphemism, oxymoron) in context and analyze their role in the text. b. Analyze nuances in the meaning of words with similar denotations.	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., hyperbole, paradox) in context and analyze their role in the text. b. Analyze nuances in the meaning of words with similar denotations.
6. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.	6. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.

Language Progressive Skills, by Grade

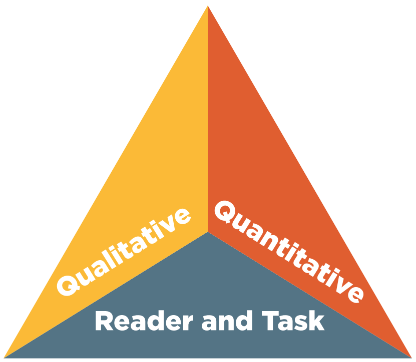
The following skills, marked with an asterisk (*) in Language standards 1–3, are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking.

Standard	Grade(s)							
	3	4	5	6	7	8	9–10	11–12
L.3.1f. Ensure subject-verb and pronoun-antecedent agreement.	■	■	■	■	■	■	■	■
L.3.3a. Choose words and phrases for effect.	■	■	■	■	■	■	■	■
L.4.1f. Produce complete sentences, recognizing and correcting inappropriate fragments and run-ons.		■	■	■	■	■	■	■
L.4.1g. Correctly use frequently confused words (e.g., *to/too/two; there/their*).		■	■	■	■	■	■	■
L.4.3a. Choose words and phrases to convey ideas precisely.[*]		■	■	■				
L.4.3b. Choose punctuation for effect.		■	■	■	■	■	■	■
L.5.1d. Recognize and correct inappropriate shifts in verb tense.			■	■	■	■	■	■
L.5.2a. Use punctuation to separate items in a series.[†]			■	■	■	■		
L.6.1c. Recognize and correct inappropriate shifts in pronoun number and person.				■	■	■	■	■
L.6.1d. Recognize and correct vague pronouns (i.e., ones with unclear or ambiguous antecedents).				■	■	■	■	■
L.6.1e. Recognize variations from standard English in their own and others' writing and speaking, and identify and use strategies to improve expression in conventional language.				■	■	■	■	■
L.6.2a. Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/parenthetical elements.				■	■	■	■	■
L.6.3a. Vary sentence patterns for meaning, reader/listener interest, and style.[‡]				■	■	■	■	
L.6.3b. Maintain consistency in style and tone.				■	■	■	■	■
L.7.1c. Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.					■	■	■	■
L.7.3a. Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.					■	■	■	■
L.8.1d. Recognize and correct inappropriate shifts in verb voice and mood.						■	■	■
L.9–10.1a. Use parallel structure.							■	■

[*] Subsumed by L.7.3a
[†] Subsumed by L.9–10.1a
[‡] Subsumed by L.11–12.3a

Standard 10: Range, Quality, and Complexity of Student Reading 6–12

Measuring Text Complexity: Three Factors



Qualitative evaluation of the text: Levels of meaning, structure, language conventionality and clarity, and knowledge demands

Quantitative evaluation of the text: Readability measures and other scores of text complexity

Matching reader to text and task: Reader variables (such as motivation, knowledge, and experiences) and task variables (such as purpose and the complexity generated by the task assigned and the questions posed)

Note: More detailed information on text complexity and how it is measured is contained in Appendix A.

Range of Text Types for 6–12

Students in grades 6–12 apply the Reading standards to the following range of text types, with texts selected from a broad range of cultures and periods.

Literature			Informational Text
Stories	**Drama**	**Poetry**	**Literary Nonfiction**
Includes the subgenres of adventure stories, historical fiction, mysteries, myths, science fiction, realistic fiction, allegories, parodies, satire, and graphic novels	Includes one-act and multi-act plays, both in written form and on film	Includes the subgenres of narrative poems, lyrical poems, free verse poems, sonnets, odes, ballads, and epics	Includes the subgenres of exposition, argument, and functional text in the form of personal essays, speeches, opinion pieces, essays about art or literature, biographies, memoirs, journalism, and historical, scientific, technical, or economic accounts (including digital sources) written for a broad audience

Texts Illustrating the Complexity, Quality, and Range of Student Reading 6–12

	Literature: Stories, Dramas, Poetry	Informational Texts: Literary Nonfiction
6–8	▪ *Little Women* by Louisa May Alcott (1869) ▪ *The Adventures of Tom Sawyer* by Mark Twain (1876) ▪ "The Road Not Taken" by Robert Frost (1915) ▪ *The Dark Is Rising* by Susan Cooper (1973) ▪ *Dragonwings* by Laurence Yep (1975) ▪ *Roll of Thunder, Hear My Cry* by Mildred Taylor (1976)	▪ "Letter on Thomas Jefferson" by John Adams (1776) ▪ *Narrative of the Life of Frederick Douglass, an American Slave* by Frederick Douglass (1845) ▪ "Blood, Toil, Tears and Sweat: Address to Parliament on May 13th, 1940" by Winston Churchill (1940) ▪ *Harriet Tubman: Conductor on the Underground Railroad* by Ann Petry (1955) ▪ *Travels with Charley: In Search of America* by John Steinbeck (1962)
9–10	▪ *The Tragedy of Macbeth* by William Shakespeare (1592) ▪ "Ozymandias" by Percy Bysshe Shelley (1817) ▪ "The Raven" by Edgar Allen Poe (1845) ▪ "The Gift of the Magi" by O. Henry (1906) ▪ *The Grapes of Wrath* by John Steinbeck (1939) ▪ *Fahrenheit 451* by Ray Bradbury (1953) ▪ *The Killer Angels* by Michael Shaara (1975)	▪ "Speech to the Second Virginia Convention" by Patrick Henry (1775) ▪ "Farewell Address" by George Washington (1796) ▪ "Gettysburg Address" by Abraham Lincoln (1863) ▪ "State of the Union Address" by Franklin Delano Roosevelt (1941) ▪ "Letter from Birmingham Jail" by Martin Luther King, Jr. (1964) ▪ "Hope, Despair and Memory" by Elie Wiesel (1997)
11–CCR	▪ "Ode on a Grecian Urn" by John Keats (1820) ▪ *Jane Eyre* by Charlotte Brontë (1848) ▪ "Because I Could Not Stop for Death" by Emily Dickinson (1890) ▪ *The Great Gatsby* by F. Scott Fitzgerald (1925) ▪ *Their Eyes Were Watching God* by Zora Neale Hurston (1937) ▪ *A Raisin in the Sun* by Lorraine Hansberry (1959) ▪ *The Namesake* by Jhumpa Lahiri (2003)	▪ *Common Sense* by Thomas Paine (1776) ▪ *Walden* by Henry David Thoreau (1854) ▪ "Society and Solitude" by Ralph Waldo Emerson (1857) ▪ "The Fallacy of Success" by G. K. Chesterton (1909) ▪ *Black Boy* by Richard Wright (1945) ▪ "Politics and the English Language" by George Orwell (1946) ▪ "Take the Tortillas Out of Your Poetry" by Rudolfo Anaya (1995)

Note: Given space limitations, the illustrative texts listed above are meant only to show individual titles that are representative of a range of topics and genres. (See Appendix B for excerpts of these and other texts illustrative of grades 6–12 text complexity, quality, and range.) At a curricular or instructional level, within and across grade levels, texts need to be selected around topics or themes that generate knowledge and allow students to study those topics or themes in depth.

STANDARDS FOR

Literacy in History/Social Studies, Science, and Technical Subjects

6–12

College and Career Readiness Anchor Standards for Reading

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade span. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Key Ideas and Details

1. Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
2. Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
3. Analyze how and why individuals, events, or ideas develop and interact over the course of a text.

Craft and Structure

4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
5. Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
6. Assess how point of view or purpose shapes the content and style of a text.

Integration of Knowledge and Ideas

7. Integrate and evaluate content presented in diverse formats and media, including visually and quantitatively, as well as in words.*
8. Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
9. Analyze how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.

Range of Reading and Level of Text Complexity

10. Read and comprehend complex literary and informational texts independently and proficiently.

*Please see “Research to Build and Present Knowledge” in Writing for additional standards relevant to gathering, assessing, and applying information from print and digital sources.

Note on range and content of student reading

Reading is critical to building knowledge in history/social studies as well as in science and technical subjects. College and career ready reading in these fields requires an appreciation of the norms and conventions of each discipline, such as the kinds of evidence used in history and science; an understanding of domain-specific words and phrases; an attention to precise details; and the capacity to evaluate intricate arguments, synthesize complex information, and follow detailed descriptions of events and concepts. In history/social studies, for example, students need to be able to analyze, evaluate, and differentiate primary and secondary sources. When reading scientific and technical texts, students need to be able to gain knowledge from challenging texts that often make extensive use of elaborate diagrams and data to convey information and illustrate concepts. Students must be able to read complex informational texts in these fields with independence and confidence because the vast majority of reading in college and workforce training programs will be sophisticated nonfiction. It is important to note that these Reading standards are meant to complement the specific content demands of the disciplines, not replace them.

Reading Standards for Literacy in History/Social Studies 6–12

RH

The standards below begin at grade 6; standards for K–5 reading in history/social studies, science, and technical subjects are integrated into the K–5 Reading standards. The CCR anchor standards and high school standards in literacy work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Key Ideas and Details		
1. Cite specific textual evidence to support analysis of primary and secondary sources.	1. Cite specific textual evidence to support analysis of primary and secondary sources, attending to such features as the date and origin of the information.	1. Cite specific textual evidence to support analysis of primary and secondary sources, connecting insights gained from specific details to an understanding of the text as a whole.
2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary of the source distinct from prior knowledge or opinions.	2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary of how key events or ideas develop over the course of the text.	2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary that makes clear the relationships among the key details and ideas.
3. Identify key steps in a text's description of a process related to history/social studies (e.g., how a bill becomes law, how interest rates are raised or lowered).	3. Analyze in detail a series of events described in a text; determine whether earlier events caused later ones or simply preceded them.	3. Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain.
Craft and Structure		
4. Determine the meaning of words and phrases as they are used in a text, including vocabulary specific to domains related to history/social studies.	4. Determine the meaning of words and phrases as they are used in a text, including vocabulary describing political, social, or economic aspects of history/social studies.	4. Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines *faction* in *Federalist* No. 10).
5. Describe how a text presents information (e.g., sequentially, comparatively, causally).	5. Analyze how a text uses structure to emphasize key points or advance an explanation or analysis.	5. Analyze in detail how a complex primary source is structured, including how key sentences, paragraphs, and larger portions of the text contribute to the whole.
6. Identify aspects of a text that reveal an author's point of view or purpose (e.g., loaded language, inclusion or avoidance of particular facts).	6. Compare the point of view of two or more authors for how they treat the same or similar topics, including which details they include and emphasize in their respective accounts.	6. Evaluate authors' differing points of view on the same historical event or issue by assessing the authors' claims, reasoning, and evidence.
Integration of Knowledge and Ideas		
7. Integrate visual information (e.g., in charts, graphs, photographs, videos, or maps) with other information in print and digital texts.	7. Integrate quantitative or technical analysis (e.g., charts, research data) with qualitative analysis in print or digital text.	7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, as well as in words) in order to address a question or solve a problem.
8. Distinguish among fact, opinion, and reasoned judgment in a text.	8. Assess the extent to which the reasoning and evidence in a text support the author's claims.	8. Evaluate an author's premises, claims, and evidence by corroborating or challenging them with other information.
9. Analyze the relationship between a primary and secondary source on the same topic.	9. Compare and contrast treatments of the same topic in several primary and secondary sources.	9. Integrate information from diverse sources, both primary and secondary, into a coherent understanding of an idea or event, noting discrepancies among sources.
Range of Reading and Level of Text Complexity		
10. By the end of grade 8, read and comprehend history/social studies texts in the grades 6–8 text complexity band independently and proficiently.	10. By the end of grade 10, read and comprehend history/social studies texts in the grades 9–10 text complexity band independently and proficiently.	10. By the end of grade 12, read and comprehend history/social studies texts in the grades 11–CCR text complexity band independently and proficiently.

Reading Standards for Literacy in Science and Technical Subjects 6–12

RST

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Key Ideas and Details		
1. Cite specific textual evidence to support analysis of science and technical texts.	1. Cite specific textual evidence to support analysis of science and technical texts, attending to the precise details of explanations or descriptions.	1. Cite specific textual evidence to support analysis of science and technical texts, attending to important distinctions the author makes and to any gaps or inconsistencies in the account.
2. Determine the central ideas or conclusions of a text; provide an accurate summary of the text distinct from prior knowledge or opinions.	2. Determine the central ideas or conclusions of a text; trace the text's explanation or depiction of a complex process, phenomenon, or concept; provide an accurate summary of the text.	2. Determine the central ideas or conclusions of a text; summarize complex concepts, processes, or information presented in a text by paraphrasing them in simpler but still accurate terms.
3. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.	3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.	3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
Craft and Structure		
4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 6–8 texts and topics.*	4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 9–10 texts and topics*.	4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 11–12 texts and topics*.
5. Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to an understanding of the topic.	5. Analyze the structure of the relationships among concepts in a text, including relationships among key terms (e.g., *force, friction, reaction force, energy*).	5. Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.
6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text.	6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, defining the question the author seeks to address.	6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, identifying important issues that remain unresolved.
Integration of Knowledge and Ideas		
7. Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).	7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.	7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.
8. Distinguish among facts, reasoned judgment based on research findings, and speculation in a text.	8. Assess the extent to which the reasoning and evidence in a text support the author's claim or a recommendation for solving a scientific or technical problem.	8. Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.
9. Compare and contrast the information gained from experiments, simulations, video, or multimedia sources with that gained from reading a text on the same topic.	9. Compare and contrast findings presented in a text to those from other sources (including their own experiments), noting when the findings support or contradict previous explanations or accounts.	9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.
Range of Reading and Level of Text Complexity		
10. By the end of grade 8, read and comprehend science/technical texts in the grades 6–8 text complexity band independently and proficiently.	10. By the end of grade 10, read and comprehend science/technical texts in the grades 9–10 text complexity band independently and proficiently.	10. By the end of grade 12, read and comprehend science/technical texts in the grades 11–CCR text complexity band independently and proficiently.

College and Career Readiness Anchor Standards for Writing

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade span. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Text Types and Purposes*

1. Write arguments to support claims in an analysis of substantive topics or texts using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details and well-structured event sequences.

Production and Distribution of Writing

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

Research to Build and Present Knowledge

7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

Range of Writing

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

*These broad types of writing include many subgenres. See Appendix A for definitions of key writing types.

Note on range and content of student writing

For students, writing is a key means of asserting and defending claims, showing what they know about a subject, and conveying what they have experienced, imagined, thought, and felt. To be college and career ready writers, students must take task, purpose, and audience into careful consideration, choosing words, information, structures, and formats deliberately. They need to be able to use technology strategically when creating, refining, and collaborating on writing. They have to become adept at gathering information, evaluating sources, and citing material accurately, reporting findings from their research and analysis of sources in a clear and cogent manner. They must have the flexibility, concentration, and fluency to produce high-quality first-draft text under a tight deadline and the capacity to revisit and make improvements to a piece of writing over multiple drafts when circumstances encourage or require it. To meet these goals, students must devote significant time and effort to writing, producing numerous pieces over short and long time frames throughout the year.

Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects 6–12

WHST

The standards below begin at grade 6; standards for K–5 writing in history/social studies, science, and technical subjects are integrated into the K–5 Writing standards. The CCR anchor standards and high school standards in literacy work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Text Types and Purposes		
1. Write arguments focused on *discipline-specific content*. a. Introduce claim(s) about a topic or issue, acknowledge and distinguish the claim(s) from alternate or opposing claims, and organize the reasons and evidence logically. b. Support claim(s) with logical reasoning and relevant, accurate data and evidence that demonstrate an understanding of the topic or text, using credible sources. c. Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), counterclaims, reasons, and evidence. d. Establish and maintain a formal style. e. Provide a concluding statement or section that follows from and supports the argument presented.	1. Write arguments focused on *discipline-specific content.* a. Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among the claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly, supplying data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form and in a manner that anticipates the audience's knowledge level and concerns. c. Use words, phrases, and clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from or supports the argument presented.	1. Write arguments focused on *discipline-specific content.* a. Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences the claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form that anticipates the audience's knowledge level, concerns, values, and possible biases. c. Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from or supports the argument presented.

Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects 6–12

WHST

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Text Types and Purposes (continued)		
2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style and objective tone. f. Provide a concluding statement or section that follows from and supports the information or explanation presented.	2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. a. Introduce a topic and organize ideas, concepts, and information to make important connections and distinctions; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with well-chosen, relevant, and sufficient facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers. e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).	2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. a. Introduce a topic and organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts. d. Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers. e. Provide a concluding statement or section that follows from and supports the information or explanation provided (e.g., articulating implications or the significance of the topic).
3. (See note; not applicable as a separate requirement)	3. (See note; not applicable as a separate requirement)	3. (See note; not applicable as a separate requirement)

Note: Students' narrative skills continue to grow in these grades. The Standards require that students be able to incorporate narrative elements effectively into arguments and informative/explanatory texts. In history/social studies, students must be able to incorporate narrative accounts into their analyses of individuals or events of historical import. In science and technical subjects, students must be able to write precise enough descriptions of the step-by-step procedures they use in their investigations or technical work that others can replicate them and (possibly) reach the same results.

Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects 6–12

WHST

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Production and Distribution of Writing		
4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed.	5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.	5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.
6. Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas clearly and efficiently.	6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.	6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.
Research to Build and Present Knowledge		
7. Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.	7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.	7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the usefulness of each source in answering the research question; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the specific task, purpose, and audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation.
9. Draw evidence from informational texts to support analysis reflection, and research.	9. Draw evidence from informational texts to support analysis, reflection, and research.	9. Draw evidence from informational texts to support analysis, reflection, and research.
Range of Writing		
10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

COMMON CORE STATE STANDARDS FOR

Mathematics

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Table of Contents

Introduction

Toward greater focus and coherence

> *Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics. Mathematical process goals should be integrated in these content areas.*
>
> — Mathematics Learning in Early Childhood, National Research Council, 2009

> *The composite standards [of Hong Kong, Korea and Singapore] have a number of features that can inform an international benchmarking process for the development of K–6 mathematics standards in the U.S. First, the composite standards concentrate the early learning of mathematics on the number, measurement, and geometry strands with less emphasis on data analysis and little exposure to algebra. The Hong Kong standards for grades 1–3 devote approximately half the targeted time to numbers and almost all the time remaining to geometry and measurement.*
>
> — Ginsburg, Leinwand and Decker, 2009

> *Because the mathematics concepts in [U.S.] textbooks are often weak, the presentation becomes more mechanical than is ideal. We looked at both traditional and non-traditional textbooks used in the US and found this conceptual weakness in both.*
>
> — Ginsburg et al., 2005

> *There are many ways to organize curricula. The challenge, now rarely met, is to avoid those that distort mathematics and turn off students.*
>
> — Steen, 2007

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is "a mile wide and an inch deep." These Standards are a substantial answer to that challenge.

It is important to recognize that "fewer standards" are no substitute for focused standards. Achieving "fewer standards" would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity.

Assessing the coherence of a set of standards is more difficult than assessing their focus. William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are:

> *articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, **but also the key ideas** that determine how knowledge is organized and generated within that discipline. This implies*

> *that to be coherent, a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (such as an understanding of the rational number system and its properties).* (emphasis added)

These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas.

In addition, the "sequence of topics and performances" that is outlined in a body of mathematics standards must also respect what is known about how students learn. As Confrey (2007) points out, developing "sequenced obstacles and challenges for students…absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise." In recognition of this, the development of these Standards began with research-based learning progressions detailing what is known today about how students' mathematical knowledge, skill, and understanding develop over time.

Understanding mathematics

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, *why* a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participaton of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

The Standards begin on page 6 with eight Standards for Mathematical Practice.

How to read the grade level standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Domain

Number and Operations in Base Ten	3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Standard

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Cluster

These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know ... should next come to learn ...." But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics | Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 - 2 = 5$. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Grade K Overview

Counting and Cardinality

- **Know number names and the count sequence.**
- **Count to tell the number of objects.**
- **Compare numbers.**

Operations and Algebraic Thinking

- **Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**

Number and Operations in Base Ten

- **Work with numbers 11–19 to gain foundations for place value.**

Measurement and Data

- **Describe and compare measurable attributes.**
- **Classify objects and count the number of objects in categories.**

Geometry

- **Identify and describe shapes.**
- **Analyze, compare, create, and compose shapes.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Counting and Cardinality K.CC

Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
   b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
   c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]
7. Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5.

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[1]Include groups with up to ten objects.
[2]Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Number and Operations in Base Ten K.NBT

Work with numbers 11–19 to gain foundations for place value.

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data K.MD

Describe and compare measurable attributes.

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Classify objects and count the number of objects in each category.

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]

Geometry K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

[3]Limit category counts to be less than or equal to 10.

Mathematics | Grade 1

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1]

(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

[1]Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Grade 1 Overview

Operations and Algebraic Thinking

- **Represent and solve problems involving addition and subtraction.**
- **Understand and apply properties of operations and the relationship between addition and subtraction.**
- **Add and subtract within 20.**
- **Work with addition and subtraction equations.**

Number and Operations in Base Ten

- **Extend the counting sequence.**
- **Understand place value.**
- **Use place value understanding and properties of operations to add and subtract.**

Measurement and Data

- **Measure lengths indirectly and by iterating length units.**
- **Tell and write time.**
- **Represent and interpret data.**

Geometry

- **Reason with shapes and their attributes.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 1.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]

2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

3. Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*

4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

Add and subtract within 20.

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ▯ – 3, 6 + 6 = ▯.*

Number and Operations in Base Ten 1.NBT

Extend the counting sequence.

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
   a. 10 can be thought of as a bundle of ten ones — called a "ten."
   b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

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[2]See Glossary, Table 1.
[3]Students need not use formal terms for these properties.

3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement and Data 1.MD

Measure lengths indirectly and by iterating length units.

1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

Tell and write time.

3. Tell and write time in hours and half-hours using analog and digital clocks.

Represent and interpret data.

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry 1.G

Reason with shapes and their attributes.

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4]
3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

[4]Students do not need to learn formal names such as "right rectangular prism."

Mathematics | Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Grade 2 Overview

Operations and Algebraic Thinking

- **Represent and solve problems involving addition and subtraction.**
- **Add and subtract within 20.**
- **Work with equal groups of objects to gain foundations for multiplication.**

Number and Operations in Base Ten

- **Understand place value.**
- **Use place value understanding and properties of operations to add and subtract.**

Measurement and Data

- **Measure and estimate lengths in standard units.**
- **Relate addition and subtraction to length.**
- **Work with time and money.**
- **Represent and interpret data.**

Geometry

- **Reason with shapes and their attributes.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 2.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten 2.NBT

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens — called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

[1]See Glossary, Table 1.
[2]See standard 1.OA.6 for a list of mental strategies.
[3]Explanations may be supported by drawings or objects.

Measurement and Data 2.MD

Measure and estimate lengths in standard units.

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3. Estimate lengths using units of inches, feet, centimeters, and meters.
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph.

Geometry 2.G

Reason with shapes and their attributes.

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

[4]See Glossary, Table 1.
[5]Sizes are compared directly or visually, not compared by measuring.

Mathematics | Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Grade 3 Overview

Operations and Algebraic Thinking

- **Represent and solve problems involving multiplication and division.**
- **Understand properties of multiplication and the relationship between multiplication and division.**
- **Multiply and divide within 100.**
- **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**

Number and Operations in Base Ten

- **Use place value understanding and properties of operations to perform multi-digit arithmetic.**

Number and Operations—Fractions

- **Develop understanding of fractions as numbers.**

Measurement and Data

- **Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**
- **Represent and interpret data.**
- **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**
- **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**

Geometry

- **Reason with shapes and their attributes.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ▯ ÷ 3, 6 × 6 = ?.*

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*
6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

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[1]See Glossary, Table 2.
[2]Students need not use formal terms for these properties.
[3]This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Number and Operations in Base Ten 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Number and Operations—Fractions[5] 3.NF

Develop understanding of fractions as numbers.

1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
   b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data 3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

[4] A range of algorithms may be used.
[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7]

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
   b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
7. Relate area to the operations of multiplication and addition.
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

[6]Excludes compound units such as cm^3 and finding the geometric volume of a container.

[7]Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).

Geometry 3.G

Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Mathematics | Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Grade 4 Overview

Operations and Algebraic Thinking

- **Use the four operations with whole numbers to solve problems.**
- **Gain familiarity with factors and multiples.**
- **Generate and analyze patterns.**

Number and Operations in Base Ten

- **Generalize place value understanding for multi-digit whole numbers.**
- **Use place value understanding and properties of operations to perform multi-digit arithmetic.**

Number and Operations—Fractions

- **Extend understanding of fraction equivalence and ordering.**
- **Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**
- **Understand decimal notation for fractions, and compare decimal fractions.**

Measurement and Data

- **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**
- **Represent and interpret data.**
- **Geometric measurement: understand concepts of angle and measure angles.**

Geometry

- **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]
3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Number and Operations in Base Ten[2] 4.NBT

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

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[1]See Glossary, Table 2.
[2]Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations—Fractions[3] 4.NF

Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
   c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
   d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
   b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)*
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Represent and interpret data.

4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles.

   b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

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[4]Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Geometry 4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Mathematics | Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Grade 5 Overview

Operations and Algebraic Thinking

- **Write and interpret numerical expressions.**
- **Analyze patterns and relationships.**

Number and Operations in Base Ten

- **Understand the place value system.**
- **Perform operations with multi-digit whole numbers and with decimals to hundredths.**

Number and Operations—Fractions

- **Use equivalent fractions as a strategy to add and subtract fractions.**
- **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

Measurement and Data

- **Convert like measurement units within a given measurement system.**
- **Represent and interpret data.**
- **Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**

Geometry

- **Graph points on the coordinate plane to solve real-world and mathematical problems.**
- **Classify two-dimensional figures into categories based on their properties.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 5.OA

Write and interpret numerical expressions.

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

Analyze patterns and relationships.

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Number and Operations in Base Ten 5.NBT

Understand the place value system.

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write, and compare decimals to thousandths.
   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
   b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Number and Operations—Fractions 5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
   a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*
   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5. Interpret multiplication as scaling (resizing), by:
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]
   a. Interpret division of a unit fraction by a non-zero whole number,

[1]Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Measurement and Data 5.MD

Convert like measurement units within a given measurement system.

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Represent and interpret data.

2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

    a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

    b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

    a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

    b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

    c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Geometry 5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

4. Classify two-dimensional figures in a hierarchy based on properties.

Mathematics | Grade 6

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and

median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Grade 6 Overview

Ratios and Proportional Relationships

- **Understand ratio concepts and use ratio reasoning to solve problems.**

The Number System

- **Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**
- **Compute fluently with multi-digit numbers and find common factors and multiples.**
- **Apply and extend previous understandings of numbers to the system of rational numbers.**

Expressions and Equations

- **Apply and extend previous understandings of arithmetic to algebraic expressions.**
- **Reason about and solve one-variable equations and inequalities.**
- **Represent and analyze quantitative relationships between dependent and independent variables.**

Geometry

- **Solve real-world and mathematical problems involving area, surface area, and volume.**

Statistics and Probability

- **Develop understanding of statistical variability.**
- **Summarize and describe distributions.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Ratios and Proportional Relationships 6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
2. Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
   c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System 6.NS

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.

2. Fluently divide multi-digit numbers using the standard algorithm.
3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

[1]Expectations for unit rates in this grade are limited to non-complex fractions.

Apply and extend previous understandings of numbers to the system of rational numbers.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

   a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.

   b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

   c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

7. Understand ordering and absolute value of rational numbers.

   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*

   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 °C > –7 °C to express the fact that –3 °C is warmer than –7 °C.*

   c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*

   d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*

8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations 6.EE

Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.

2. Write, read, and evaluate expressions in which letters stand for numbers.

   a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6 s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.

3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Geometry 6.G

Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability 6.SP

Develop understanding of statistical variability.

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
5. Summarize numerical data sets in relation to their context, such as by:
   a. Reporting the number of observations.
   b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
   c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
   d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Mathematics | Grade 7

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Grade 7 Overview

Ratios and Proportional Relationships

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Statistics and Probability

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Ratios and Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.*
2. Recognize and represent proportional relationships between quantities.
   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
   c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*
   d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.
3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

The Number System 7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
   c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
   d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

3. Solve real-world and mathematical problems involving the four operations with rational numbers.[1]

Expressions and Equations 7.EE

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

   a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

   b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Geometry 7.G

Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

[1]Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability 7.SP

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

    a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

    b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

    a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

    b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

    c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Mathematics | Grade 8

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Grade 8 Overview

The Number System

- **Know that there are numbers that are not rational, and approximate them by rational numbers.**

Expressions and Equations

- **Work with radicals and integer exponents.**
- **Understand the connections between proportional relationships, lines, and linear equations.**
- **Analyze and solve linear equations and pairs of simultaneous linear equations.**

Functions

- **Define, evaluate, and compare functions.**
- **Use functions to model relationships between quantities.**

Geometry

- **Understand congruence and similarity using physical models, transparencies, or geometry software.**
- **Understand and apply the Pythagorean Theorem.**
- **Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.**

Statistics and Probability

- **Investigate patterns of association in bivariate data.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Number System 8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$*, show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations 8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$*.*
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.

   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

   c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Functions 8.F

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:

   a. Lines are taken to lines, and line segments to line segments of the same length.

   b. Angles are taken to angles of the same measure.

   c. Parallel lines are taken to parallel lines.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

[1]Function notation is not required in Grade 8.

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Statistics and Probability 8.SP

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Mathematics Standards for High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

> (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Mathematics | High School—Number and Quantity

Numbers and Number Systems. During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Number and Quantity Overview

The Real Number System

- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers.

Quantities

- Reason quantitatively and use units to solve problems

The Complex Number System

- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

Vector and Matrix Quantities

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Real Number System — N-RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities* — N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System — N-CN

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus* 2 *and argument* 120°.
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities N-VM

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v).
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. (+) Add and subtract vectors.
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
5. (+) Multiply a vector by a scalar.
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
   b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12. (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Mathematics | High School—Algebra

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Algebra Overview

Seeing Structure in Expressions

- **Interpret the structure of expressions**
- **Write expressions in equivalent forms to solve problems**

Arithmetic with Polynomials and Rational Expressions

- **Perform arithmetic operations on polynomials**
- **Understand the relationship between zeros and factors of polynomials**
- **Use polynomial identities to solve problems**
- **Rewrite rational expressions**

Creating Equations

- **Create equations that describe numbers or relationships**

Reasoning with Equations and Inequalities

- **Understand solving equations as a process of reasoning and explain the reasoning**
- **Solve equations and inequalities in one variable**
- **Solve systems of equations**
- **Represent and solve equations and inequalities graphically**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Seeing Structure in Expressions — A-SSE

Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.*
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**

Arithmetic with Polynomials and Rational Expressions — A-APR

Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

[1] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations* — A-CED

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Reasoning with Equations and Inequalities — A-REI

Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$.

8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.

9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Mathematics | High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Functions Overview

Interpreting Functions

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

Building Functions

- Build a function that models a relationship between two quantities
- Build new functions from existing functions

Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

Trigonometric Functions

- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Interpreting Functions F-IF

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.*
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
   a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models* F-LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

Prove and apply trigonometric identities

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Mathematics | High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

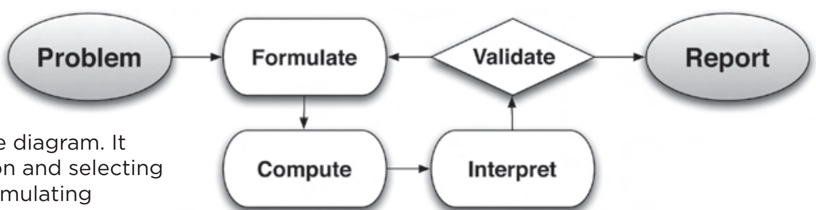
A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol* (★).

Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Geometry Overview

Congruence

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

Circles

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects

Modeling with Geometry

- Apply geometric concepts in modeling situations

Mathematical Practices

1. Apply geometric concepts in modeling situations
2. Mathematical Practices
3. Make sense of problems and persevere in solving them.
4. Reason abstractly and quantitatively.
5. Construct viable arguments and critique the reasoning of others.
6. Model with mathematics.
7. Use appropriate tools strategically.
8. Attend to precision.
9. Look for and make use of structure.
10. Look for and express regularity in repeated reasoning.

Congruence G-CO

Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems

9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, and Trigonometry — G-SRT

Understand similarity in terms of similarity transformations

1. Verify experimentally the properties of dilations given by a center and a scale factor:
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity

4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Apply trigonometry to general triangles

9. (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles — G-C

Understand and apply theorems about circles

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations — G-GPE

Translate between the geometric description and the equation for a conic section

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.
3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Geometric Measurement and Dimension — G-GMD

Explain volume formulas and use them to solve problems

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Visualize relationships between two-dimensional and three-dimensional objects

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry — G-MG

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Mathematics | High School—Statistics and Probability★

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Statistics and Probability Overview

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments and observational studies

Conditional Probability and the Rules of Probability

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

Using Probability to Make Decisions

- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Interpreting Categorical and Quantitative Data — S-ID

Summarize, represent, and interpret data on a single count or measurement variable

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Summarize, represent, and interpret data on two categorical and quantitative variables

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
   b. Informally assess the fit of a function by plotting and analyzing residuals.
   c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions — S-IC

Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

Conditional Probability and the Rules of Probability S-CP

Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model

6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model.
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions S-MD

Calculate expected values and use them to solve problems

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decisions

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
   b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Note on courses and transitions

The high school portion of the Standards for Mathematical Content specifies the mathematics all students should study for college and career readiness. These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. To that end, sample high school pathways for mathematics – in both a traditional course sequence (Algebra I, Geometry, and Algebra II) as well as an integrated course sequence (Mathematics 1, Mathematics 2, Mathematics 3) – will be made available shortly after the release of the final Common Core State Standards. It is expected that additional model pathways based on these standards will become available as well.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, states may handle the transition to high school in different ways. For example, many students in the U.S. today take Algebra I in the 8th grade, and in some states this is a requirement. The K-7 standards contain the prerequisites to prepare students for Algebra I by 8th grade, and the standards are designed to permit states to continue existing policies concerning Algebra I in 8th grade.

A second major transition is the transition from high school to post-secondary education for college and careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as Grades 6-8. It is important to note as well that cut scores or other information generated by assessment systems for college and career readiness should be developed in collaboration with representatives from higher education and workforce development programs, and should be validated by subsequent performance of students in college and the workforce.

Glossary

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: $^3/_4$ and – $^3/_4$ are additive inverses of one another because $^3/_4$ + (– $^3/_4$) = (– $^3/_4$) + $^3/_4$ = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction $^A/_B$ where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form $^a/_b$ where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

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[1]Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

[2]Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

Integer. A number expressible in the form *a* or –*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[4] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or $-\,a/b$ for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

[3]Adapted from Wisconsin Department of Public Instruction, *op. cit.*
[4]To be more precise, this defines the *arithmetic mean*.

one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ….

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit*.

TABLE 1. Common addition and subtraction situations.[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

TABLE 2. Common multiplication and division situations.[7]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

TABLE 3. The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

TABLE 4. The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

TABLE 5. The properties of inequality. Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

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Appendix B1-4

International Benchmarking of Common Core Standards

The following is information on international benchmarking of the Common Core State Standards provided by the Council of Chief State School Officers.

International Benchmarking and the Common Core

The Common Core State Standards (CCSS) are designed to be **college- and career-ready** and **internationally benchmarked**. To that end, the development process included the review and consideration of many sources, including research studies, existing standards from the U.S and abroad, and the professional judgment of teachers, content area experts, and college faculty. This paper will briefly describe how international benchmarking was used to develop the CCSS.

What documents were used to ensure that the CCSS were internationally benchmarked?
To ensure that the standards prepare students to be globally competitive, the development team used a number of sources, including: the frameworks for PISA and TIMSS; the International Baccalaureate syllabi; the American Institutes for Research report, **Informing Grades 1-6 Mathematics Standards Development: What Can Be Learned From High-Performing Hong Kong, Korea, and Singapore** and; the A+ Composite found in **A Coherent Curriculum: The Case for Mathematics** ***by Bill Schmidt, Richard Houang, and Leland Cogan.***

In addition, the development team looked to the standards of a number of individual countries and provinces to inform the content, structure and language of the CCSS. In *mathematics*, twelve set of standards were selected to help guide the writing of the standards: Belgium, Canada [Alberta], China, Chinese Taipei, England, Finland, Hong Kong, India, Ireland, Japan, Korea, and Singapore.[3] In *English language arts*, the writing team looked closely at ten sets of standards from Australia (New South Wales and Victoria), Canada (Alberta, British Columbia, and Ontario), England, Finland, Hong Kong, Ireland, and Singapore.[4]

How were the international benchmarks used to inform the development of the CCSS?
The goal of the international benchmarking in the common core state standards development process was to ensure that the CCSS are as rigorous as comparable standards in the high-performing and other countries. However, the use of international benchmarks as evidence is no easy feat; it is not simply a matter of identifying the “best” source and copying it, or of aggregating all viable sources to find some set of shared expectations. Rather, international benchmarks were used to guide critical decisions in the following areas:

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[3] Eight of these were high-performers on either TIMSS, PISA or both: Belgium, Canada [Alberta], Chinese Taipei, Finland, Hong Kong, Japan, Korea, and Singapore. England and Ireland, which have uneven performances on international assessments, were included because of their cultural links to the United States. China and India were included because of their growing global competitiveness.

[4] Differences in language have a greater impact on the teaching and learning of language arts than of mathematics, so the teams looked primarily at English-speaking countries. All were high-performers on PISA except Singapore, which did not participate, and England, which as in mathematics was selected partly for its cultural links to the United States.

- *Whether particular content should be included:* One of the principal ways international standards were used in this development process was as a guide when making tough decisions about whether content should be included or excluded.
- *When content should be introduced and how that content should progress*: The progression of topics in the international mathematics standards helped the development team make decisions about when to introduce topics in the CCSS as well as when to stop focusing on them.
- *Ensuring focus and coherence*: Standards from other countries tend to be very focused, including only what is absolutely necessary.
- *Organizing and formatting the standards*: Certain organizational aspects or characteristics of international standards that promoted clarity and ease of reading and use served as a model for the CCSS.
- *Determining emphasis on particular topics in standards*: Where emphasis on particular topics was found repeatedly in international standard, this was instructive in determining their importance for inclusion in the CCSS.

In addition to the above summary, attached are the following documents which further demonstrate the international benchmarking of the Common Core State Standards as well as their college and career-readiness design:

- *Benchmarking for Success: Ensuring U.S. Students Receive a World Class Education, a report by the National Governors Association, the Counsel of Chief School Officers and Achieve, Inc.*
    - This report summarizes the recommendations of the International Benchmarking Advisory Group, including the group's recommendation to upgrade state standards by adopting a common core of internationally benchmarked standards in math and language arts for grades K-12 to ensure that students are equipped with the necessary knowledge and skills to be globally competitive.
- *Reaching Higher, The Common Core State Standards Validation Committee, a report from the National Governors Association Center for Best Practices & The Council of Chief State School Officers*.
    - This report from the Common Core State Standards Validation Committee highlights the international benchmarking of the Common Core State Standards as one of the Validation Committee's findings as part of their review of the Standards. The Validation Committee also found that the Common Core State Standards are reflective of the core knowledge and skills in English language arts and mathematics that students need to be college- and career-ready.

- *Common Core State Standards for English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects, Appendix, Research Supporting Key Elements of the Standards.*
    - The Appendix to the Common Core State Standards for English Language Arts includes a specific note which explains how, in developing these standards, the writing team for the Standards consulted numerous international models.
- *Common Core State Standards for Mathematics*
    - In the Common Core State Standards for Mathematics, international benchmarking was a key component as noted in the Introduction to the Standards as well as the Standards' explanation of grade placements for specific topics. The Sample of Works consulted also reflects analysis and benchmarking against international standards.

Benchmarking for Success:

Ensuring U.S. Students Receive a World-Class Education

A report by the National Governors Association, the Council of Chief State School Officers, and Achieve, Inc.

National Governors Association

Founded in 1908, the National Governors Association (NGA) is the collective voice of the nation's governors and one of Washington, D.C.'s most respected public policy organizations. Its members are the governors of the 50 states, three territories and two commonwealths. NGA provides governors and their senior staff members with services that range from representing states on Capitol Hill and before the Administration on key federal issues to developing and implementing innovative solutions to public policy challenges through the NGA Center for Best Practices. For more information, visit www.nga.org.

Council of Chief State School Officers

The Council of Chief State School Officers (CCSSO) is a nonpartisan, nationwide, nonprofit organization of public officials who head departments of elementary and secondary education in the states, the District of Columbia, the Department of Defense Education Activity, and five U.S. extra-state jurisdictions. CCSSO provides leadership, advocacy, and technical assistance on major educational issues. The Council seeks member consensus on major educational issues and expresses their views to civic and professional organizations, federal agencies, Congress, and the public.

Achieve, Inc.

Created by the nation's governors and business leaders, Achieve is a bipartisan, non-profit organization that helps states raise academic standards, improve assessments and strengthen accountability to prepare all young people for postsecondary success. At the 2005 National Education Summit, Achieve launched the American Diploma Project (ADP) Network, a coalition that has grown to 34 states, educating nearly 85% of public school students in the United States. The ADP Network is committed to aligning high school expectations with the demands of college, career and life. To learn more about Achieve, visit www.achieve.org.

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Benchmarking for Success:

Ensuring U.S. Students Receive a World-Class Education

A report by the National Governors Association, the Council of Chief State School Officers, and Achieve, Inc.

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Foreword

We are living in a world without borders. To meet the realities of the 21st century global economy and maintain America's competitive edge into the future, we need students who are prepared to compete not only with their American peers, but with students from all across the globe for the jobs of tomorrow.

States have voluntarily taken the lead in developing standards-based education, but policymakers lack a critical tool for moving forward—international benchmarking. This report is intended to help states take the next steps toward ensuring that American students receive a world-class education that positions them to compete and innovate in the 21st century.

International benchmarking will help state policymakers identify the qualities and characteristics of education systems that best prepare students for success in the global marketplace. The stakes are high, and improving our educational system will require commitment and insight not just from state leaders but many other stakeholders as well. With this in mind, the National Governors Association, the Council of Chief State School Officers, and Achieve, Inc. have joined to provide to states a roadmap for benchmarking their K-12 education systems to those of top-performing nations.

The partners' recommendations were informed by an International Benchmarking Advisory Group consisting of education experts representing education institutions, the business community, researchers, former federal officials, and current state and local officials. The Advisory Group's expertise and experience helped the partners identify the need for international comparisons and provide guidance for benchmarking state education system practices in areas such as standards, accountability, educator workforce, and assessments. The partner organizations will work with states to develop and implement these recommendations.

Governors recognize that new economic realities mean it no longer matters how one U.S. state compares to another on a national test; what matters is how a state's students compare to those in countries around the globe. America must seize this moment to ensure that we have workers whose knowledge, skills, and talents are competitive with the best in the world.

Governor Janet Napolitano
Arizona

Governor Sonny Perdue
Georgia

Craig R. Barrett
Chairman of the Board
Intel Corporation

Co-Chairs, International Benchmarking Advisory Group

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International Benchmarking Advisory Group

To develop this report, the National Governors Association (NGA), Council of Chief State School Officers (CCSSO), and Achieve, Inc. invited national, state, and local education and policy leaders to serve on an International Benchmarking Advisory Group. The Advisory Group provided the three partner organizations with valuable insights and helped frame this bipartisan Call to Action. They collectively support the recommendations herein for internationally benchmarking state K-12 education systems.

Co-Chairs:

Governor Janet Napolitano, Arizona
Governor Sonny Perdue, Georgia
Craig R. Barrett, Chairman of the Board, Intel Corporation

Members:

Steven A. Ballmer, Chief Executive Officer, Microsoft Corporation

Governor Donald L. Carcieri, Rhode Island

Mitchell Chester, Commissioner of Education, Massachusetts Department of Elementary and Secondary Education

Christopher Edley, Jr., Dean and Professor of Law, University of California–Berkeley

Chester E. Finn, Jr., President, Thomas B. Fordham Institute

Beverly L. Hall, Superintendent, Atlanta Public Schools

James B. Hunt, Jr., Chairman, the James B. Hunt, Jr. Institute for Educational Leadership and Policy at the University of North Carolina–Chapel Hill and former Governor of North Carolina

Kati Haycock, President, The Education Trust

Dwight Jones, Commissioner of Education, Colorado Department of Education

Governor Tim Kaine, Virginia

Janet Murguía, President and Chief Executive Officer, National Council of La Raza

Thomas Payzant, Professor of Practice, Harvard Graduate School of Education

Charles B. Reed, Chancellor, California State University

Richard W. Riley, Senior Partner with EducationCounsel LLC, former U.S. Secretary of Education, and former Governor of South Carolina

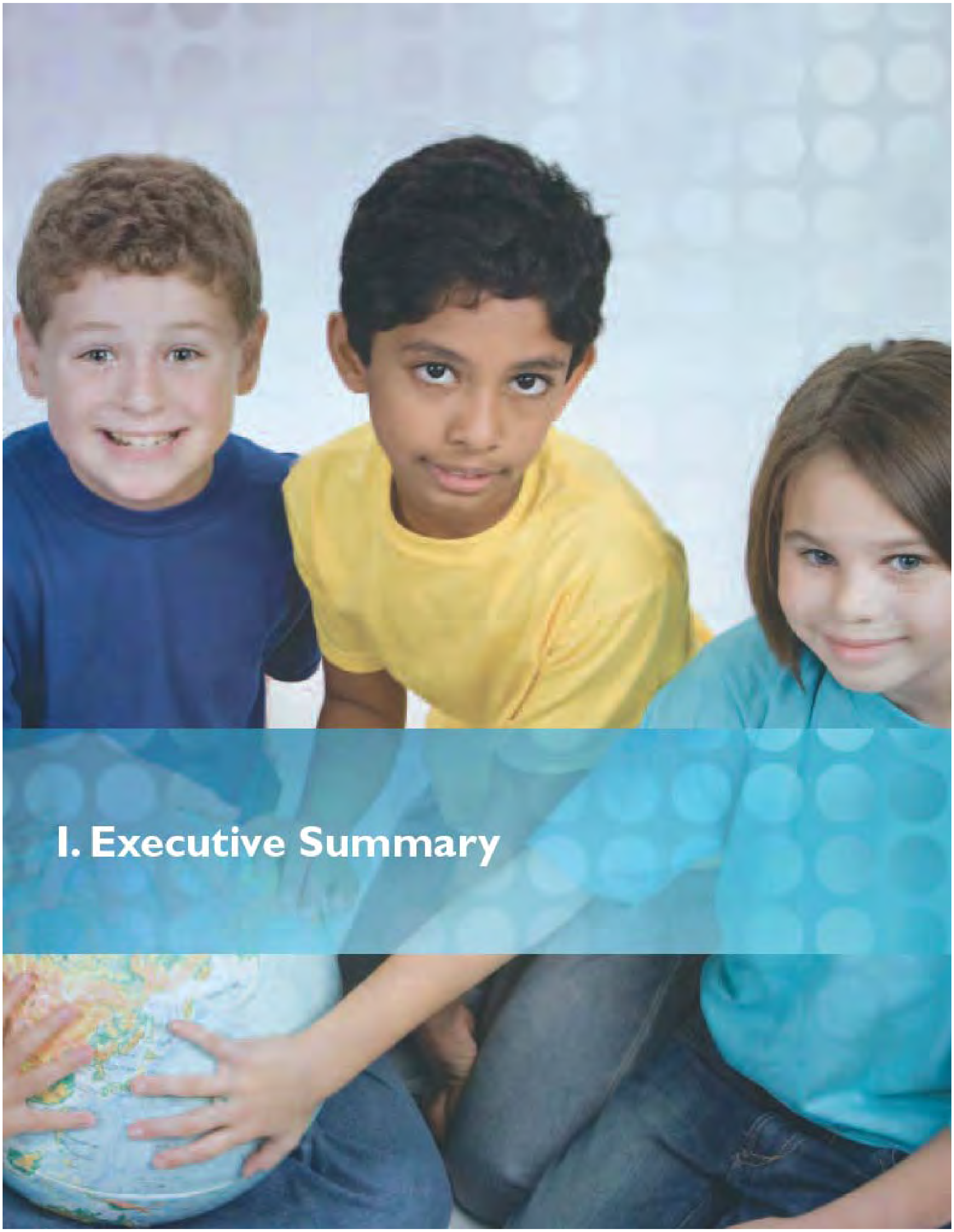
Andreas Schleicher, Head of the Indicators and Analysis Division, Directorate for Education, Organisation for Economic Co-Operation and Development

William H. Schmidt, University Distinguished Professor, Michigan State University

Vivien Stewart, Vice President for Education, Asia Society

Phillip Uri Treisman, Executive Director, The Charles A. Dana Center at the University of Texas at Austin

Bob Wise, President, Alliance for Excellent Education and former Governor of West Virginia



I. Executive Summary

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Around the globe, governments are eagerly comparing their educational outcomes to the best in the world. The goal is not just to see how they rank, but rather to identify and learn from top performers and rapid improvers—from nations and states that offer ideas for boosting their own performance. This process, known as "international benchmarking," has become a critical tool for governments striving to create world-class education systems.

In American education, "benchmarking" often simply means comparing performance outcomes or setting performance targets (or "benchmarks"). But in business and among education leaders in other countries, it means much more. The American Productivity and Quality Center puts it this way: "Benchmarking is the practice of being humble enough to admit that someone else has a better process and wise enough to learn how to match or even surpass them."

Countries and states have good reason to make the effort. Technological, economic, and political trends have combined to increase demand for higher skills while heightening competition for quality jobs. Rule-bound jobs on factory floors and in offices are being automated and outsourced. The world's knowledge-and-innovation economy favors workers who have postsecondary education or training, strong fundamental skills in math and reading, and the ability to solve unfamiliar problems and communicate effectively.

At the same time, new technologies and corporate strategies have opened the global labor market to billions of people from places like Eastern Europe, India, China, and Brazil who had been left out. An increasing variety of work tasks can be digitized and performed nearly anywhere in the world. More jobs are going to the best educated no matter where they live, which means that Americans will face more competition than ever for work.

International trade agreements, such as China's membership in the World Trade Organization in 2001, have hastened this transformation. Since 1980, global trade has grown 2.5 times faster than the global gross domestic product (GDP). Recent estimates put today's world exports at $12.5 trillion, nearly 20 percent of world GDP.

The global economy is here to stay, with recent research suggesting that it is evolving and its impact intensifying at a stunning pace. "Globalization is happening faster than people think," says Vivek Wadhwa, Wertheim Fellow at Harvard Law School's Labor and Worklife program and Duke University Executive in Residence. His recent research shows that companies are no longer just outsourcing production but are farming out *innovation* as well. "Having India and China conduct such sophisticated research and participate in drug discovery was unimaginable even five years ago," he says.

Education is a tremendously important lever for ensuring competitiveness and prosperity in the age of globalization, albeit not the only one. Recent economic studies show that high skills lead to better wages, more equitable distributions of income, and substantial gains in economic productivity. Higher math performance at the end of high school translates into a 12 percent increase in future earnings. If the United States raised students' math and science skills to globally competitive levels over the next two decades, its GDP would be an additional 36 percent higher 75 years from now.

The race is on among nations to create knowledge-fueled innovation economies. In Singapore, Germany, China, Brazil, Korea, and other countries around the world, educational improvement is viewed as a critical part of that mission. Nations and states are therefore working hard to benchmark their education systems to establish a solid foundation for economic development in the 21st century. Some are finding innovative ways to measure their students' progress internationally. Others are examining high-performing and fast-improving nations to learn about best practices that they then adapt or adopt to improve their own systems.

American education has not adequately responded to these new challenges. The United States is falling behind other countries in the resource that matters most in the new global economy: human capital. American 15-year-olds ranked 25th in math and 21st in science achievement on the most recent international assessment conducted in 2006. At the same time, the U.S. ranked high in inequity, with the third largest gap in science scores between students from different socioeconomic groups.

The U.S. is rapidly losing its historic edge in educational attainment as well. As recently as 1995, America still tied for first in college and university graduation rates, but by 2006 had dropped to 14th. That same year it had the second-highest college dropout rate of 27 countries.

State leaders already are deeply engaged in efforts to raise standards, advance teaching quality, and improve low-performing schools. International benchmarking provides an additional tool for making that process more effective, offering insights and ideas that cannot be garnered solely from looking within and across state lines. To that end, the partner organizations and International Benchmarking Advisory Group call on state leaders to take the following actions:

State leaders also should tackle "the equity imperative" by creating strategies for closing the achievement gap between students from different racial and socioeconomic backgrounds in each of the action steps above. Reducing inequality in education is not only socially just, it's essential for ensuring that the United States retain a competitive edge.

Research shows that education systems in the United States tend to give disadvantaged and low-achieving students a watered down curriculum and place them in larger classes taught by less qualified teachers—exactly opposite of the educational practices of high-performing countries.

Action 1: Upgrade state standards by adopting a common core of internationally benchmarked standards in math and language arts for grades K-12 to ensure that students are equipped with the necessary knowledge and skills to be globally competitive.

Action 2: Leverage states' collective influence to ensure that textbooks, digital media, curricula, and assessments are aligned to internationally benchmarked standards and draw on lessons from high-performing nations and states.

Action 3: Revise state policies for recruiting, preparing, developing, and supporting teachers and school leaders to reflect the human capital practices of top-performing nations and states around the world.

Action 4: Hold schools and systems accountable through monitoring, interventions, and support to ensure consistently high performance, drawing upon international best practices.

Action 5: Measure state-level education performance globally by examining student achievement and attainment in an international context to ensure that, over time, students are receiving the education they need to compete in the 21st century economy.

The federal government can play an enabling role as states engage in the critical but challenging work of international benchmarking. First, federal policymakers should offer funds to help underwrite the cost for states to take the five action steps described above. At the same time, policymakers should boost federal research and development (R&D) investments to provide state leaders with more and better information about international best practices, and should help states develop streamlined assessment strategies that facilitate cost-effective international comparisons of student performance.

As states reach important milestones on the way toward building internationally competitive education systems, the federal government should offer a range of tiered incentives to make the next stage of the journey easier, including increased flexibility in the use of federal funds and in meeting federal educational requirements and providing more resources to implement world-class educational best practices. Over the long term, the federal government will need to update laws to align national education policies with lessons learned from state benchmarking efforts and from federally funded research.

Nations around the world are facing a new education imperative, and many are seizing the historical moment to provide their citizens with better opportunities and stronger economies.

America must seize this moment too, with states leading the way. Many states already are working hard to improve standards, teaching quality, and accountability, but policymakers lack a critical tool—international benchmarking.

The U.S. can take pride in many aspects of its education system, from the high performance of its teenagers on international civics tests to the strength of its higher education institutions.

But if state leaders want to ensure that their citizens and their economies remain competitive, they must look beyond America's borders and benchmark their education systems with the best in the world. The state mandate to educate all students remains, but the world that students will enter after school has changed.

For Andreas Schleicher, head of the Indicators and Analysis Division at the Organisation for Economic Co-Operation and Development's Directorate for Education, the case for adopting a global view to improving education is undeniable:

> It is only through such benchmarking that countries can understand relative strengths and weaknesses of their education system and identify best practices and ways forward. The world is indifferent to tradition and past reputations, unforgiving of frailty and ignorant of custom or practice. Success will go to those individuals and countries which are swift to adapt, slow to complain, and open to change.

II. The Need for Action

II. The Need for Action

Around the globe, governments are eagerly comparing their educational outcomes to the best in the world. The goal is not just to see how they rank, but rather to identify and learn from top performers and rapid improvers—from nations and states that offer ideas for boosting their own performance. This process, known as "international benchmarking," has become a critical tool for governments striving to create world-class education systems.

In American education, "benchmarking" often simply means comparing performance outcomes or setting performance targets (or "benchmarks"). But in business and among education leaders in other countries, it means much more: Comparing outcomes to identify top performers or fast improvers, learning how they achieve great results, and applying those lessons to improve one's own performance. The American Productivity and Quality Center puts it this way: "Benchmarking is the practice of being humble enough to admit that someone else has a better process and wise enough to learn how to match or even surpass them."[1]

A Skills-Driven Global Economy

Governments have good reason to benchmark and improve their education systems. Technological, economic, and political trends have increased demand for higher skills while heightening competition for quality jobs. In the U.S., outsourcing and automation have dramatically altered the mix of jobs in the labor force. The proportion of American workers in blue-collar and administrative support jobs plummeted from 56 percent to 39 percent between 1969 and 1999, and the share of jobs requiring more education and specialized skills—work that is managerial, professional, and technical in nature—increased from 23 percent to 33 percent over the same period.[2]

Skill demands *within* jobs are rising as well. A study that analyzed typical tasks in the American workplace found that routine manual and cognitive tasks that follow a set of prescribed rules are rapidly being taken over by computers or workers in other countries. But more sophisticated tasks are on the rise, specifically those that require workers to "bring facts and relationships to bear in problem solving, the ability to judge when one problem-solving strategy is not working and another should be tried, and the ability to engage in complex communication with others," along with "foundational skills" in math and reading.[3]

Technology is changing not just how work gets done, but also where it can be done. Advances in telecommunications allow companies to digitize work tasks and products so that jobs can be performed virtually anywhere in the world. And new management software has enabled firms to shift from "vertical" production—where all tasks are done in sequence in the same place—to "horizontal" production in which tasks are carved up and shipped out to wherever they can be done best and cheapest. The result, according to a blue-ribbon commission report released last year, "is a world in which it is just as easy to create work teams on four continents as it is to create work teams composed of people from four divisions of the same firm located in the same city."[4]

While all these changes took place, political and economic developments opened the doors of this new global economy to more than a billion new workers from Russia, Eastern Europe, China, India, and other developing countries who now compete for jobs with those in developed nations. Harvard economist Richard Freeman calls this "The Great Doubling" of the global workforce. At first, low-skilled, low-paying jobs were outsourced to these workers, but now some higher skilled jobs—from analyzing X-rays to tutoring high school students to preparing tax returns—are migrating abroad, too.[5] The twin forces of globalization and computerization mean that any job reducible to a set of scripted rules is vulnerable to outsourcing or automation.[6]

International trade agreements, such as China's membership in the World Trade Organization in 2001, have sped this transformation along. Although some firms have long had global links, globalization is now pervasive: More nations are joining the marketplace, more goods and services are traded globally, and more of the production process is interconnected in a worldwide supply web. Since 1980, global trade has grown 2.5 times faster than the global gross domestic product (GDP). Recent estimates put today's world exports at $12.5 trillion, nearly 20 percent of world GDP.[7]

Recent research suggests that globalization is not only here to stay, it is evolving and intensifying at a rapid pace. In June, Harvard and Duke University researchers published the first in a series of studies documenting how corporations are no longer just outsourcing production; they are beginning to outsource *innovation* as well. For example, big pharmaceutical companies such as Merck, Eli Lilly, and Johnson & Johnson are relying on India and China not only for manufacturing and clinical trials, but also for advanced research and development. As a result, scientists in those countries are rapidly increasing their ability to innovate and create their own intellectual property; the global share of pharmaceutical patent applications originating in India and China increased fourfold from 1995 to 2006.[8]

"Globalization is happening faster than people think," says Vivek Wadhwa, the researcher and former entrepreneur who led the study. "Having India and China conduct such sophisticated research and participate in drug discovery was unimaginable even five years ago."[9] Wadhwa's team is finding the same kind of rapid change in a wide range of industries—from telecommunications and computer networking to aerospace and computers. Indeed, the National Academy of Engineering recently noted that nearly all of the top 20 U.S.-based semiconductor companies have opened design centers in India, nine of them since 2004.[10] "Our take is that the global technology landscape has changed dramatically over the last decade," says Wadhwa, "and that we're at the beginning of a new wave of globalization."[11]

Education for Economic Growth

As a result of these trends, American workers are competing not only with skilled workers here, but with those living in far-away places. Labor economists Frank Levy and Richard Murnane argue that "over the long run, better education is the best tool we have to prepare the population for a rapidly changing job market."[12] Studies show that higher math performance at the end of high school translates into substantially higher future earnings; an increase of one standard deviation in math scores translates into a 12 percent boost in wages.[13] Family income for households headed by someone with a college degree grew by nearly 40 percent from 1973 to 2006, compared with less than 6 percent for families headed by someone with only a high school diploma.[14]

Fortune may favor the prepared mind, but it also favors the prepared *place*—whether that place is a nation, a region, or an individual state. To lay a solid foundation for widespread economic growth, governments around the world are adopting policies aligned with a 21st century economy that is increasingly knowledge-fueled, innovation-driven, and global in scope. The Organisation for Economic Co-Operation and Development (OECD) estimates that each additional year of schooling among the adult population raises a nation's economic output by between 3 percent and 6 percent.[15] New studies by Stanford economist Eric Hanushek and others have found strong evidence that high skills lead to elevated individual wages, a more equitable distribution of income, and substantial gains in economic productivity.[16]

Indeed, Hanushek estimates that if the U.S. improved enough to become a top-performing nation on international assessments between 2005 and 2025, by 2037 its GDP would be an additional 5 percent higher than if skills stayed the same. Improving human capital pays off even more handsomely over a longer time horizon: By 2080, America's GDP would be *36 percent higher* than would be the case if the U.S. remained mediocre in math and science.[17]

The implications are clear: In today's world, high wages follow high skills, and long-term economic growth increasingly depends on educational excellence. Unfortunately, American education has not adequately responded to these challenges. As other countries seize the opportunity to improve their education systems so their citizens can benefit from new economic opportunities, the United States is rapidly losing its leading edge in the resource that matters most for economic success: human capital.

Four decades ago America had the best high school graduation rate in the world, but by 2006 it had slipped to 18th out of 24 industrialized countries.[18] For most of the 20th century, the U.S. set the standard for quality in higher education—and, in many respects, it still does. But other countries learned from our success and are now catching up or pulling ahead. As recently as 1995 America was still tied for first in the proportion of young adults with a college degree, but by 2000 it had slipped to 9th and by 2006 to 14th—below the OECD average for the first time.[19] According to the latest OECD figures, the U.S. has one of the highest college dropout rates in the industrialized world.[20]

Even if the U.S. improves its high school and postsecondary graduation rates, it will be difficult if not impossible to maintain its historic dominance in the supply of educated workers. Already, America's share of the world's college students has dropped from 30 percent in 1970 to less than half that today.[21] And because of their sheer size, China and India will surpass both Europe and the United States in the number of secondary and postsecondary graduates produced over the next decade.[22] Many experts have concluded that since the U.S. can no longer compete in *quantity* of human capital, it will have to compete in *quality* by providing its young people with the highest level of math, science, reading, and problem-solving skills in the world.

But so far American education has not adequately responded to the skills challenge either. Out of 30 industrialized countries participating in the OECD's Programme for International Student Assessment (PISA) in 2006, the U.S. ranked 25th in math and 21st in science achievement (**Figure 1**). The performance gap between the United States and top-performing nations is huge: American students lag about a full year behind their peers in the countries that perform best in mathematics.[23] Even our "best and brightest" cannot compete with excellent students elsewhere. According to the OECD, "the United States does not just have more students performing badly—it also has many fewer students performing well."[24] America's best math students performed worse than the best math students in 22 other OECD nations. Moreover, only 1.3 percent of U.S. 15-year-olds performed at the highest PISA level in mathematics, while among the top 10 countries the share of high performers was three to seven times as large.[25]

American students seemed to perform better on the most recent Trends in International Mathematics and Science Study (TIMSS), conducted in 2003. For example, fourth-graders scored "above average" in mathematics among participating countries while eighth-graders scored either above average or about average depending on the calculation.[26] However, when compared only with more developed nations that are America's economic competitors, U.S. performance on TIMSS looks more like its performance on PISA. In 2005, the American Institutes for Research (AIR) analyzed a group of industrialized nations participating in both TIMSS and PISA; among that group, U.S. students consistently performed below average across international assessments. "U.S. performance is below the 12-country average at both low- and high-skill levels and low and high-levels of item difficulty."[27]

American students tend to perform better on international assessments of reading than they do in math and science. But U.S. 15-year-olds perform only about average among industrialized countries, and fourth graders' reading scores have stagnated while other countries have made sizeable gains. "Reforms aimed at improving reading achievement seem to have propelled Russia, Hong Kong, and Singapore from middle to top rankings [on the Progress in International Reading Literacy Study (PIRLS)]," *Education Week* reported last year, "even as U.S. performance stood still."[28]

Moreover, a 2003 PISA assessment of students' ability to solve real-world problems found that fewer than half of U.S. 15-year-olds are analytical problem-solvers who can communicate well about solutions. Among 29 industrialized nations, the U.S. had the fifth highest percentage of very weak problem-solvers and the sixth lowest percentage of strong problem-solvers.[29] Such results suggest that U.S. schools not only are failing to provide many students with strong foundational skills in subjects like math and science, but they also are not providing enough students with the broader skills that the modern workplace increasingly demands.

Schools also must find ways to provide students with the "global awareness" that the globalization of work requires.[30] To collaborate on international work teams, manage employees from other cultures and countries, and communicate with colleagues and clients abroad, Americans will need to know and understand much more about the rest of the world than they do now.[31] "A pervasive lack of knowledge about foreign cultures and foreign languages threatens the security of the United States as well as its ability to compete in the global marketplace and [to] produce an informed citizenry," the National Academy of Sciences warned last year.[32]

The Equity Imperative

Some might argue that it is enough to produce the next generation of elite "rocket scientists" who can invent new technologies and spur innovation. There is a widespread belief that providing America's top students with a world-class education is the single most important way to boost economic growth. This notion is often paired with a conviction that focusing on educational equity for all sacrifices excellence for the few who are already advanced. But these are myths. Our national commitment to closing achievement gaps is not only compatible with a global competitiveness agenda, it is essential for realizing that agenda.

Figure 1: U.S. 15-Year-Old Performance Compared with Other Countries

Programme for International Student Assessment (PISA)

Average is measurably higher than the U.S.
Average is measurably lower than the U.S.

Mathematics (2006)

Rank		Score
1	Finland	548
2	Korea	547
3	Netherlands	531
4	Switzerland	530
5	Canada	527
6	Japan	523
7	New Zealand	522
8	Belgium	520
9	Australia	520
10	Denmark	513
11	Czech Republic	510
12	Iceland	506
13	Austria	505
14	Germany	504
15	Sweden	502
16	Ireland	501
17	France	496
18	United Kingdom	495
19	Poland	495
20	Slovak Republic	492
21	Hungary	491
22	Luxembourg	490
23	Norway	490
24	Spain	480
25	United States	474
26	Portugal	466
27	Italy	462
28	Greece	459
29	Turkey	424
30	Mexico	406
	OECD average	498

Science (2006)

Rank		Score
1	Finland	563
2	Canada	534
3	Japan	531
4	New Zealand	530
5	Australia	527
6	Netherlands	525
7	Korea	522
8	Germany	516
9	United Kingdom	515
10	Czech Republic	513
11	Switzerland	512
12	Austria	511
13	Belgium	510
14	Ireland	508
15	Hungary	504
16	Sweden	503
17	Poland	498
18	Denmark	496
19	France	495
20	Iceland	491
21	United States	489
22	Slovak Republic	488
23	Spain	488
24	Norway	487
25	Luxembourg	486
26	Italy	475
27	Portugal	474
28	Greece	473
29	Turkey	424
30	Mexico	410
	OECD average	500

Reading (2003)

Rank		Score
1	Finland	543
2	Korea	534
3	Canada	528
4	Australia	525
5	New Zealand	522
6	Ireland	515
7	Sweden	514
8	Netherlands	513
9	Belgium	507
10	Norway	500
11	Switzerland	499
12	Japan	498
13	Poland	497
14	France	496
15	United States	495
16	Denmark	492
17	Iceland	492
18	Germany	491
19	Austria	491
20	Czech Republic	489
21	Hungary	482
22	Spain	481
23	Luxembourg	479
24	Portugal	478
25	Italy	476
26	Greece	472
27	Slovak Republic	469
28	Turkey	441
29	Mexico	400
	OECD average	494

Problem Solving (2003)

Rank		Score
1	Korea	550
2	Finland	548
3	Japan	547
4	New Zealand	533
5	Australia	530
6	Canada	529
7	Belgium	525
8	Switzerland	521
9	Netherlands	520
10	France	519
11	Denmark	517
12	Czech Republic	516
13	Germany	513
14	Sweden	509
15	Austria	506
16	Iceland	505
17	Hungary	501
18	Ireland	498
19	Luxembourg	494
20	Slovak Republic	492
21	Norway	490
22	Poland	487
23	Spain	482
24	United States	477
25	Portugal	470
26	Italy	469
27	Greece	448
28	Turkey	408
29	Mexico	384
	OECD average	500

Source: Organisation for Economic Co-Operation and Development and U.S. Department of Education.

Recent studies offer compelling evidence that educational equity is just as important for economic competitiveness as it is for social justice. Hanushek and colleagues specifically analyzed economic data to answer this question: "Which is more important for growth—having a substantial cadre of high performers or bringing everyone up to a basic level of performance?" They found that to truly maximize growth, it is not enough to produce a high-achieving elite; a nation's economic success also depends on closing achievement gaps to ensure that all students attain a solid foundation of knowledge and skills.[33] Another recent study of 14 developed countries concluded that "increasing the average level of literacy will have a greater effect on growth than increasing the percentage of individuals who achieve high levels of literacy skills."[34]

But the U.S. has a long way to go before it achieves that goal. While American 15-year-olds rank in the *bottom-third* of developed nations in overall performance in math and science, they rank in the *top-third* when it comes to gaps between students from different family backgrounds.[35] In fact, the difference in science scores between students from different socioeconomic backgrounds is bigger in the United States than in almost any other country.[36] Fortunately, international assessments also show that it is possible to realize high average performance alongside more equitable performance. Across several continents, countries like Japan, Korea, Finland, and Canada demonstrate that students from disadvantaged backgrounds need not automatically perform poorly in school.[37]

Learning how some countries achieve performance that is both higher and more equitable has tremendous implications in this country given America's long-term demographic outlook. Demographers now predict that "minorities" will constitute the majority of schoolchildren by 2023 and of working-age Americans by 2039.[38] In 2006, U.S. Hispanic 15-year-olds performed below the average of every OECD country except Turkey and Mexico in science literacy, and black students performed even worse (**Figure 2**).[39] America cannot remain competitive if half of its population graduates from high school so poorly prepared that it is unable to thrive in the global knowledge economy. States that plan to grow their economies *must* find ways to close their achievement gaps.

Of course, some critics of international assessments claim that America's disappointing performance is inevitable precisely because of its demographic challenges. But the data do not support such beliefs: Overall, U.S. 15-year-olds are slightly above the international average when it comes to families' social, economic, and cultural status.[40] The problem is that America's education system does a poor job supporting students and offering equal learning opportunities. According to OECD, in 2006, the U.S. ranked fourth out of 30 countries in the relative *impact* that socioeconomic background had on students' PISA science achievement.[41] Another recent study measuring the impact of family background on TIMSS results found a similar pattern: "The U.S. falls in the top quarter of the most unequal countries."[42]

Figure 2: U.S. Minority Performance Below Averages of Most Industrialized Nations

Source: Baldi, S., Y. Jin, M. Skemer, P.J. Green, and D. Herget. Highlights from PISA 2006: Performance of U.S. 15-Year-Old Students in Science and Mathematics Literacy in an International Context. Washington, DC: U.S. Department of Education, National Center for Education Statistics, December 2007, pp. 6 & 15.

Other Countries Pulling Ahead

America's global position is slipping not because U.S. schools are getting worse. Rather, America is losing ground because its educational outcomes have mostly stagnated while those in other countries have surged. Nations that formerly lagged far behind the U.S. have caught up with and in some cases even surpassed it.

Korea, for instance, has gone from well behind to significantly ahead of the United States in high school attainment in just a few generations—an education triumph that has helped fuel the country's tremendous progress (**Figure 3**). In 1960, Mexico's economic productivity was twice as large as Korea's, but by 2003 Korea's GDP was twice as large as Mexico's. According to the World Bank, "the contribution of knowledge … was a key factor in Korea's miracle of rapid economic growth."[43]

Other countries have made rapid strides in building competitive knowledge-and-innovation economies. "At the end of World War II, a single nation stood atop Mount Innovation, and it was the United States," notes former Harvard Business School professor John Kao in his 2007 book *Innovation Nation*. "Now, powerful new climbers have emerged to challenge U.S. supremacy. … Some may be surprising—Brazil, Denmark, Estonia, Finland, New Zealand, Singapore, and Taiwan."[44] Not surprisingly, some of those same nations also top the list of countries achieving high performance or seeing big gains on international assessments.

"Young Chinese, Indians, and Poles are not racing us to the bottom," *New York Times* columnist Thomas Friedman observed in 2005. "They do not want to work for us; they don't even want to be us. They want to dominate us—in the sense that they want to be creating the companies of the future …."[45]

These governments are giving their people an edge by making major efforts to improve K-12 education. Between 2000 and 2006, Poland increased its PISA reading achievement by 29 points—almost a year's worth of learning—while decreasing the proportion of achievement variation across schools from 51 percent to 12 percent. Improving average skills while decreasing the achievement gap is no accident: Poland's major education reforms are now bearing fruit.[46]

Some countries are working hard to compare their performance internationally and to use those comparisons to drive improvement. Mexico plans to link its national assessment to PISA and has set presidential targets for 2012 and for 2030. Brazil has benchmarked every secondary school against PISA so that each one receives two scores—one benchmarked to the national metric and one benchmarked to PISA. The goal is to have all Brazilian secondary schools achieving at the international average by 2021. "Instead of spending years complaining that they don't do well, they turned it around to talk about what to do about it and to measure progress," says Andreas Schleicher, head of the Indicators and Analysis Division at OECD's Directorate for Education.[47]

Many nations are going beyond performance to benchmark their policies and practices with the world's top performers—and making major strategic changes as a result. When Germany received disappointing results on the PISA 2000 assessment, leaders commissioned a team of experts from high-performing and innovative countries to investigate best practices and provide advice. In 2003, the German government launched a $4.6 billion package of education reforms, including a program to expand learning time by introducing 10,000 all-day schools across the country.[48] And by 2004, Germany's 16 *Länder* (states) began to adopt common, jointly developed "national education standards"—something that previously had been considered politically daunting if not impossible.[49]

Figure 3: Korea's Education Advancement

Source: Organisation for Economic Co-Operation and Development. Education at a Glance 2008. Paris: OECD, September 2008, p. 43, Table A1.2a.

Germany is not alone in its response to international assessment results. A recent evaluation of the policy impact of PISA found that the assessment has had a major influence on educational policy and practice in many OECD countries, most notably on educational standards and curricula as well as on systems of evaluation and accountability.[50]

Countries have responded to TIMSS and PIRLS results as well. A 2005 study found that 10 out of 18 developing nations had changed their science curricula in response to the TIMSS 1999 results, and eight had changed their math curricula—including "relocating into grade 8 topics that had been taught later."[51] Hong Kong's reading reforms, which boosted its fourth-grade PIRLS achievement from significantly below the U.S. to significantly above it, were enacted in response to disappointing results on the 2001 assessment.[52] Singapore's impressive math and science performance on TIMSS assessment is hardly a mistake; rather, the outcomes resulted from major education reforms the country launched in response to poor performance on the Second International Science Study (a precursor of TIMSS) in the mid-1980s.[53]

Vivien Stewart, vice president of the Asia Society, says she is often impressed by the openness and eagerness of education leaders in other countries to learn from and apply international best practice. "Singapore is currently at the top and China is rapidly improving and India is just beginning to improve, but they are all very interested in using international best practices," she says. "China, before it engages in any reforms, will send teams to examine best practices around the world. Although this is mostly done at the national level, it's increasingly done at the province level too. China is doing this with a vengeance because they traditionally have been cut off from the rest of the world, and they want to catch up quickly. A lot of the Chinese curriculum reforms are based on looking at systems in other parts of the world."[54]

China's educational efforts are well matched with its economic aspirations. In 2006, the country's Eleventh Five-Year-Plan put technological innovation squarely at the center, emphasizing the need to develop a "rich talent base" and calling for the government to "cultivate talents with creativity and completely improve our capacity of self-innovation so top universities in China will become an important force for the establishment of an innovation nation."[55] A July 2008 study found that the University of California, Berkeley had been displaced by not one but two Chinese universities as the top undergraduate feeder institutions for U.S. Ph.D. programs.[56] In addition, while America could once expect talented foreigners studying here to stay and contribute to the U.S. economy after graduation, foreign-born specialists educated in this country are increasingly returning home to take advantage of new economic opportunities in their own countries.

Many other regions and nations are working to benchmark and improve education to attract high-skilled, high-paying jobs. In 2000, the European Union (EU) heads of state adopted the goal of becoming "the most competitive and dynamic knowledge-based economy in the world," encouraging member nations to introduce a host of education and other reforms. Since then, the EU has adopted educational goals that are internationally benchmarked, and publishes an annual report that allows national leaders to compare results within Europe as well as with the U.S. and other countries around the world. The 2008 edition emphasizes the critical role of international benchmarking: "All Member States can learn from the best performers in the Union. … This is why the Council asked for the three best performing countries (leaders) in specific policy areas to be identified."[57]

Such attitudes stand in stark contrast to the United States, which so far has largely ignored the international benchmarking movement in education. "The U.S. education system in general is very introverted," observes Sir Michael Barber, a former top education official in Great Britain who now focuses on international benchmarking at McKinsey and Company, a global management consulting firm.[58] The U.S. participates in far fewer international benchmarking studies than do many other countries, especially compared with those working hardest to improve. In June, a group of governors attending an NGA- and Hunt Institute-sponsored seminar on educational competitiveness learned that the U.S. is the only OECD country with a federal-style education system where most state leaders have no regular and reliable information to compare student performance internationally.

Barber argues that will need to change if the U.S. wants to remain competitive. "All around the world," he says, "governments are seeking insights into how to improve education systems, and many understand that the only way for a country or a state to keep up globally is to look at what's happening with best practice around the world."[59]

Of course, the U.S. education system has strengths as well as weaknesses, and plenty to teach other countries. For example, U.S. ninth-graders scored well above average on the 1999 Civic Education Study, ranking sixth out of 28 countries overall and first in students' ability to critically interpret political information. Moreover, the U.S. was one of only two countries whose students scored above average not only in civics content, but also on measures of positive civic engagement and attitudes.[60] Clearly, educators in emerging democracies can look to the U.S. for lessons in how to prepare students for active civic engagement.

Many countries also find much to admire about America's higher education system and reforms around the globe have been informed by the U.S. "You have created a public-private partnership in tertiary education that is amazingly successful," Singapore's Education Minister Tharman Shanmugaratnam told *Newsweek* in 2006. "The government provides massive funding, and private and public colleges compete, raising everyone's standards." Moreover, some Asian countries have looked to U.S. schools for ideas on how to encourage innovation and risk taking. "America has a culture of learning that challenges conventional wisdom, even if it means challenging authority," says Shanmugaratnam. "These are the areas where Singapore must learn from America."[61]

But the U.S. cannot afford to rest on its past accomplishments. The global knowledge economy is here, and if state leaders want to ensure that their citizens can compete in it, they must seize the initiative, looking beyond America's borders and benchmarking their education systems with the best in the world. The state mandate to educate all students remains, but the world that schools are preparing students for has changed—and will continue to change—dramatically.

OECD's Schleicher says the case for adopting a global perspective on improving education is undeniable:

> It is only through such benchmarking that countries can understand relative strengths and weaknesses of their education system and identify best practices and ways forward. The world is indifferent to tradition and past reputations, unforgiving of frailty and ignorant of custom or practice. Success will go to those individuals and countries which are swift to adapt, slow to complain, and open to change.[62]

Myths and Realities about International Comparisons

Myth: *Other countries test a more select, elite group of students.*

Reality: That might have been true for early international assessments, but it is no longer true today. According to Jim Hull, who examined international assessments for the National School Boards Association, "Since the 1990s, due to better sampling techniques and a move by more countries to universal education, the results represent the performance of the whole student population, including students who attend public, private, and vocational schools, students with special needs, and students who are not native speakers of their nation's language."[63]

While the U.S. still sets a relatively high age for compulsory education among OECD nations, that does not automatically translate into higher rates of school enrollment. U.S. enrollment rates in primary and secondary education are the same as or below those in other industrialized nations. For example, among OECD member nations, the U.S. ranks only 22nd in school enrollment of 5- to 14-year-olds and 23rd in enrollment of 15- to 19-year-olds.[64]

Moreover, on the most recent PISA assessment, OECD member nations on average tested a *higher* proportion of 15-year-olds than did the U.S. (97 percent versus 96 percent of those enrolled in schools, and 89 percent versus 86 percent of the entire 15-year-old population), which refutes the idea that the U.S. was disadvantaged by testing a broader population.[65] While no assessment is perfect, PISA, TIMSS, and PIRLS all have tight quality-control mechanisms, including very strict and transparent guidelines for sampling students and administering assessments. All exclusions must be thoroughly documented and justified, and total exclusions must fall below established thresholds.

Myth: *The U.S. performs poorly because of poverty and other family factors.*

Reality: According to the U.S. Department of Education, the U.S. looks about average compared with other wealthy nations on most measures of family background.[66] Among the OECD's 30 member nations, U.S. 15-year-olds are slightly *above* the international average on a composite index of economic, social, and cultural status (ESCS); only 11 percent of U.S. students fall within the lowest 15 percent of the ESCS internationally.[67] Moreover, America's most affluent 15-year-olds ranked only 23rd in math and 17th in science on the 2006 PISA assessment when compared with affluent students in other industrialized nations.[68] In fact, when the OECD uses statistical methods to estimate how PISA scores would look if the ESCS index were equalized across all countries—a leveling of the playing field—U.S. performance actually looks *worse* rather than better.[69]

This is not to say that demographics are unimportant in American schools: The U.S. ranks high in the impact that family background has on student achievement (fourth out of 30 countries),[70] in part because its education system does a particularly poor job supporting students and equalizing learning opportunities. For example, a 2006 study published in the *European Journal of Political Economy* found that out of 18 developed nations, the U.S. is the only country where weaker students are more likely to be enrolled in larger classes.[71] Another study found that the U.S. has one of the largest gaps in access to qualified teachers between students of high and low socioeconomic status.[72]

Myth: *Cultural factors prevent U.S. students from performing as well as those in other nations, particularly Asian countries.*

Reality: U.S. 15-year-olds reported spending *more* time on self study or homework in science, math, and reading than did students on average across the 30 OECD nations taking the 2006 PISA assessment, including those in Japan and, except for math, in Korea.[73] Moreover, high-performing nations and states can be found all over the world, not just in Asia. For example, the five top-scoring nations in the 2006 PISA science assessment were located on four different continents, reflecting a range of cultures: Europe (Finland), North America (Canada), Asia (Japan), and Oceania (New Zealand and Australia).

Singapore is often singled out for its top performance on the TIMSS math assessment, which some say must be due to an unusually strong work ethic. But that belief was challenged in a 2005 study by the American Institutes for Research (AIR): "Singaporean students are hardworking, but if Singapore's success is attributable only to work ethic, how can we account for the fact that its high achievement is a comparatively recent development? On the Second International Science Study in the mid-1980s, Singaporean fourth graders scored only 13th out of 15 participating nations, and Singaporean eighth graders did no better than their U.S. counterparts .... In response to these poor scores, Singapore's Ministry of Education re-engineered and strengthened the education system, reforming both the science and mathematics curriculum."[74]

Countries such as Finland, Korea, and Hong Kong have achieved major improvements in learning outcomes over time without changing their national cultures. In fact, as recently as the mid-1980s Finnish students performed only about average among OECD nations on tests used at the time.[75] Hong Kong instituted numerous reading reforms that boosted its fourth-graders' performance from significantly below the U.S. in 2001 to significantly above it in 2006.[76]

Of course, cultural attitudes can play a role in achievement. Studies conducted in the 1980s found that mothers and students in some Asian countries were likely to attribute success in math more to effort than to innate ability, while the reverse was true for Americans.[77] But experimental studies have shown that students' beliefs can be changed in ways that positively impact learning; the National Mathematics Panel recommended that such strategies be used more widely in American classrooms.[78]

Myth: *Other countries are less diverse.*

Reality: The U.S. is a diverse nation, but that diversity should not prevent states from improving student achievement. Among the 11 other OECD countries that like the U.S. had more than 10 percent immigrant students, all of them performed higher in math and nine performed higher in science.[79] And Singapore, which scored at the top of the most recent TIMSS math assessment, is not as homogeneous as many assume. According to the 2005 AIR report, "Arguments about Singapore's homogeneity are not persuasive. ... Singapore has three major ethnic groups. About three-fourths of Singapore's population is Chinese, but almost a quarter is Malay or Indian. Like the United States, Singapore experienced serious ethnic strife in the 1960s."[80]

Cultural homogeneity has been cited as a factor in Finland's high achievement in that it lends itself to a great deal of agreement about education and education reform. But Finland's success also is attributable to very different educational policies and practices in areas like teacher recruitment and student support.[81]

Myth: *Wealthier countries spend more than the U.S. on education.*

Reality: The U.S. is wealthier and spends more on education than most other countries. Among the OECD's 30 member nations, the U.S. ranks highest in GDP per capita and second highest in educational expenditures.[82] A report on the U.S. economy published by OECD last year observed, "On average, and relative to other OECD countries, U.S. students come from well-educated, wealthy families and ... go to schools that are unusually well-financed. Given any of these factors, U.S. students might be expected to be among the world leaders."[83] However, while the U.S. ranks high in education spending, it ranks only near the middle of OECD nations in its "effort" to fund education when expenditures are compared with wealth (gross national product).[84]

Myth: *U.S. attainment rates cannot be compared with other countries' because the U.S. tries to educate many more students.*

Reality: The U.S. does rank higher than average on access to higher education, but that does not explain its very low college-completion rates. While America's entry rate for four-year and advanced postsecondary programs exceeds the OECD average by 10 percentage points (64 percent to 54 percent), its college "survival rate" trails the OECD average by 17 points (54 percent to 71 percent).[85] According to OECD, "Comparatively high drop out rates in the United States are [negatively] contributing to the United States' relative standing against other countries" in educational attainment.[86]

Myth: *Education does not really affect the economy anyway.* A Nation at Risk *warned that America's economy would suffer, but that never happened.*

Reality: While *A Nation at Risk* erred in linking the recession of the early 1980s to educational stagnation (other factors such as the business cycle are more important over the short term), the report was correct that improving education is critical to America's economic competitiveness. New research based on extensive data from many countries over several decades confirms that cognitive skills as measured by international tests strongly influence long-term economic growth.[87]

Other factors matter too, of course. In fact, America's historic advantages in other areas have made up for its students' mediocre skills and allowed the U.S. to grow its economy without significantly improving its schools. First, the sheer size of the U.S. and its much earlier investment in mass secondary and postsecondary education gave it a significant numerical advantage in human capital. Second, its open and agile economy, flexible labor markets, and intellectual property protections enabled industry to make better use of the human capital available.[88]

But those historic advantages are eroding as other countries imitate the U.S. example. America already has lost its lead in educational attainment, and many countries are instituting economic reforms. "Eventually, our competitors will narrow our economic lead as they learn how to create their own versions of agility and scale," says economist Anthony Carnevale. "At that point, the competition will really come down to who has the best human capital."[89]


III. Five Steps Toward Building Globally Competitive Education Systems

III. Five Steps Toward Building Globally Competitive Education Systems

States have both the authority and the responsibility to provide students with a high-quality education, and state leaders *already* are deeply engaged in efforts to raise standards, improve teaching quality, and help low-performing schools and students improve. For example, 34 states now belong to the American Diploma Project Network, an initiative dedicated to making sure that every high school graduate is prepared for college or work. In those states, governors, state superintendents of education, business executives, and college leaders are working to improve high school standards, assessments, and curricula by aligning expectations with the demands of postsecondary education and work.

International benchmarking provides an additional tool for making every state's existing education policy and improvement process more effective, offering insights and ideas that cannot be garnered by examining educational practices only within U.S. borders. State leaders can use benchmarking to augment their "database of policy options" by adding strategies suggested by international best practice to the range of ideas already under consideration. Indeed, international benchmarking should not be a stand-alone project, but rather should function as a critical and well-integrated component of the regular policy planning process.

The following action steps were carefully chosen to help states focus their efforts on the policy areas that have both a high impact on student performance and also a high potential for best practice learning—in other words, where existing research has shown significant differences in how high-performing nations or states organize education compared with traditional approaches in most U.S. states. However, this should not be viewed as a static checklist. Benchmarking is a process of discovery as well as adaptation, and state leaders should keep an open mind as they collect information on practices abroad to expand their policy toolkits.

For example, action steps two through four address the major elements of what can be thought of as the "instructional delivery system"—the people, tools, and processes that translate educational expectations into teaching and, ultimately, into learning for students. Other countries have shown that all of these elements can be tightly aligned and focused through systematic reform, so they should not be considered in isolation. And because benchmarking is meant to broaden the policy lens, revealing lessons that might not be apparent in a limited state or national context, state leaders should be attuned to all the ways that other nations are delivering instruction more efficiently and effectively—from educational technology to school finance to governance.

Finally, higher education leaders should be asked to join international benchmarking efforts as full participants so existing initiatives are better coordinated with pre–K-12 and higher education policies through P-16 councils and other mechanisms. For example, higher education plays a key role in the recruitment and training of teachers and an increasingly important role in ensuring that high school graduation standards reflect college- and career-readiness requirements. Partnering with higher education also will facilitate a robust discussion about college graduation rates, which are very low in the United States and have contributed to the erosion of America's preeminence in higher education. Since the responsibility probably lies both with K-12 preparation and with higher education practice, leaders from both sectors should work together to ensure that attainment rates are internationally competitive.

The Action Steps

Action 1: *Upgrade state standards by adopting a common core of internationally benchmarked standards in math and language arts for grades K-12 to ensure that students are equipped with the necessary knowledge and skills to be globally competitive.*

Research has revealed striking similarities among the math and science standards in top-performing nations, along with stark differences between those world-class expectations and the standards adopted by most U.S. states. According to Bill Schmidt, a Michigan State University researcher and expert on international benchmarking, standards in the best-performing nations share the following three characteristics that are not commonly found in U.S. standards:

Focus. World-class content standards cover a smaller number of topics in greater depth at every grade level, enabling teachers to spend more time on each topic so that all students learn it well before they advance to more difficult content. In contrast, state content standards in the U.S. typically cover a large number of topics in each grade level—even first and second grade. U.S. schools therefore end up using curricula that are "a mile wide and an inch deep."

Rigor. By the eighth grade, students in top-performing nations are studying algebra and geometry, while in the U.S., most eighth-grade math courses focus on arithmetic. In science, American eighth-graders are memorizing the parts of the eye, while students in top-performing nations are learning about how the eye actually works by capturing photons that are translated into images by the brain.[90] In fact, the curriculum studied by the typical American eighth-grader is two full years behind the curriculum being studied by eighth-graders in high-performing countries.[91]

Coherence. Math and science standards in top-performing countries lay out an orderly progression of topics that follow the logic of the discipline, allowing thorough and deep coverage of content. In contrast, standards in many U.S. states resemble an arbitrary "laundry list" of topics, resulting in too much repetition across grades. "In the United States the principle that seems to guide our curriculum development is that you teach everything everywhere," says Michigan researcher Schmidt, "because then somehow somebody will learn something somewhere."[92]

To upgrade state standards, leaders will be able to leverage the Common State Standards Initiative, an upcoming joint project of NGA, CCSSO, Achieve, the Alliance for Excellent Education, and the James B. Hunt, Jr. Institute for Educational Leadership and Policy. The initiative will enable all states to adopt coherent and rigorous standards in K-12 math, reading, and language arts that are fully aligned with college and career expectations and also internationally benchmarked against leading nations. Achieve is developing an important tool for the initiative: a set of voluntary, globally competitive reference standards based on the existing American Diploma Project (ADP) framework. Because of how it was originally developed, the ADP framework *already* reflects the skills necessary to succeed in college and in well-paying jobs in today's labor market. Achieve is now working to further calibrate the framework to reflect international expectations as well as recent research on college and career readiness.

A key goal of the initiative will be to ensure that standards reflect all three of the critical dimensions exemplified by high-performing nations—not only rigor but also focus and coherence. In a study published last year, Schmidt and a colleague found that trying to cover too many topics per grade clearly has a negative influence on student learning, even when the order of topics is otherwise coherent. At the eighth-grade level, the researchers found "a decrease of fifty in the number of intended topics and grade combinations would predict an increase in achievement of almost three-fourths of a standard deviation. … The amount of 'clutter' created by covering too many topics … must be kept small."[93] Therefore, the internationally benchmarked common core of standards should not be seen as an addition to existing standards, but rather the foundation for states to establish rigorous standards that also are fewer and clearer (**Figure 4**).

Figure 4: Mathematics Topics in Content Standards of 21 States

Topic	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8
Whole number meaning	●	●	●	●	⊙	○		
Whole number operations	●	●	●	●	⊙	○		
Measurement units	⊙	●	●	⊙	●	●	●	⊙
Common fractions	○	⊙	⊙	⊙	●	⊙	○	○
Equations and formulas	○	○	⊙	⊙	⊙	●	●	●
Data representation and analysis	●	●	●	●	●	●	●	⊙
2-D geometry: basics	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙
Polygons and circles	●	●	●	●	●	●	●	⊙
Perimeter, area and volume		○	○	○	⊙	●	●	⊙
Rounding and significant figures								
Estimating computations	○	○	⊙	⊙	⊙	⊙	⊙	⊙
Properties of whole number operations	○	○	○	○				
Estimating quantity and size			○					
Decimal fractions			○	⊙	⊙	⊙	○	○
Relationship of common and decimal fractions				○	○	○		
Properties of common and decimal fractions								
Percentages					○	⊙	⊙	○
Proportionality concepts						⊙	○	
Proportionality problems						⊙	⊙	⊙
2-D coordinate geometry			○	⊙	○	○	○	⊙
Geometry: transformations	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙
Negative numbers, integers, and their properties						○	⊙	○
Number theory					⊙	○	○	○
Exponents, roots and radicals						○	○	●
Exponents and orders of magnitude							○	○
Measurement estimation and errors	○	○	⊙	○	⊙	⊙	⊙	○
Constructions w/ straightedge/ruler and compass								
3-D geometry	●	●	●	⊙	●	⊙	●	⊙
Congruence and similarity					○	⊙	⊙	○
Rational numbers and their properties						⊙	⊙	○
Patterns, relations, and functions	⊙	●	●	●	⊙	●	●	●
Slope and trigonometry								

Intended by 67 percent of the 21 states ○
Intended by 83 percent of the 21 states ⊙
Intended by *all* of the 21 states ●

Bold yellow line shows content coherence typical of top-performing countries

Source: Schmidt, W.H., C.H. Wang, and C.C. McKnight. Curriculum Coherence: An Examination of U.S. Mathematics and Science Content Standards from an International Perspective. Journal of Curriculum Studies 37, no. 5, 2005, pp. 525-559. (p. 541, Figure 4)

***Action 2:** Leverage states' collective influence to ensure that textbooks, digital media, curricula, and assessments are aligned to internationally benchmarked standards and draw on lessons from high-performing nations and states.*

Research shows that top-performing countries support rigorous, coherent standards with a wide range of tightly aligned instructional tools—from assessments to classroom curriculum materials. In the U.S., while each state retains its own authority to make decisions in those areas, states can more efficiently reflect international best practice by working cooperatively on ways to upgrade those elements of their standards-based education systems.

Assessment offers a good example. Top-performing countries administer assessments that are more rigorous and better aligned with standards than the tests U.S. students typically take. For example, AIR found that Singapore's math assessments expect greater rigor and depth in mathematical knowledge; to test that knowledge, they employ fewer multiple choice questions and more problems that require multistep solutions and finding unknowns. In fact, Singapore's sixth-grade assessment proved more challenging than the eighth-grade math tests given in seven states as well as the eighth-grade National Assessment of Educational Progress.[94]

Such assessments typically are more expensive to develop and administer than the multiple-choice exams commonly used in the U.S. However, states can save time and money by sharing resources and expertise to develop high-quality voluntary assessments or a common pool of assessment items. That kind of collective effort also can ensure the availability of voluntary assessments or assessment items that are aligned with the internationally benchmarked standards to be developed through the Common State Standards Initiative.

The same is true when it comes to the components of the curriculum. Schmidt and colleagues found that the coherence typical of math standards in high-performing countries "is translated into textbooks, workbooks, diagnostic tests for teacher use, and other classroom materials that enable teachers to bring the curriculum into the classroom in a relatively consistent, effective way. In turn, the curriculum serves as an important basis for the nation's preservice teacher education and for ongoing professional development."[95]

While textbooks are only one of many kinds of instructional tools, they usefully illustrate the power of state collaboration to address international best practice. Researchers have found that U.S. textbooks, compared with those used in high-performing countries, are less aligned with standards and much less focused and coherent in the topics they cover. "If you look at U.S. textbooks," Schmidt and colleagues observe, "you'll find there is no textbook in the world that has as many topics as our mathematics textbooks, bar none."[96] For example, common elementary math textbooks in the U.S. cover almost twice as many topics per grade as do Singapore's. As a result, math textbooks in Singapore expect students to complete about one thorough lesson on a single topic per week, while U.S. students are expected to complete about one lesson on a narrowly focused topic each day.[97]

The problem is not simply a lack of focus and coherence in individual state standards, but also a lack of agreement *across* state standards. Publishers of math textbooks market them nationally by cramming them with enough topics to cover states' widely divergent standards. The Common State Standards Initiative partly solves this problem by providing a more focused and coherent set of expectations around which to develop textbooks and digital media. By working in concert to address concerns about length, focus, and coherence with commercial publishers, states can ensure that new expectations for textbooks, digital media, and other instructional materials are being addressed by the industry.

Finally, states can pool resources to develop entirely new tools, such as replacement units or diagnostic assessments that align with internationally benchmarked standards. In doing so, leaders should collaborate to ensure that curriculum supports take advantage of the newest technologies, including multimedia strategies, to support instruction. Harvard Business School professor Clayton Christensen predicts that by 2019 half of all high school courses will be delivered online.[98] Some research indicates that countries are pursuing a wide range of strategies and goals to encourage the use of computers and information technology for instruction, suggesting that there might be much to learn in this area from international benchmarking.[99]

Action 3: *Revise state policies for recruiting, preparing, developing, and supporting teachers and school leaders to reflect the human capital practices of top-performing nations and states around the world.*

Beyond establishing world-class educational standards, high-performing nations also adopt policies to ensure that students receive the best instruction possible. Recent studies have identified major differences in how top-performers and fast-improvers recruit, train, and support their teachers and school leaders compared with the policies in place in most U.S. states. Tackling these challenges can yield big dividends. Studies by U.S. researchers have found that assigning students to strong teachers for three years in a row can boost their test scores by as much as 50 percentile points above what they would gain with three ineffective teachers in a row.[100]

According to a study by Sir Michael Barber and Mona Mourshed of McKinsey and Company, the best-performing nations begin by recruiting top talent to the teaching profession: Korea recruits from the top 5 percent of graduates, Finland the top 10 percent, and Singapore the top 30 percent. The McKinsey researchers found that some countries accomplish this by setting a high initial bar and limiting access to teacher training to prevent an oversupply of candidates—especially weak ones—which, along with other strategies, raises the status of the profession and aids in recruitment.[101] "Finns have come to cherish good educators as Texans do ace quarterbacks," Kao writes in *Innovation Nation*.[102]

In contrast, the U.S. teacher pipeline seems to discourage individuals with competitive academic skills from entering and remaining in the profession. College students with high SAT and ACT scores are less likely to train to become teachers, less likely to take a teaching job, and less likely to stay in the classroom after a few years.[103] The likelihood that a highly talented female in the top 10 percent of her graduating class would become a teacher shrank by half, from about 20 percent to about 10 percent, between 1964 and 2000.[104]

Top-performing nations and provinces also use a range of strategies to provide teachers with excellent training and ongoing professional development—both of which are mostly mediocre in the United States. An international study released last year by the International Association for the Evaluation of Educational Achievement (IEA) and Michigan State University found that college students preparing to be teachers have weaker knowledge of mathematics and take less rigorous math courses than those in other countries. "What's most disturbing is that one of the areas in which U.S. future teachers tend to do the worst is algebra, and algebra is the heart of middle school math," say Bill Schmidt, who directed the study.[105]

Top-performing nations are going well beyond recruitment and initial training to build a 21st century teaching force, however. According to Schleicher and Stewart, "These countries are abandoning the traditional factory model, with teachers at the bottom of the production line receiving orders from on high, to move toward a professionalized model of teachers as knowledge workers. In this model, teachers are on a par with other professionals in terms of diagnosing problems and applying evidence-based practices and strategies to address the diversity in students' interests and abilities."[106] Such countries recognize that quality of classroom instruction is the most critical element of any education system, and they work to build cultures that combine high expectations with strong support and empowerment of teachers.

However, bolstering teacher professionalism does not mean asking teachers to create everything from scratch. Korea's Institute for Curriculum and Evaluation operates a Teaching and Learning Center that offers information about the national curriculum; promotes aligned instructional practices; and provides educators with a wide range of teaching materials, guidelines, and assessment tools.[107] The New Zealand Ministry of Education has supported development of tools for formative assessment, including Assessment Tools for Teaching and Learning, which can be used to assess literacy and numeracy of upper elementary and lower secondary students, as well as national curriculum exemplars in all subject areas. Teachers use the tools to evaluate the impact of instruction on student learning and adjust teaching to better meet students' needs.[108]

Based on conversations with many local educators across the United States, Education Trust President Kati Haycock underscores that benchmarking efforts should consider the immediate concerns of classroom teachers: "What do the leading countries do with children who arrive behind? What is international best practice for improving the performance of language minorities? How do teachers differentiate instruction without losing sight of rigorous standards?"[109] Since educators ultimately will be responsible for ensuring that students meet the new globally competitive standards, policymakers should take care to incorporate such questions into their benchmarking research.

Top nations and states also focus on developing excellent school leaders and charge principals with ensuring that teachers provide consistently high-quality instruction. The state of Victoria in southeastern Australia recently implemented an intensive strategy to improve educational leadership that has been dubbed "cutting edge" by international experts. The strategy is closely aligned with the state's comprehensive effort to improve schools and includes a rigorous principal selection process; mentoring programs for new principals and a coaching program for experienced ones; a "balanced scorecard" approach to principal performance management; an accelerated program for high-potential leaders; and a program to develop high-performing principals. The government has established 19 separate leadership-development opportunities, each firmly rooted in research and best practice (**Figure 5**).[110]

Singapore's approach to developing leaders is widely admired too. Singapore screens prospective school leaders using a rigorous process and then provides a six-month training program run by the National Institute of Education. The program includes management and leadership courses from leading executive training programs; one day per week spent in schools to come up with innovative solutions to practical problems; group projects; two-week overseas placements with major corporations; and rigorous evaluation.[111] Great Britain recently revamped its national approach to developing principals based on a careful study of that model.[112]

Sir Michael Barber emphasizes that there are important lessons for improving teaching and leadership that can be adapted and applied across nations—and vigorous policy efforts can result in rapid improvements. When the British government surveyed adults aged 24 to 35 in the year 2000 about switching jobs, teaching ranked 92nd out of 150 career choices. But in a follow-up survey conducted in 2005, after improvements to teacher training coupled with a vigorous marketing campaign, teaching came out on top.[113] "Our benchmarking suggests that the same broad policies are effective in different systems irrespective of the cultural context in which they are applied," Barber and Mourshed conclude in their report.[114] U.S. state leaders could learn much from such examples; particularly during the current economic downturn, there might be many adults with strong content backgrounds who could be induced to switch to a career in teaching.

In the U.S., costs related to human capital account for the vast majority of education spending. The goal for international benchmarking should be to ensure the most effective and efficient use of funds for preparation, recruitment, training, ongoing development, and support. This will require a careful examination of how higher education institutions and systems in top-performing countries are structured to encourage young people to enter the teaching field and prepare them to become quality instructors at the elementary and secondary level.

Figure 5: Leadership Development Opportunities in Victoria, Australia

Name of Programme	Open to	Description	Aspirant leaders	Assistant principals	Principals
Master in School Leadership	All after 5 years teaching	Taught modules, in-school elements and mentoring or shadowing; 2 years	√	√	√
Building capacity for improvement	Teams of teachers	Briefing, residential and day workshops, coaching support and feedback; 1 year	√	√	√
Building the capacity of school leadership teams	School leadership teams	Three-day residential, action research in school, 3 coaching sessions, follow-up workshop; 1 year	√	√	√
Leading across effective small schools	Small school teams	Three 1-day forums, action learning project, Web-based support, mentor with small school experience; 1 year	√	√	√
Leading in effective schools (strategic planning)	High potential leaders	Briefing, preparatory activities and 360-degree feedback, two workshops, 4 coaching sessions and ongoing e-mail contact; 1 year	√	√	
Preparing for leadership	Experienced teachers	Two-day conference, four-day workshops, background reading, pre- & post-programme 360-degree feedback, school based project, shadowing; 1 year	√		
Leading for student learning	Expert teachers	Five days workshops, reading and data collection, 360-degree feedback, peer learning groups; 1 year	√		
Leading professional learning	Professional development coordinators	One year part-time programme	√	√	
Scholarships for postgraduate study	Postgraduate teachers	Range of postgraduate courses	√	√	
Eleanor Davies school leadership programme	Female leading teachers / APs	Five months including mentoring, reading, seminars, school based project	√	√	
Leaders in the making	Assistant principals	One year with workshops and strategic planning project	√	√	
Stepping up to the principalship	Assistant principals	One year, including data-collection, workshop, shadowing, reviews		√	
Educational leadership: shaping pedagogy	APs and principals	One year, including preparation, intensive workshop, review, feedback, action planning		√	√
Human leadership: developing people	APs and principals	One year, development and implementation of a professional learning plan		√	√
Technical leadership: thinking and planning strategically	APs and principals	One year, including strategic planning project		√	√
Mentoring for first time principals	First time principals	One year			√
Coaching to enhance the capabilities of experienced principals	Experienced principals	One year with assigned coach			√
Development programme for high performing principals	Principals	Over a two-year period including contribution to system development and individual professional development			√
Building the capacity of the principals of small schools	Principals of small schools	One year			√
Teachers professional leave	All teachers	30 days	√		

Source: Matthews, P., H. Moorman, and D. Nusche. In Pont, B., D. Nusche, and D. Hopkins (Eds.), Improving School Leadership, Volume 2: Case Studies on System Leadership. Organisation for Economic Co-Operation and Development, Paris: OECD, 2008, pp. 179–213. (p. 196, Box 7.5)

Action 4: *Hold schools and systems accountable through monitoring, interventions, and support to ensure consistently high performance, drawing upon international best practices.*

Top-performing nations exhibit a wide range of different approaches to the functions commonly defined in the U.S. under the rubric of "accountability." But recent research suggests that such nations share several key strategic priorities and employ a broader range of tools for managing those priorities than is evident in this country.

First, most high-performing nations use multiple mechanisms to monitor school performance, including annual student assessments in key grades and whole-school reviews or "inspections." Such inspections evaluate the performance of a school against a broad set of criteria, including, but not limited to, student achievement and also examine the school *practices* that contribute to student results. Inspections take many different forms in different countries, including annual reviews conducted by an external agency; annual self evaluations complemented by an external review every few years; and self reviews coupled with external reviews on a much more occasional basis, often initiated by schools themselves.[115] New York City recently adopted a system of school inspections based on the British model.[116]

One advantage of such an approach is that leaders can more precisely diagnose the root causes of underperformance and, consequently, better match interventions with specific needs. According to a benchmarking report commissioned by Achieve for the state of Ohio, the British system "takes account of each school's day-to-day working and its capacity for change. … When [the Office for Standards in Education] finds poor student outcomes and poor quality leadership, for instance, it calls for stronger measures than it would for a school with bad test scores but competent leadership."[117]

Second, some top-performing countries have adopted policies to ensure that every student succeeds by monitoring students' progress and intervening to prevent them from falling too far behind. In Finland, every school employs "special education teachers" who receive additional training to provide individual or small-group support to students who need it, mainly in Finnish language arts and mathematics. On average, about 30 percent of students receive such additional help every year, sometimes even the best students. The goal is to identify any student who is having difficulty at a particular point in time and get that student caught up and able to handle a rigorous classroom curriculum.[118]

In Singapore, schools use a national examination to identify upper elementary grade students who are having difficulty in math. Those students then receive special instruction based on an adapted curriculum framework taught by trained Mathematics Support Teachers. Importantly, they also receive about 30 percent *more* math instruction than their peers so that they can cover the same rigorous content, only at a slower pace.[119]

According to Schleicher and Stewart, many of the countries that perform well on PISA have established strong norms and mechanisms to support students. Teachers in such countries "don't have the option of making students repeat the school year—retention is not permitted—or transferring students to schools with lower performance requirements," they say. "Even where retention or transfers are technically possible, incentive structures for teachers and schools encourage teachers to address and solve challenges rather than hand them to others."[120]

Moreover, a thoughtful approach to accountability can help ensure that students experience a curriculum consistent with state standards and also that academic expectations do not vary too much across schools and classrooms. Even though Finland has an educational culture that greatly values the autonomy granted to local educators, its government recently tightened the national core curriculum after evaluations revealed too many gaps between students' classroom grades and their assessment results. "Another reason for the new approach is the fact that students use their final school reports in basic education when applying to upper secondary education institutions," says Reijo Laukkanen of the Finnish National Board of Education. "Thus, the new rules also safeguard the equality of students."[121]

Finally, top-performing nations balance accountability with greater school autonomy. A number of studies based on PISA, TIMSS, and PIRLS have found that students perform better in systems that give schools greater freedom to hire and reward teachers, purchase supplies and make other school-specific budget allocations, and choose curriculum materials and teaching methods.[122] Those studies also show that decentralization works best when it is combined with various forms of accountability. According to one team of researchers, the positive impact of school autonomy coupled with choice and accountability amounts to more than one-and-a-half grade-level equivalents on the PISA assessment.[123]

In general, however, there is still much to learn about forms of accountability in other nations. One area that states might examine closely as part of their benchmarking work is how other nations use assessment for accountability. What kinds of assessments do they administer in which grades and subjects? What content and skills do those tests measure? What kinds of questions do they use—multiple choice or more open-ended problems? How are assessments scored? And how are the results published and used for accountability purposes?

Action 5: *Measure state-level education performance globally by examining student achievement and attainment in an international context to ensure that, over time, students are receiving the education they need to compete in the 21st century economy.*

As states establish world-class standards and adopt other policies based on international best practice, leaders will want information on whether students are benefiting from the changes and are meeting higher expectations. "States are no longer competing with just the states next door but with countries around the world," argues Vivien Stewart. "Their students are competing with students in Singapore, Shanghai, and Salzburg; it's important to have a sense of whether they are being prepared to thrive in a global, knowledge-based economy."[124] Over time such data also can help prevent newly upgraded, internationally benchmarked state standards from slipping back below globally competitive levels.

In most industrialized countries with a federal-style education system, state leaders already have access to that kind of information because most take part in PISA at state levels and some also participate in TIMSS. In the U.S., governors and chief state school officers would welcome the opportunity to compare student performance internationally. However, state leaders are concerned about the number of tests students already are required to take for various purposes as well as the costs of administering additional assessments. Currently the U.S. is characterized by an overly cumbersome and fragmented testing system in which the federal government, states, districts, and schools together administer many different assessments to meet a wide variety of purposes.

Therefore, states can best address this action step through cooperative action to find a streamlined and cost-effective solution for generating international student achievement comparisons. Since all states already are required to participate in the National Assessment of Educational Progress (NAEP), leaders can use their collective leverage to work with the National Assessment Governing Board (NAGB) to explore the feasibility of upgrading NAEP to yield results that are comparable with existing international assessments such as TIMSS, PIRLS, and PISA. The strategy should permit states to secure representative school-level samples to analyze the relationship between school-level practices and student achievement, which in turn would enable leaders to craft policies promoting more widespread use of effective practices.

Adapting NAEP to yield internationally comparable results will be easier to accomplish in the case of TIMSS and PIRLS. TIMSS is more closely aligned with NAEP, and they both assess students in math and science in grades four and eight. Similarly, PIRLS tests students in reading in grade four, though a recent U.S. Department of Education study found that PIRLS incorporates easier reading passages than NAEP while also assessing some kinds of reading tasks that NAEP does not.[125]

Since PISA assesses 15-year-olds in participating nations, NAGB would need to explore how to adjust NAEP samples to include a comparable group of young people, as well as how to incorporate the more open-ended assessment items that characterize PISA. (PISA relies on "constructed response" items over multiple choice questions by a margin of two to one, while the reverse is true for TIMSS and NAEP.[126]) However, many consider PISA to be an important complement to TIMSS and PIRLS because, while the majority of countries participating in TIMSS are low-and middle-income countries, PISA focuses on the lead industrialized countries that are the main economic competitors of the United States (**Appendix A, pg. 41**). In addition, PISA assesses students near the end of compulsory education on whether they can *apply* what they have learned in math, science, and reading to solve real-world problems.

Governors, chief state school officers, and other leaders also should work to develop assessments that indicate whether students are on track for college readiness. The best example of such an initiative is California's Early Assessment Program (EAP), a collaborative effort among the California State Board of Education, the California Department of Education, and California State University (CSU). EAP allows students to take an additional component of the Grade 11 California Standards Test in reading and mathematics. The results provide an "early warning" that signals the student's college-readiness status; students who meet the benchmark are exempt from having to take the CSU placement test, which is normally given to students after they enroll.[127] Fourteen states in the American Diploma Project Network are developing a common end-of-course exam for Algebra II that is intended to serve the same purpose.

Of course, each state has the authority to make its own decisions regarding assessment and leaders always can choose to administer one or more of the existing international tests. For many policymakers, the most significant difference between TIMSS and PISA is in the type of content and skills each assesses. According to an analysis by the U.S. Department of Education, "TIMSS and NAEP appear to have the most in common, with a focus on material that is more likely to be taught through the school curriculum than PISA, which is more situation and phenomena-based. … TIMSS and PISA differ in a number of respects, including a greater focus on factual knowledge in mathematics and science in TIMSS than in PISA, and a greater focus on problem solving and the critical evaluation of information in PISA than in TIMSS. Moreover, PISA has a greater focus on data analysis, statistics and probability in mathematics than either TIMSS or NAEP [**Table 1**]."[128]

Some U.S. states already have participated in the TIMSS assessment, including Massachusetts and Minnesota in 2007. The IEA and the U.S. Department of Education are working to develop cost models for various levels of state participation in the next administrations of TIMSS and PIRLS in 2011. While no U.S. state has yet participated in PISA, most federal education systems around the world—including Australia, Belgium, Canada, Germany, Italy, Mexico, Spain, Switzerland, and the United Kingdom—have worked with OECD to report PISA results for states or provinces. Across OECD nations, state-level results are generated using a variety of strategies, offering U.S. states several proven models to consider.

A few nations and states have experimented with approaches that do not require students to take the full international assessment every few years. One option is to embed a selection of PISA or TIMSS items into existing state assessments. Another is to generate a statistical "link" using NAEP tests that can then be used to estimate state PISA or TIMSS performance. Such options are less expensive, and in practice are less burdensome on schools that must administer the tests, but what they save in dollars, time, and effort, they sacrifice in depth of data, since policymakers will not be able to dig beneath overall averages.

In addition to achievement, state leaders should gather information to compare educational *attainment* with top-performing and fast-improving nations, starting with indicators published by the OECD in its annual *Education at a Glance* report. Many of the raw data necessary are already collected by federal statistical agencies. For the OECD's 2008 report, the United States provided comparable data on the following key indicators:

- Percentage of 25- to 34-year-olds who have attained at least a high school degree;
- Percentage of 25- to 34-year-olds who have attained a postsecondary degree;
- Upper secondary graduation rate;
- Postsecondary entry rate;
- Postsecondary graduation and completion rates; and
- Number of postsecondary science degree holders per 100,000 employed among 25- to 34-year-olds.

Finally, state leaders should create an explicit plan to ensure that their investment yields more than a new set of numbers—including a strategy for communicating the results; a strategy for analyzing the results to dig beneath averages and identify significant patterns, strengths, and weaknesses; and the designation

Table 1. The Three Major International Assessments

	PISA	TIMSS	PIRLS
Sponsor	Organisation for Economic Co-Operation and Development	International Association for the Evaluation of Educational Achievement	International Association for the Evaluation of Educational Achievement
Grades or ages tested	15-year-olds	Fourth and eighth graders	Fourth graders
Subjects tested	Math, science, and reading every three years; special problem solving assessment in 2003	Math and science	Reading
Content tested	Ability to apply math, science, and reading to solve real-world problems	Attainment of knowledge and skills in math and science curriculum	Reading comprehension skills
Testing cycle	Every 3 years	Every 4 years	Every 5 years
Last administration	2006	2007	2006
Next administration	2009	2011	2011
Cost for state participation	2009: $250,000 to $550,000 depending on level of participation	2007: $600,000 for full participation including *both* 4th and 8th grades, or $350,000 for a full sample in just one grade 2011: To be determined	2011: To be determined
Type of test questions	About two-thirds constructed response and one-third multiple choice	About one-third constructed response and two-thirds multiple choice	About one-half constructed response and one-half multiple choice
Sub-topics for which scores are reported	**Math (2003):** Quantity; space and shape; change and relationships; uncertainty **Science (2006):** Overall knowledge; knowledge about earth and space; knowledge about living systems; knowledge about physical systems; identifying scientific issues; explaining phenomena scientifically; using scientific evidence **Reading (2000):** Retrieving information; interpreting texts; reflection and evaluation	**Math:** Grade 4–Number; patterns and relationships; measurement; geometry; data. Grade 8–Number; algebra; measurement; geometry; data **Science:** Grade 4–Life science; physical science; earth science. Grade 8–Life science; chemistry; physics; earth science; environmental science	Reading for literary purposes; reading for informational purposes; retrieving and straightforward inferencing; interpreting, integrating, and evaluating
Technical alignment with NAEP: Can scores be equated to NAEP?	Little alignment; not enough to crosswalk scales and scores	Significant alignment; enough for some researchers to crosswalk scales and scores*	Unknown
Nations participating	*Please refer to Appendix A for a complete list of countries participating in each.*		

* See for example Phillips, G.W. (2007). *Chance Favors the Prepared Mind: Mathematics and Science Indicators for Comparing States and Nations.* Washington, DC: American Institutes for Research.

of an agency or agencies responsible for collecting additional information and making recommendations for improvement.

Addressing the Equity Imperative

Rather than addressing equity as an isolated action step, state leaders should approach it as an overarching or "interdisciplinary" imperative as they tackle each of the action areas described above. Recent research shows that other nations arrange their education systems more equitably. For example, the U.S. falls short across the following dimensions:

- An opportunity gap in access to qualified teachers that is among the largest in the world;[129]
- The only country where lower performing students and children with less-educated parents are likely to be taught in *larger* classes;[130] and
- Math teachers less likely than those in high-performing countries to include conceptual strategies along with basic computation for low-achieving students.[131]

In other words, education systems in the United States tend to give disadvantaged and low-achieving students a watered down curriculum in larger classes taught by less qualified teachers—*exactly the opposite of what high-performing countries do.*

States could greatly improve their repertoire of policy strategies for promoting academic equity by examining specific strategies in other countries. Korea, for example, has two major policies for encouraging more equal access to qualified teachers. First, teachers are rotated within districts on a regular basis every five years. Second, the government offers educators a wide range of attractive incentives to teach in remote areas and regions with disadvantaged populations, including smaller class size, less in-class teaching time, salary stipends, the chance to choose the next school placement, and a competitive advantage when seeking administrative positions.[132]

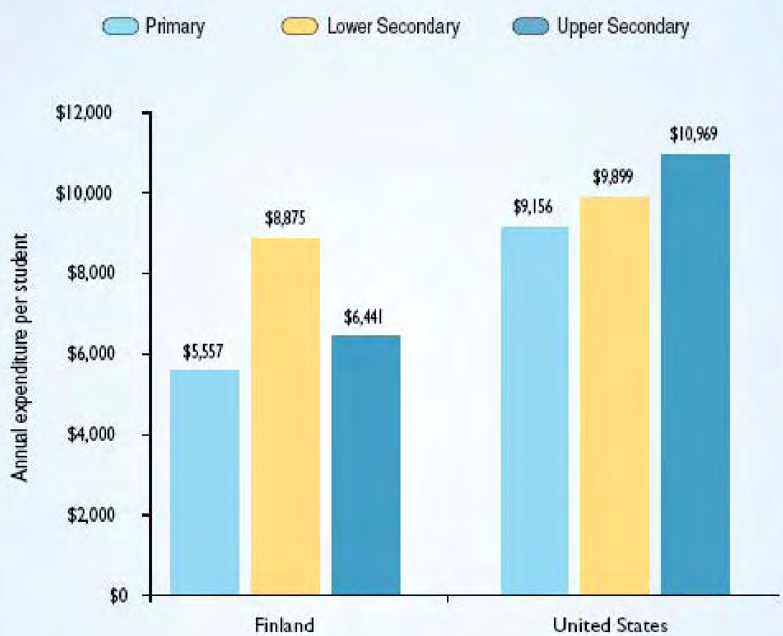
Many high-performing countries also provide intensive, targeted academic supports to students, such as the Finnish and Singaporean intervention strategies described above. The Finnish example is particularly interesting in that it is one of four overlapping "layers" of intensifying interventions for students who fall behind. The first line of attack is formed by regular classroom teachers who receive intensive training to deal with diverse learning challenges through teacher preparation internships, which might deal with "students performing at different levels to the special needs of immigrant children to more difficult cases of fetal alcohol syndrome or attention deficit hyperactivity disorder."[133]

The second line of attack is made up of classroom teaching aides who often work with individuals or small groups of students, followed by the highly trained "special education" teachers described above. Finally, students whose lack of progress is due to family or social difficulties outside of school can be referred to "multi-disciplinary teams."[134] According to a recent case study by the OECD, "Overall, these approaches to minimizing the number of students falling behind display two features: intensification (providing more time by more instructors) and alternative approaches (rather than 'more of the same') ... But they do so in consistent ways, working with the classroom teacher on the specific subjects students are having trouble with, rather than relying on a grab bag of after-school programs and tutoring efforts randomly distributed by grade levels and subjects."[135]

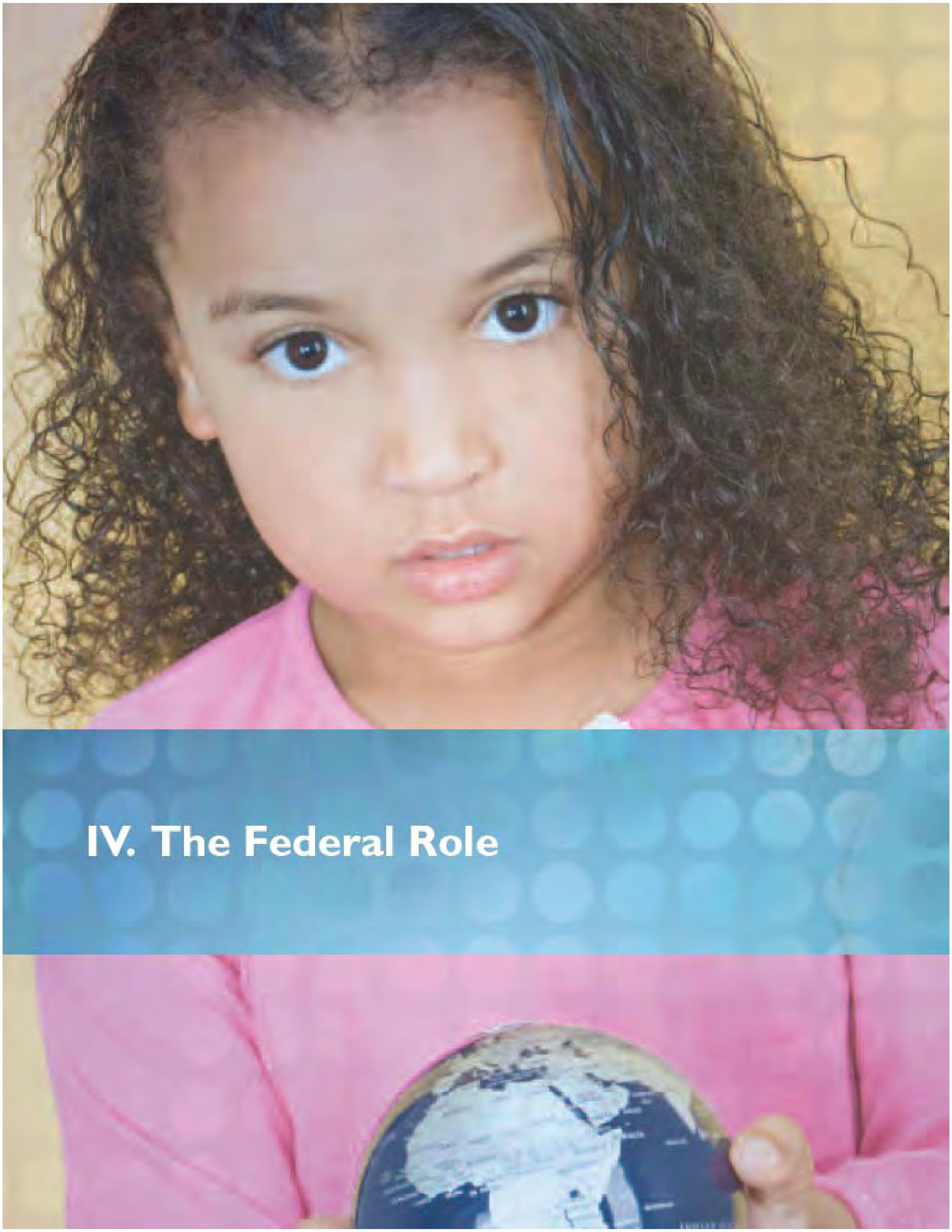
Such supports continue through lower secondary education, including a "class teacher" who follows a particular group of students for three years to monitor individual progress.[136] Indeed, when Finland ended early tracking of students and moved toward a more equitable system in the 1980s, leaders realized that lower secondary education would be a problem spot in the pipeline where vulnerable students might fall off track, so they specifically targeted greater funding toward the lower secondary grades—and continue to do so today (**Figure 6**).[137]

Some would argue that the U.S. cannot learn from Finland because it is a more equitable country socially and economically. However, it is telling that Finland's commitment to equity does not stop at the schoolhouse door; rather, the education system itself has been carefully constructed to maximize equity and ensure consistently high levels of performance for all students. According to an OECD report on educational equity best practices published last year, "Many countries could usefully follow the successful Finnish approach to learning difficulties, offering a sequence of intensifying interventions which draw back into the mainstream those who fall behind."[138]

Figure 6: Finland Targets Funds Toward Lower Secondary Where Needs Are Greatest



Source: Organisation for Economic Co-Operation and Development. Education at a Glance 2008. Paris: OECD, September 2008, p. 219, Table B1.1a. Figures represent annual expenditure on educational institutions per full-time equivalent students for all services in 2005, in equivalent U.S. dollars converted using purchasing power parity for gross domestic product.



IV. The Federal Role

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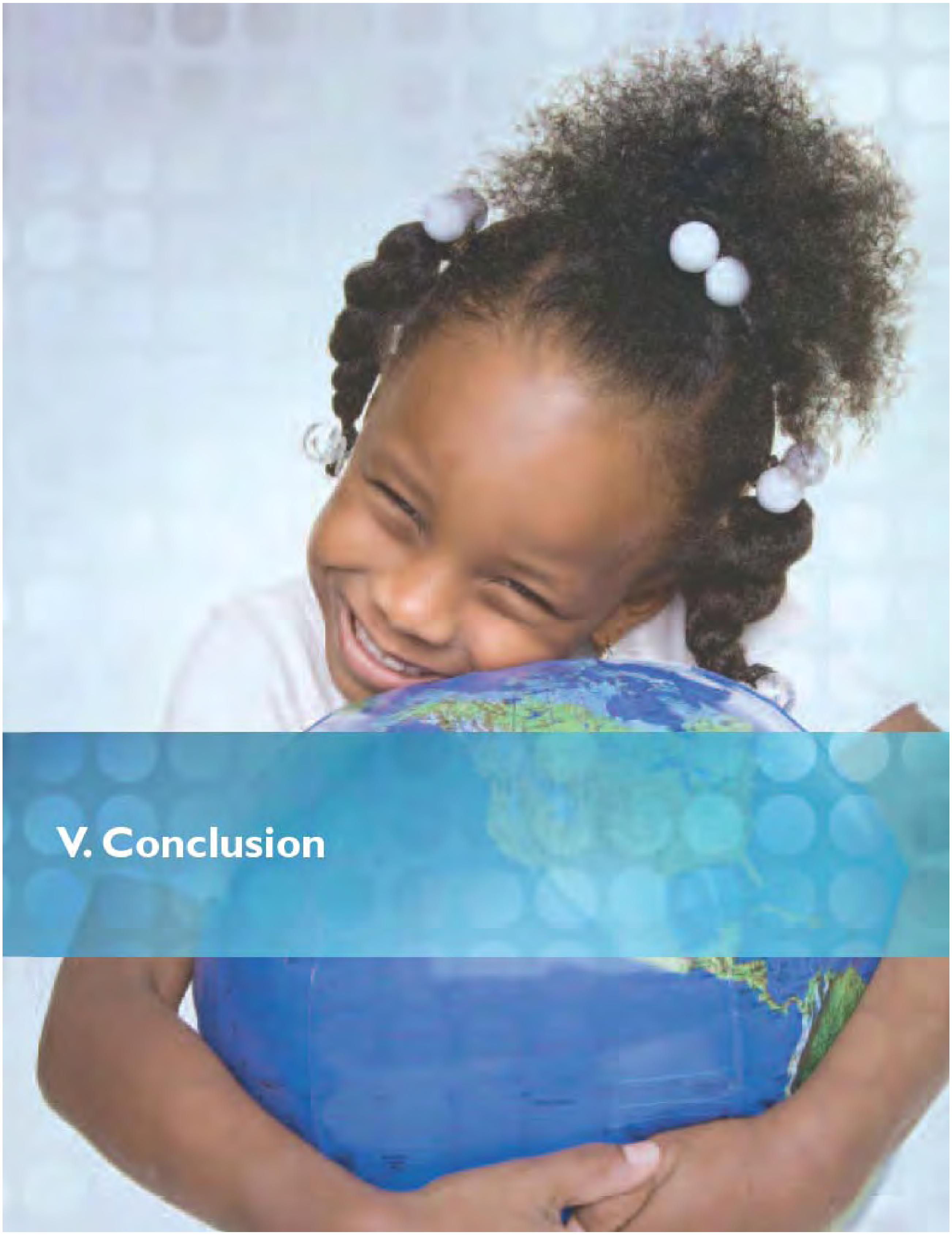
If benchmarking were only about measuring and comparing outcomes, the federal government might be able to play a leading role. However, because benchmarking is also—and most critically—about improving policy, states must take the lead. States have primary authority over the policy areas that other nations are most eager to benchmark and improve: standards, assessments, curriculum, and the education workforce. States already have led in raising standards, with 16 having adopted a common core of college- and career-ready expectations in math and reading for high school graduation.

The United States is not alone in this regard. Countries such as Canada, Australia, Germany, and Spain have federal-style education systems where states retain a great deal of authority over education. And in many of those countries, states are taking a leading role in benchmarking educational performance and policies. For example, the public outcry over mediocre results on the 2000 PISA assessment led to a historic new partnership between Germany's federal government and its 16 *Länder* (states), with the *Länder* taking responsibility for the establishment of shared education standards and assessments for schools across the nation while the federal government provided support for those and other state reforms.

America can learn from that example, too: While states must take the lead, the federal government can help. And the federal government can do that best by playing an *enabling* role grounded in a new vision for the historic state–federal partnership in education—one that is *less* restrictive and mandate-driven and *more* encouraging of innovation. As states take on the important work of benchmarking their education systems to the best in the world, the federal government can assist states in specific ways at each stage of the journey:

- As soon as possible, the federal government should offer new funding or allow existing funds to be used to help underwrite the cost for states to take the five action steps described above related to standards and assessment, curriculum, human capital, and accountability.
- At the same time, the federal government should increase its own investment or focus existing resources toward better research and development in this area to provide state leaders with more and better information about tools for benchmarking and international best practice in education. For example, the U.S. Department of Education should:
  1) Support efforts to collect and share international achievement and attainment data relevant to states; help state leaders identify good comparison nations or provinces for benchmarking; and collect and disseminate information about best practices of high-performing and fast-improving nations and provinces around the world; and
  2) Convene a technical advisory committee on assessment to make recommendations for generating internationally benchmarked results by state without adding significantly to costs and testing time. The committee should disseminate useful technical information about existing assessments, share policy options for improving and streamlining state assessment systems, and review the feasibility of adapting NAEP to generate international comparisons as described above.
- As states reach important milestones on the way toward building internationally competitive education systems, the federal government should offer a range of tiered incentives to make the next stage of the journey easier. With accountability at the core for greater results, such incentives could include:
  1) Increased flexibility in the use of federal funds;
  2) Increased flexibility in meeting requirements of existing federal education laws so that states are not thwarted in their efforts to adapt and adopt international best practices; and
  3) Additional funds to help states implement world-class practices.
- Over the *long term*, the federal government should change existing federal laws to align national education policies with the lessons learned from state benchmarking efforts and from federally funded research.

Over time, the combination of better information, additional support, and more flexibility for innovation would greatly accelerate state progress in developing and implementing world-class education systems. And that, in turn, will benefit all Americans, safeguarding U.S. economic security and ensuring continued prosperity in the new global economy.


V. Conclusion

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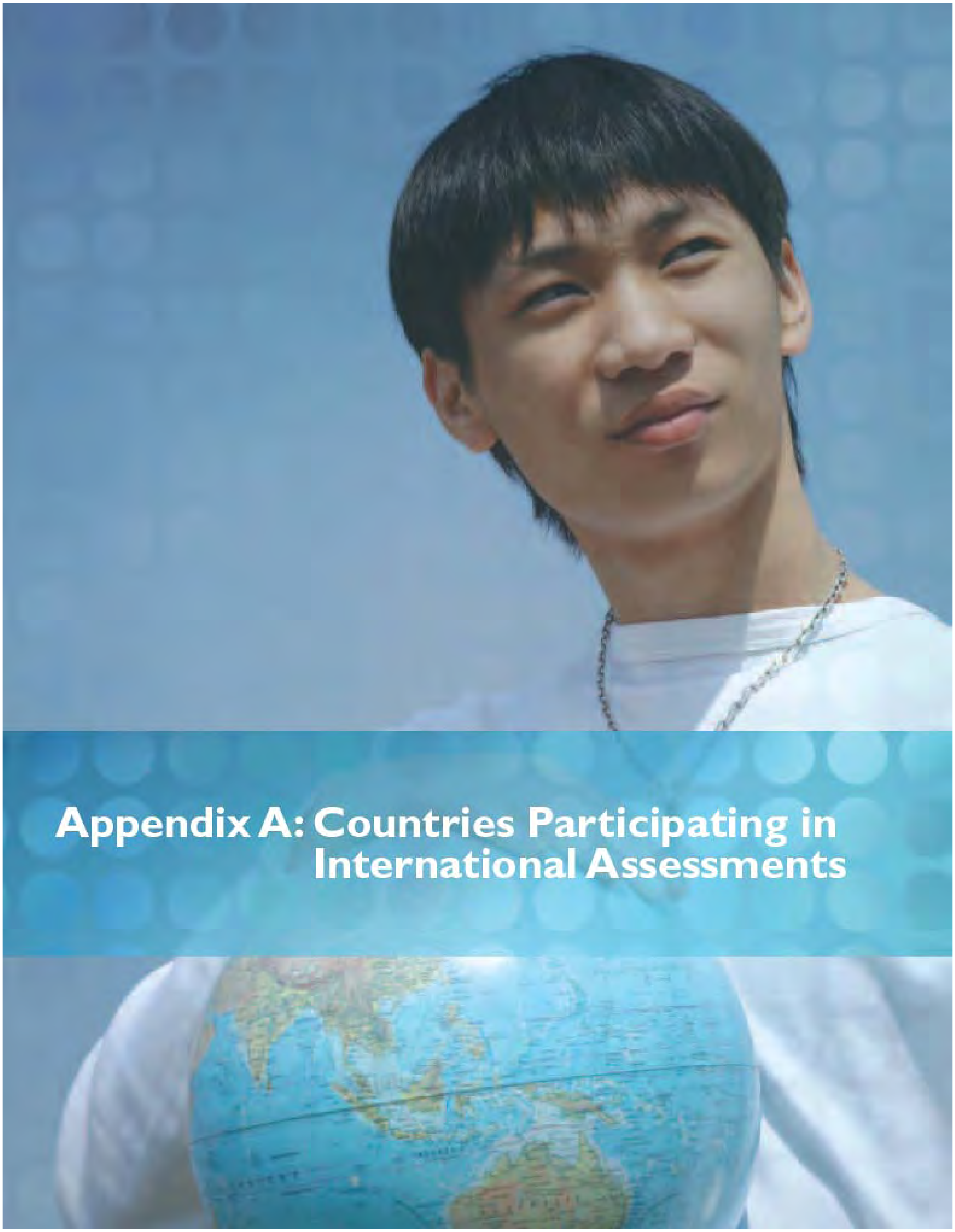
Other nations have benefited from America's historic example by expanding educational opportunities for their own citizens. Now it is time for U.S. leaders to ensure that Americans develop the skills they need to compete—and help the U.S. remain competitive—in a rapidly changing world.

The federal government can help, but states must lead. They must look beyond their borders and America's shores to fully understand how to benchmark expectations for student learning. They must significantly broaden the policy lens by drawing lessons from the highest performing, most equitable, and fastest advancing nations and states around the globe and adapting the very best educational practices to incorporate here at home.

If states in other countries can shape the response to the global education imperative, states in America must do so as well. And state leaders have both the authority and an obligation to ensure that students attend globally competitive schools and school districts. America cannot maintain its place in the world—economically, socially, or culturally—unless all of its students gain the skills that allow them to compete on a global scale. The United States will only achieve true international competitiveness when state education policies and institutions are restructured to meet 21st century realities.


Appendix A: Countries Participating in International Assessments

Appendix A: Countries Participating in International Assessments

Table reflects the most recent test year for which participation information is available.

	PISA 2009	TIMSS 2007 4th	TIMSS 2007 8th	PIRLS 2006
Africa				
Algeria		X	X	
Botswana			X	
Djibouti		X		
Egypt			X	
Ghana			X	
Morocco		X	X	X
South Africa			X	X
Tunisia	X	X	X	
Asia				
Azerbaijan	X			
Bahrain			X	
Chinese Taipei	X	X	X	X
Dubai (UAE)	X			
Hong Kong SAR	X	X	X	X
Indonesia	X		X	X
Iran, Islamic Republic		X	X	X
Israel	X		X	X
Japan	X	X	X	
Jordan	X		X	
Kazakhstan	X			
Korea, Republic of	X		X	
Kuwait		X	X	X
Kyrgyzstan	X			
Lebanon			X	
Macao-China	X			
Malaysia			X	
Mongolia		X	X	
Oman			X	
Palestinian Authority			X	
Qatar	X	X	X	X
Saudi Arabia			X	
Shanghai (China)	X			
Singapore	X	X	X	X
Syria			X	
Thailand	X		X	
Turkey	X		X	
Uzbekistan			X	
Yemen		X		
South America				
Argentina	X			
Brazil	X			
Chile	X			
Colombia	X	X	X	
Dominican Republic	X			
Panama	X			
Peru	X			
Trinidad and Tobago	X			
Uruguay	X			
Oceania				
Australia	X	X	X	
New Zealand	X	X		X
Europe				
Albania	X			
Armenia		X	X	
Austria	X	X		X
Belgium	X			X
Bosnia & Herc			X	
Bulgaria	X		X	X
Croatia	X			
Cyprus		X	X	
Czech Republic	X	X	X	
Denmark	X	X		X
England	X	X	X	X
Estonia	X			
Finland	X			
France	X			X
Georgia			X	X
Germany	X	X		X
Greece	X			
Hungary	X	X		X
Iceland	X			X
Ireland	X			
Italy	X	X	X	X
Latvia	X		X	X
Liechtenstein	X			
Lithuania	X	X	X	X
Luxembourg	X			X
Macedonia, Republic of				X
Malta		X		
Moldova, Republic of	X	X	X	X
Montenegro, Republic of	X			
Netherlands, The	X	X		X
Norway	X	X	X	X
Poland	X			X
Portugal	X			
Romania	X		X	X
Russian Federation	X	X	X	X
Scotland	X	X	X	X
Serbia, Republic of	X		X	
Slovak Republic	X	X		X
Slovenia	X	X	X	X
Spain	X		Basque	X
Sweden	X	X	X	X
Switzerland	X			
Ukraine		X	X	
North America				
Belize				
Canada	X	X	X	X
El Salvador		X	X	
Honduras		X	X	
Mexico	X			
Trinidad and Tobago				X
United States	X	X	X	X
Totals	*68*	*40*	*55*	*40*

Source: National Center for Education Statistics and Organisation for Economic Co-Operation and Development.

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The Common Core State Standards Validation Committee

A REPORT FROM THE NATIONAL GOVERNORS ASSOCIATION CENTER FOR BEST PRACTICES & THE COUNCIL OF CHIEF STATE SCHOOL OFFICERS

June 2010

COMMON CORE
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National Governors Association Center for Best Practices

Founded in 1908, the National Governors Association (NGA) is the collective voice of the nation's governors and one of Washington, D.C.'s most respected public policy organizations. Its members are the governors of the 50 states, three territories, and two commonwealths. NGA provides governors and their senior staff members with services that range from representing states on Capitol Hill and before the Administration on key federal issues to developing and implementing innovative solutions to today's most pressing public policy challenges through the NGA Center for Best Practices (NGA Center). The NGA Center is the only research and development firm that directly serves the nation's governors. For more information, visit www.nga.org.

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The Council of Chief State School Officers (CCSSO) is a nonpartisan, nationwide, nonprofit organization of public officials who head departments of elementary and secondary education in the states, the District of Columbia, the Department of Defense Education Activity, and five U.S. extra-state jurisdictions. CCSSO provides leadership, advocacy, and technical assistance on major educational issues. The Council seeks member consensus on major educational issues and expresses their views to civic and professional organizations, federal agencies, Congress, and the public. For more information, visit www.ccsso.org.

Acknowledgment

The NGA Center and CCSSO wish to acknowledge the work of Denis P. Doyle, chief academic officer of Schoolnet, Inc., for the preparation of this report. Doyle, a nationally and internationally known education writer and consultant, is co-author, with Susan Pimentel, of the book *Raising the Standard: An eight-step action guide for schools and communities* (Corwin Press, 1999).

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Reaching Higher

The Common Core State Standards Validation Committee

A REPORT FROM THE NATIONAL GOVERNORS ASSOCIATION CENTER FOR BEST PRACTICES &
THE COUNCIL OF CHIEF STATE SCHOOL OFFICERS

June 2010

The Common Core State Standards Initiative Validation Committee

Co-Chairs

David Conley—Professor and Director of the Center for Educational Policy Research, Educational Methodology, Policy, and Leadership at the University of Oregon's College of Education.

Brian Gong—Executive Director of the National Center for the Improvement of Educational Assessment.

Members

Bryan Albrecht—President of Gateway Technical College, Kenosha, Wis.

Arthur Applebee—Distinguished Professor of Education and Director of the Center on English Learning & Achievement at the University at Albany–State University of New York.

Sarah Baird—Mathematics Specialist/Teacher, Kyrene Elementary School District, Tempe, Ariz.

Jere Confrey—Senior Research Fellow and Joseph D.Moore Distinguished Professor at The William & Ida Friday Institute for Educational Innovation, North Carolina State University's College of Education.

Kristin Buckstad Hamilton—Nationally Board Certified Teacher, Battlefield Senior High School, National Education Association

Linda Darling-Hammond—Charles Ducommon Professor of Education and Co-Director of the School Redesign Network at Stanford University's School of Education.

Alfinio Flores—Hollowell Professor of Mathematics Education in the Department of Mathematical Sciences and School of Education at the University of Delaware's College of Education & Public Policy.

Kenji Hakuta—Lee L. Jacks Professor of Education at Stanford University's School of Education.

Feng-Jui Hsieh—Associate Professor in the Mathematics Department at the National Taiwan Normal University.

Mary Ann Jordan—Teacher, New York City Dept. of Education, American Federation of Teachers

Jeremy Kilpatrick—Regents Professor of Mathematics Education at the University of Georgia.

Dr. Jill Martin—Principal, Pine Creek High School

R. James Milgram—Emeritus Professor at Stanford University's Department of Mathematics.

David Pearson—Professor and Dean of the Graduate School of Education at the University of California–Berkeley.

Steve Pophal—Principal, D.C. Everest Junior High

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Andreas Schleicher—Head of the Indicators and Analysis Division with the Organisation for Economic Co-operation and Development's Directorate for Education.

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Catherine Snow—Henry Lee Shattuck Professor of Education, Harvard Graduate School of Education.

Christopher Steinhauser—Superintendent of Schools, Long Beach Unified School District, California.

Sandra Stotsky—Endowed Chair in Teacher Quality at the University of Arkansas's Department of Education Reform and Chair of the Sadlier Mathematics Advisory Board.

Dorothy Strickland—Distinguished Research Fellow at the National Institute for Early Education Research and the Samuel DeWitt Proctor Chair in Education at Rutgers University.

Martha Thurlow—Director, National Center on Educational Outcomes.

Norman L. Webb—Senior Research Scientist with the Wisconsin Center for Education Research and the National Institute for Science Education, both based at the University of Wisconsin–Madison's School of Education.

Dylan William—Director, Learning and Teaching Research Center at the Educational Testing Service.

INTRODUCTION

The National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) commissioned this report to chronicle the work of the Common Core State Standards Validation Committee, a key element of the Common Core State Standards Initiative (CCSSI).

BACKGROUND

The CCSSI is a historic effort designed to advance nationwide education reform. Coordinated by the NGA Center and CCSSO, 48 states, the District of Columbia, Puerto Rico, and the Virgin Islands have voluntarily come together to create shared common core standards in English language arts (ELA) and mathematics. The ultimate goal is for all American children to graduate from high school ready for college, career pathways, and success in a global economy.

Work groups comprised of representatives from higher education, K-12 education, teachers, and researchers drafted the Common Core State Standards. The work groups consulted educators, administrators, community and parent organizations, higher education representatives, the business community, researchers, civil rights groups, and states for feedback on each of the drafts. A list of work groups and expert members is available at www.corestandards.org.

Unlike past standards setting efforts, the Common Core State Standards are based on best practices in national and international education, as well as research and input from numerous sources including scholars, assessment developers, professional organizations, and educators representing all grade levels from kindergarten through postsecondary instruction. Contributions from the states and the public also served to inform the standards development process.

The Common Core State Standards represent what American students need to know and do to be successful in college and careers. Once the standards are adopted and implemented, states will determine how best to measure and hold students accountable for meeting these standards.

THE VALIDATION COMMITTEE

The NGA Center and CCSSO, as part of the CCSSI, convened a 25-member Validation Committee (VC) composed of leading figures in the education standards community. The committee was charged with providing independent, expert validation of the process of identifying the Common Core State Standards as part of the CCSSI. The VC:

» Reviewed the process by which evidence was used to create K-12 and college- and career-readiness standards; and

» Determined that the standards-development principles were adhered to by examining the standards for:
  - evidence of the knowledge and skills students need to be college- and career-ready,
  - a proper level of clarity and specificity,
  - evidence that the standards are comparable with other leading countries' expectations, and
  - a grounding in available evidence and research.

The VC also provided the work groups with invaluable input on the draft standards.

The Common Core State Standards represent what American students need to know and do to be successful in college and careers.

SELECTION OF THE VALIDATION COMMITTEE

Six governors and chief state school officers from states involved in the CCSSI selected the individual VC members based on nominations from national organizations and states. The individuals selected for the VC have experience in the development or implementation of national or international standards in education or have a demonstrated record of exceptional or unique expertise in English language arts, mathematics, or a related field, such as special education, English language learners, assessments, or curriculum development.

The VC's charge was to:

» Review the process used to develop the Common Core State Standards and provide input and feedback on that process; and

» Validate the sufficiency of the evidence supporting the Common Core State Standards.

As a free-standing committee—independent of standard-setting responsibility—the VC's role was to observe and validate the process of identifying Common Core State Standards and assess the evidentiary base for the standards.

PROCESS

The VC was appointed in September 2009 and first met in Washington, D.C., in December 2009. The committee's second meeting was held in April 2010. Staffed by NGA Center and CCSSO personnel, these meetings were augmented with intermittent telephone conference calls and e-mail exchanges among the committee members.

FINDINGS

On April 7, 2010, the VC met in Washington, D.C., to discuss the strengths and areas for additional consideration in the publicly released draft standards. The day-long conversation resulted in rich, substantive feedback that informed the final content of the Common Core State Standards. The VC generally praised the writers of the standards, as well as NGA Center and CCSSO staff for the tremendous overall progress.

The VC was, for the most part, pleased with the content of the standards and with the direction they were headed. The committee also provided suggestions for areas of the draft that would benefit from additional consideration. In mathematics, these areas included: strengthening the use of technology; paying attention to the specifics of the standards; providing a clearer explanation for the science, technology, engineering, and math (STEM) label; and streamlining all of the learning progressions. In English language arts, the committee focused on implementation issues, ranging from greater attention to the assessment of the standards to stronger guidance on the intent of the standards.

In May 2010, the VC received an embargoed copy of the final content of the Common Core State Standards for review and certification. The VC, in reviewing the processes employed to develop the standards, ultimately found them to be:

These certified research- and evidenced-based standards—aligned with college and career expectations—respect unique state contexts and the authority of each state to govern its public education system.

- Reflective of the core knowledge and skills in ELA and mathematics that students need to be college- and career-ready;
- Appropriate in terms of their level of clarity and specificity;
- Comparable to the expectations of other leading nations;
- Informed by available research or evidence;
- The result of processes that reflect best practices for standards development;
- A solid starting point for adoption of cross-state common core standards; and
- A sound basis for eventual development of standards-based assessments.

THE FUTURE: A WORK IN PROGRESS

The certification and release of the Common Core State Standards is a historic milestone; however, it does not mark the end of the work. Standard setting is an iterative process; there is no finish line. Alignment of curricula and assessments to the Common Core State Standards—the next great task facing the states—will be essential to the staying power and lasting impact of the standards.

The vital task of maintaining the continuity of the standards over time is another challenge. The standards will need to be continuously updated through processes that may, on occasion, pull stakeholders in opposite directions as consistency competes with the inevitable calls for changes or adjustments.

Effective long-term governance and organization of the CCSSI—sustaining a complex and demanding process over the span of years—is critical. The NGA Center and CCSSO are committed to ensuring that the Common Core State Standards remain state-driven and state-based. This includes encouraging states to lead cycles of revisions to the standards as new knowledge, best practices, and research emerge. The role of the VC or a similar structure will also need to be incorporated into the structure of long-term standards revision and governance.

In conclusion, it is worth remembering that the CCSSI was built on the following foundations:

- While the states are autonomous, when they work together on matters such as education, the collective knowledge can yield significant improvements.
- When the states voluntarily move in the same direction, they demonstrate their ability to achieve national goals.

These certified research- and evidenced-based standards—aligned with college and career expectations—respect unique state contexts and the authority of each state to govern its public education system. The lasting hallmark of this process on student achievement, then, will be clear, easy, and straightforward comparability over time—standard by standard, assessment by assessment, and state by state. These common standards are an important step in bringing about a real and meaningful transformation of the education system for the benefit of all students.

The CCSSI is more than a symbol of interstate collaboration; it could usher in a new era of cooperation on important public policy matters. It reflects 21st Century America as a place where states have come together on a shared vision of excellence.

CERTIFICATION

Based on the deliberations of, and review by, the Validation Committee, the National Governors Association Center for Best Practices and the Council of Chief State School Officers accept the Validation Committee's certification that the Common Core State Standards in English language arts and mathematics are consistent with the criteria established in the charge to the Validation Committee.

Signed,

Bryan Albrecht

Arthur Applebee

Sarah Baird

Jere Confrey

David T. Conley

Linda Darling-Hammond

Brian Gong

Kenji Hakuta

Kristin Buckstad Hamilton

Feng-Jui Hsieh

Mary Ann Jordan

Jeremy Kilpatrick

Dr. Jill Martin

David Pearson

Steve Pophal

Stanley Rabinowitz

Lauren Resnick

Andreas Schleicher

William Schmidt

Catherine Snow

Christopher Steinhauser

Dorothy Strickland

Martha Thurlow

Norman Webb

APPENDIX I

Validation Committee Members Biographical Sketches

Bryan Albrecht
President of Gateway Technical College, a two-year college in Kenosha, Wisconsin
Albrecht also serves as president of the Association for Career and Technical Education and on the president's advisory council for the National Coalition of Advanced Technology Centers. He has a background in secondary and postsecondary education, serves on numerous workforce-related boards, and participates in related standards and assessment organizations that bridge the education and economic and workforce development fields.

Arthur Applebee
Distinguished Professor of Education and Director of the Center on English Learning & Achievement at the University at Albany–State University of New York
Applebee is a nationally recognized authority on English language arts and a long-time advisor to the National Assessment of Educational Progress (NAEP). He received the David A. Russell Award for Distinguished Research in the Teaching of English from the National Council of Teachers of English for his book, *Curriculum as Conversation: Transforming Traditions of Teaching and Learning*, which re-conceptualized the role of curriculum in American schools. He has also advised policymakers at the national, state, and district levels on curriculum, instruction, and assessment.

Sarah Baird
Mathematics Specialist/Teacher with Kyrene Elementary School District in Tempe, Arizona
Baird, a National Board for Professional Teaching Standards–certified instructor, was named Arizona Teacher of the Year in 2009 and is a member of Arizona's Science, Technology, Engineering, and Mathematics Academy. Baird, who is the math coach for two district schools, serves K-5 students in roughly 60 classrooms.

Jere Confrey
Senior Research Fellow and Joseph D. Moore Distinguished Professor at The William & Ida Friday Institute for Educational Innovation at North Carolina State University's College of Education
Confrey has co-authored research for the National Research Council's Scientific Research in Education; produced extensive work in formative assessment in mathematics; and participates in the Generating Increased Science and Math Opportunities (GISMO) lab.

David Conley
Professor and Director of the Center for Educational Policy Research, Educational Methodology, Policy, and Leadership at the University of Oregon's College of Education
Conley has conducted extensive research on standards-based education and systemic school reform. He also directed the Standards for Success project for the Association of American Universities and The Pew Charitable Trusts, which documented the knowledge and skills students need to succeed in entry-level university courses. His recent book, *College Knowledge*, expands on this line of research.

Linda Darling-Hammond
Charles Ducommon Professor of Education and Co-Director of the School Redesign Network at Stanford University's School of Education
Darling-Hammond's focus is on international curriculum studies, school restructuring, teacher quality, and educational equity. She is former president of the American Educational Research Association and former executive director of the National Commission on Teaching & America's Future, whose 1996 report, *What Matters Most: Teaching for America's Future*, led to sweeping teaching and teacher education policy changes.

Alfinio Flores
Hollowell Professor of Mathematics Education in the Department of Mathematical Sciences and School of Education at the University of Delaware's College of Education & Public Policy
Flores is a nationally recognized expert in mathematics education and mathematics teaching and learning, curriculum development, and pre- and in-service preparation of teachers of mathematics.

Brian Gong
Executive Director of the National Center for the Improvement of Educational Assessment
Gong is a nationally recognized expert in standards and assessment and a research scientist. He has served as associate commissioner for the Kentucky Department of Education. As a former member of the technical advisory committee for the New England Common Assessment Program, Gong possesses a keen understanding of the working relationship between standards and large-scale assessment programs.

Kenji Hakuta
Lee L. Jacks Professor of Education at Stanford University's School of Education
Hakuta's studies focus on bilingualism and the acquisition of English in immigrant students. He co-authored the National Academy of Sciences report, *Improving Schooling for Language-Minority Children: A Research Agenda*; co-edited a book on affirmative action in higher education, *Compelling Interest: Examining the Evidence on Racial Dynamics in Colleges and Universities*; has testified before Congress on topics such as language policy, the education of language-minority students, affirmative action in higher education, and the improvement of quality in educational research; and was founding dean of the University of California–Merced School of Social Sciences, Humanities and Arts.

Kristin Buckstad Hamilton
Nationally Board Certified Teacher, Battlefield Senior High School, National Education Association

Feng-Jui Hsieh
Associate Professor in the Mathematics Department at the National Taiwan Normal University
Hsieh researches mathematics learning, mathematics teaching, teacher education, and pre-service and in-service teacher professional development. He served as chairman of Taiwan's first evaluation committee of the junior high school mathematics textbook, as a member of the first evaluation committee of the elementary school mathematics textbook, and as a member of the committee to develop the first national curriculum standards for private textbook publishers at the senior high school level. He has received grants from Taiwan's National Science Council and Ministry of Education. Hsieh serves as Taiwan's representative on two international studies administered by the International Association for the Evaluation of Educational Achievement and the National Science Foundation.

Mary Ann Jordan
Teacher, New York City Dept. of Education, American Federation of Teachers

Jeremy Kilpatrick
Regents Professor of Mathematics Education at the University of Georgia
Kilpatrick, a charter member of the National Research Council's (NRC) Mathematical Sciences Education Board, has received numerous education awards, including the Felix Klein Medal in 2007 for lifetime achievement in mathematics education from the International Commission on Mathematical Instruction and the 2003 Lifetime Achievement Award for Distinguished Service to Mathematics Education from the National Council of Teachers of Mathematics. He chaired the committee that produced *Adding It Up: Helping Children Learn Mathematics*, a 2001 NRC report on arithmetic proficiency.

Dr. Jill Martin
Principal, Pine Creek High School

R. James Milgram
Emeritus Professor at Stanford University's Department of Mathematics
Milgram, one of the authors of the California Mathematics Standards and the California Mathematics Framework, has worked with a number of states, and with the Achieve Mathematics Advisory Panel, on standards in education. As a member of the National Board for Education Sciences, he has worked with the U.S. Department of Education on the math that pre-service K-8 teachers need to know and understand.

David Pearson
Professor and Dean of the Graduate School of Education at the University of California–Berkeley
Pearson, a literacy expert, conducts research on practice and policy in literacy instruction and assessment. He was president of the National Reading Conference and served on the board of directors for the International Reading Association, National Reading Conference, and American Association of Colleges for Teacher Education, all of which bestowed on Pearson several honors and awards for his service.

Steve Pophal
Principal, D.C. Everest Junior High

Stanley Rabinowitz
Director of Assessment & Standards Development Services at WestEd in San Francisco, California
At WestEd, Rabinowitz oversees program activities and directs assessment development for the Kentucky and Nevada statewide assessment programs. He also directs the WestEd/CRESST Assessment and Accountability Comprehensive Center. He has authored papers on the use of integrated standards and assessment systems in high-stakes state programs and worker-training initiatives and directed the statewide assessment program for the New Jersey Department of Education.

Lauren Resnick
Professor and Director of the Institute for Learning at the University of Pittsburgh
Resnick is an internationally known scholar in the cognitive science of learning and instruction whose research involves the learning and teaching of literacy, math, and science. She is former director of the Learning Research and Development Center at the University of Pittsburgh. Resnick also co-founded the New Standards Project, a nearly 10-year effort to develop performance-based standards and assessments that widely influenced state and school district practices.

Andreas Schleicher
Head of the Indicators and Analysis Division with the Organisation for Economic Co-operation and Development's (OECD) Directorate for Education
Schleicher also directs OECD's Programme for International Student Assessment (PISA) and the Indicators of Education Systems program. She previously served as director for analysis at the Institute for Educational Research in the Netherlands. She was awarded the Theodor Heuss prize for exemplary democratic engagement in association with the public debate on PISA.

William Schmidt
University Distinguished Professor and Co-Director of Michigan State University's Education Policy Center
Schmidt also co-directs the US–China Center for Research on Educational Excellence and the Promoting Rigorous Outcomes in Math and Science Education Project. He has provided recommendations for internationally competitive K-12 math standards for Minnesota and participated in the Third International Mathematics and Science Study.

Catherine Snow
Henry Lee Shattuck Professor of Education at the Harvard Graduate School of Education
An international expert on literacy instruction, Snow has chaired several major committees on literacy, has studied low-income students, and has written about bilingualism and its relation to language-policy issues.

Christopher Steinhauser
Superintendent of Schools for the Long Beach Unified School District in California
Steinhauser possesses extensive knowledge about California's ELA standards and their implementation across a very large school district. During his previous tenure as deputy superintendent, students in all major racial and ethnic groups throughout the district made unprecedented gains on rigorous state tests.

Sandra Stotsky
Endowed Chair in Teacher Quality at the University of Arkansas's Department of Education Reform and Chair of the Sadlier Mathematics Advisory Board
Stotsky has abundant experience in developing and reviewing ELA standards. As senior associate commissioner of the Massachusetts Department of Education, she helped revise pre–K-12 standards. She also served on the 2009 steering committee for NAEP reading and on the 2006 National Math Advisory Panel.

Dorothy Strickland
Distinguished Research Fellow at the National Institute for Early Education Research and the Samuel DeWitt Proctor Chair in Education at Rutgers University
A national expert on literacy, Strickland assisted the New Jersey Department of Education with developing its ELA standards for young children. She has served on many national panels and task forces over her multi-decade career in education, most recently on the National Center for Learning Disabilities professional advisory board and on a Head Start research and evaluation board at the U.S. Department of Health and Human Services.

Martha Thurlow
Director of the National Center on Educational Outcomes
Thurlow focuses on implications of contemporary U.S. policy and practice for students with disabilities and English language learners, including national and statewide assessment policies and practices, standards-setting efforts, and graduation requirements. She has conducted research and published extensively on assessment, learning disabilities, early childhood education, dropout prevention, effective classroom instruction, and integration of students with disabilities in general education settings.

Norman L. Webb
Senior Research Scientist with the Wisconsin Center for Education Research and the National Institute for Science Education, both based at the University of Wisconsin–Madison's School of Education
Webb works on strategies for evaluating reform and rethinking aspects of math and science education. He also conducts research on assessing students' knowledge of math and science and aligning standards and assessments, and directs evaluations of curriculum and professional-development projects.

Dylan William
Director of the Learning and Teaching Research Center at the Educational Testing Service
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COMMON CORE STATE STANDARDS FOR

English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects

Appendix A:

Research Supporting Key Elements of the Standards

Glossary of Key Terms

A Note on International Sources for the Standards

In the course of developing the Standards, the writing team consulted numerous international models, including those from Ireland, Finland, New Zealand, Australia (by state), Canada (by province), Singapore, the United Kingdom, and others. Several patterns emerging from international standards efforts influenced the design and content of the Standards:

(1) *Other nations pay equal attention to what students read and how they read.* Many countries set standards for student reading by providing a reading list. The United Kingdom has standards for the "range and content" of student reading. While lacking the mandate to set particular reading requirements, the Standards nonetheless follow the spirit of international models by setting explicit expectations for the range, quality, and complexity of what students read along with more conventional standards describing how well students must be able to read.

(2) *Students are required to write in response to sources.* In several international assessment programs, students are confronted with a text or texts and asked to gather evidence, analyze readings, and synthesize content. The Standards likewise require students to "draw evidence from literary or informational texts to support analysis, reflection, and research" (Writing CCR standard 9).

(3) *Writing arguments and writing informational/explanatory texts are priorities.* The Standards follow international models by making writing arguments and writing informational/explanatory texts the dominant modes of writing in high school to demonstrate readiness for college and career.

COMMON CORE STATE STANDARDS FOR

Mathematics

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Introduction

Toward greater focus and coherence

> *Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics. Mathematical process goals should be integrated in these content areas.*
>
> — Mathematics Learning in Early Childhood, National Research Council, 2009

> *The composite standards [of Hong Kong, Korea and Singapore] have a number of features that can inform an international benchmarking process for the development of K–6 mathematics standards in the U.S. First, the composite standards concentrate the early learning of mathematics on the number, measurement, and geometry strands with less emphasis on data analysis and little exposure to algebra. The Hong Kong standards for grades 1–3 devote approximately half the targeted time to numbers and almost all the time remaining to geometry and measurement.*
>
> — Ginsburg, Leinwand and Decker, 2009

> *Because the mathematics concepts in [U.S.] textbooks are often weak, the presentation becomes more mechanical than is ideal. We looked at both traditional and non-traditional textbooks used in the US and found this conceptual weakness in both.*
>
> — Ginsburg et al., 2005

> *There are many ways to organize curricula. The challenge, now rarely met, is to avoid those that distort mathematics and turn off students.*
>
> — Steen, 2007

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is "a mile wide and an inch deep." These Standards are a substantial answer to that challenge.

It is important to recognize that "fewer standards" are no substitute for focused standards. Achieving "fewer standards" would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity.

Assessing the coherence of a set of standards is more difficult than assessing their focus. William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are:

> *articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline,* ***but also the key ideas*** *that determine how knowledge is organized and generated within that discipline. This implies*

How to read the grade level standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Domain

Number and Operations in Base Ten **3.NBT**

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Standard

Cluster

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A *must be taught before* topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know ... should next come to learn ...." But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

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Appendix B1-5

State Documentation of Adoption of Final Common Core State Standards

Illinois State Board of Education

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Jesse H. Ruiz
Chairman

Christopher A. Koch, Ed.D.
State Superintendent of Education

Certificate of Emergency Amendment

The State Board of Education
(Name of Agency, Board, Commission or Department)

certifies that the attached hereto is a true and correct copy of:

Heading of Part: Public Schools Evaluation, Recognition and Supervision

Code Citation: 23 Ill. Adm. Code 1

Sections Involved: 1.Appendix D

which was duly adopted by emergency action by this agency on June 24, 2010.

Statutory Authority: 105 ILCS 5/2-3.6

The public interest is best served by Illinois' securing funding under the federal RTTT competitive grant program. Illinois is requesting $400 million for various school improvement and reform efforts, with a particular emphasis on the State's lowest performing districts and schools. Under RTTT, states that adopt the common core standards by August 2 will receive additional points in the proposal evaluation process. The ordinary rulemaking process requires a 45-day public comment period, which will prevent the State Board of Education agency from promulgating the standards by the RTTT deadline. Thus, it is critical that this rule be in effect as soon as possible in order to strengthen Illinois' application for funding under the RTTT initiative.

Christopher Koch
Signature of Officer

State Superintendent of Education
Title of Officer

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TITLE 23: EDUCATION AND CULTURAL RESOURCES
SUBTITLE A: EDUCATION
CHAPTER I: STATE BOARD OF EDUCATION
SUBCHAPTER a: PUBLIC SCHOOL RECOGNITION

PART 1
PUBLIC SCHOOLS EVALUATION, RECOGNITION AND SUPERVISION

SUBPART A: RECOGNITION REQUIREMENTS

SUBPART B: SCHOOL GOVERNANCE

1.285 Requirements for the Use of Isolated Time Out and Physical Restraint
1.290 Absenteeism and Truancy Policies

SUBPART C: SCHOOL DISTRICT ADMINISTRATION

Section
1.310 Administrative Qualifications and Responsibilities
1.320 Evaluation of Certified Staff in Contractual Continued Service
1.330 Hazardous Materials Training

SUBPART D: THE INSTRUCTIONAL PROGRAM

Section
1.410 Determination of the Instructional Program
1.420 Basic Standards
1.430 Additional Criteria for Elementary Schools
1.440 Additional Criteria for High Schools
1.445 Required Course Substitute
1.450 Special Programs (Repealed)
1.460 Credit Earned Through Proficiency Examinations
1.462 Uniform Annual Consumer Education Proficiency Test
1.465 Ethnic School Foreign Language Credit and Program Approval
1.470 Adult and Continuing Education
1.480 Correctional Institution Educational Programs

SUBPART E: SUPPORT SERVICES

Section
1.510 Transportation
1.515 Training of School Bus Driver Instructors
1.520 School Food Services (Repealed)
1.530 Health Services
1.540 Pupil Personnel Services (Repealed)

SUBPART F: STAFF CERTIFICATION REQUIREMENTS

Section
1.610 Personnel Required to be Qualified
1.620 Accreditation of Staff (Repealed)
1.630 Noncertificated Personnel
1.640 Requirements for Different Certificates (Repealed)
1.650 Transcripts of Credits

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SUBPART G: STAFF QUALIFICATIONS

Section

AUTHORITY: Implementing Sections 2-3.25, 2-3.25g, 2-3.43, 2-3.44, 2-3.96, 10-17a, 10-20.14, 10-22.43a, 14C-8, 21-0.01, 26-13, 27-3.5, 27-12.1, 27-13.1, 27-20.3, 27-20.4, 27-20.5, 27-22, 27-23.3, and 27-23.8 and authorized by Section 2-3.6 of the School Code [105 ILCS 5/2-

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3.25, 2-3.25g, 2-3.43, 2-3.44, 2-3.96, 10-17a, 10-20.14, 10-22.43a, 14C-8, 21-0.01, 26-13, 27-3.5, 27-12.1, 27-13.1, 27-20.3, 27-20.4, 27-20.5, 27-22, 27-23.3, 27-23.8, and 2-3.6].

SOURCE: Adopted September 21, 1977; codified at 7 Ill. Reg. 16022; amended at 9 Ill. Reg. 8608, effective May 28, 1985; amended at 9 Ill. Reg. 17766, effective November 5, 1985; emergency amendment at 10 Ill. Reg. 14314, effective August 18, 1986, for a maximum of 150 days; amended at 11 Ill. Reg. 3073, effective February 2, 1987; amended at 12 Ill. Reg. 4800, effective February 26, 1988; amended at 14 Ill. Reg. 12457, effective July 24, 1990; amended at 15 Ill. Reg. 2692, effective February 1, 1991; amended at 16 Ill. Reg. 18010, effective November 17, 1992; expedited correction at 17 Ill. Reg. 3553, effective November 17, 1992; amended at 18 Ill. Reg. 1171, effective January 10, 1994; emergency amendment at 19 Ill. Reg. 5137, effective March 17, 1995, for a maximum of 150 days; amended at 19 Ill. Reg. 6530, effective May 1, 1995; amended at 19 Ill. Reg. 11813, effective August 4, 1995; amended at 20 Ill. Reg. 6255, effective April 17, 1996; amended at 20 Ill. Reg. 15290, effective November 18, 1996; amended at 22 Ill. Reg. 22233, effective December 8, 1998; emergency amendment at 24 Ill. Reg. 6111, effective March 21, 2000, for a maximum of 150 days; amended at 24 Ill. Reg. 12985, effective August 14, 2000; amended at 25 Ill. Reg. 8159, effective June 21, 2001; amended at 25 Ill. Reg. 16073, effective November 28, 2001; amended at 26 Ill. Reg. 1157, effective January 16, 2002; amended at 26 Ill. Reg. 16160, effective October 21, 2002; amended at 28 Ill. Reg. 8486, effective June 1, 2004; emergency amendment at 28 Ill. Reg. 13637, effective September 27, 2004, for a maximum of 150 days; amended at 29 Ill. Reg. 1891, effective January 24, 2005; amended at 29 Ill. Reg.11811, effective July 13, 2005; amended at 29 Ill. Reg. 12351, effective July 28, 2005; amended at 29 Ill. Reg. 15789, effective October 3, 2005; amended at 29 Ill. Reg. 19891, effective November 23, 2005; amended at 30 Ill. Reg. 8480, effective April 21, 2006; amended at 30 Ill. Reg. 16338, effective September 26, 2006; amended at 30 Ill. Reg. 17416, effective October 23, 2006; amended at 31 Ill. Reg. 5116, effective March 16, 2007; amended at 31 Ill. Reg. 7135, effective April 25, 2007; amended at 31 Ill. Reg. 9897, effective June 26, 2007; amended at 32 Ill. Reg. 10229, effective June 30, 2008; amended at 33 Ill. Reg. 5448, effective March 24, 2009; amended at 33 Ill. Reg. 15193, effective October 20, 2009; amended at 34 Ill. Reg. 2959, effective February 18, 2010; emergency amendments at 34 Ill. Reg. 9533, effective JUN 24 2010, for a maximum of 150 days.

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Section 1.Appendix D State Goals for Learning
EMERGENCY

The State Goals for Learning are broad statements of what students should know and be able to do as a result of their public education. The Illinois Learning Standards provide more specific definition of the essential knowledge and skills desired of Illinois students. The state assessment is designed to measure students' mastery of the Illinois Learning Standards, so that a clear connection will emerge between students' learning and the goals and standards of the State of Illinois.

ENGLISH LANGUAGE ARTS AND LITERACY IN HISTORY/SOCIAL STUDIES, SCIENCE, AND TECHNICAL SUBJECTS

There are no State Goals for Learning in this area. The applicable standards shall be the "Common Core State Standards for English Language Arts and Literacy in History/Social Studies, Science, and Technical Subjects" (2010) published by the Common Core State Standards Initiative and posted at http://www.corestandards.org/the-standards/english-language-arts-standards. No later amendments to or editions of these standards are incorporated by this Section.

MATHEMATICS

There are no State Goals for Learning in this area.The applicable standards shall be the "Common Core State Standards for Mathematics" (2010) published by the Common Core State Standards Initiative and posted at http://www.corestandards.org/the-standards/mathematics. No later amendments to or editions of these standards are incorporated by this Section.

SCIENCE

State Goal 11: Understand the processes of scientific inquiry and technological design to investigate questions, conduct experiments and solve problems.

Standards:

Know and apply the concepts, principles and processes of scientific inquiry.

Know and apply the concepts, principles and processes of technological design.

State Goal 12: Understand the fundamental concepts, principles and interconnections of the life, physical and earth/space sciences.

Standards:

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Know and apply concepts that explain how living things function, adapt and change.

Know and apply concepts that describe how living things interact with each other and with their environment.

Know and apply concepts that describe properties of matter and energy and the interactions between them.

Know and apply concepts that describe force and motion and the principles that explain them.

Know and apply concepts that describe the features and processes of the Earth and its resources.

Know and apply concepts that explain the composition and structure of the universe and Earth's place in it.

State Goal 13: Understand the relationships among science, technology and society in historical and contemporary contexts.

Standards:

Know and apply the accepted practices of science.

Know and apply concepts that describe the interaction between science, technology and society.

SOCIAL SCIENCE

State Goal 14: Understand political systems, with an emphasis on the United States.

Standards:

Understand and explain basic principles of the United States government.

Understand the structures and functions of the political systems of Illinois, the United States and other nations.

Understand election processes and responsibilities of citizens.

Understand the roles and influences of individuals and interest groups in the political systems of Illinois, the United States and other nations.

Understand United States foreign policy as it relates to other nations and international issues.

Understand the development of United States political ideas and traditions.

State Goal 15: Understand economic systems, with an emphasis on the United States.

Standards:

Understand how different economic systems operate in the exchange, production, distribution and consumption of goods and services.

Understand that scarcity necessitates choices by consumers.

Understand that scarcity necessitates choices by producers.

Understand trade as an exchange of goods or services.

Understand the impact of government policies and decisions on production and consumption in the economy.

State Goal 16: Understand events, trends, individuals and movements shaping the history of Illinois, the United States and other nations.

Standards:

Apply the skills of historical analysis and interpretation.

Understand the development of significant political events.

Understand the development of economic systems.

Understand Illinois, United States and world social history.

Understand Illinois, United States and world environmental history.

State Goal 17: Understand world geography and the effects of geography on society, with an emphasis on the United States.

Standards:

Locate, describe and explain places, regions and features on the Earth.

Analyze and explain characteristics and interactions of the Earth's physical systems.

Understand relationships between geographic factors and society.

Understand the historical significance of geography.

State Goal 18: Understand social systems, with an emphasis on the United States.

Standards:

Compare characteristics of culture as reflected in language, literature, the arts, traditions and institutions.

Understand the roles and interactions of individuals and groups in society.

Understand how social systems form and develop over time.

PHYSICAL DEVELOPMENT AND HEALTH

State Goal 19: Acquire movement skills and understand concepts needed to engage in health-enhancing physical activity.

Standards:

Demonstrate physical competency in individual and team sports, creative movement and leisure and work-related activities.

Analyze various movement concepts and applications.

Demonstrate knowledge of rules, safety and strategies during physical activity.

State Goal 20: Achieve and maintain a health-enhancing level of physical fitness based upon continual self-assessment.

Standards:

Know and apply the principles and components of health-related fitness.

Assess individual fitness levels.

Set goals based on fitness data and develop, implement and monitor an individual fitness improvement plan.

State Goal 21: Develop team-building skills by working with others through physical activity.

Standards:

Demonstrate individual responsibility during group physical activities.

Demonstrate cooperative skills during structured group physical activity.

State Goal 22: Understand principles of health promotion and the prevention and treatment of illness and injury.

Standards:

Explain the basic principles of health promotion, illness prevention and safety.

Describe and explain the factors that influence health among individuals, groups and communities.

Explain how the environment can affect health.

State Goal 23: Understand human body systems and factors that influence growth and development.

Standards:

Describe and explain the structure and functions of the human body systems and how they interrelate.

Explain the effects of health-related actions on the body systems.

Describe factors that affect growth and development.

State Goal 24: Promote and enhance health and well-being through the use of effective communication and decision-making skills.

Standards:

Demonstrate procedures for communicating in positive_ways, resolving differences and preventing conflict.

Apply decision-making skills related to the protection and promotion of individual health.

Demonstrate skills essential to enhancing health and avoiding dangerous situations.

FINE ARTS

State Goal 25: Know the language of the arts.

Standards:

Understand the sensory elements, organizational principles and expressive qualities of the arts.

Understand the similarities, distinctions and connections in and among the arts.

State Goal 26: Through creating and performing, understand how works of art are produced.

Standards:

Understand processes, traditional tools and modern technologies used in the arts.

Apply skills and knowledge necessary to create and perform in one or more of the arts.

State Goal 27: Understand the role of the arts in civilizations, past and present.

Standards:

Analyze how the arts function in history, society and everyday life.

Understand how the arts shape and reflect history, society and everyday life.

(Source: Amended by emergency rulemaking at 34 Ill. Reg. 9533, effective JUN 24 2010, for a maximum of 150 days)

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