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Research Summary:
Cognition: Math (COG: MATH) Domain in the DRDP-K (2015) and KIDS (2015) Kindergarten Assessment Instruments

The **Cognition: Math (COG: MATH)** domain focuses on observation, exploration of people and objects, and investigation of objects and concepts. The COG: MATH domain includes the following knowledge or skill areas: classification, number sense of quantity, number sense of math operations, measurement, patterning, and shapes.

COG: MATH 1: Classification

"Classification is the systematic arrangement of objects into groups according to established criteria" (California Department of Education, 2010a, p. 161). This measure highlights how children show an increasing ability to compare, match, and sort objects into groups according to their attributes. Research distinguishes between two types of categorization made by young children: perceptual and conceptual (Mandler, 2000). Perceptual categorization involves similarities or differences that can be observed based on physical characteristics, such as similarities in visual appearance. Conceptual categorization refers to grouping based on what objects do or how they act.

Young children develop the ability to sort consistently based on a single attribute (e.g., size, color, shape). They also learn to re-classify by different attributes, understanding that objects can belong to more than one group. Kindergarten-age children are able to explain the related attributes of grouped objects. Major advances for the next level involve the ability to sort based on multiple attributes, and according to conceptual criteria. The research literature on the development of classification emphasizes children's initial differentiation of broad categories of prototypical and concrete objects, later extension to atypical exemplars, and distinctions based upon functional attributes. Children also learn to use sorting skills to gather information. They may sort and count the number of objects in each group, for example, separate red and yellow apples in two bowls, and report which group has more apples.

COG: MATH 2: Number Sense of Quantity

This measure highlights a child's developing understanding of number and quantity. Research suggests that children start developing number sense in early infancy (Feigenson, Dehaene, & Spelke, 2004).

Toddlers' performance on numeracy matching tasks indicates that, as they grow older, they become better at representing quantity abstractly. This achievement appears to be related to the acquisition of number words (Mix, 2008). "Children's understanding of numbers is initially qualitative as they gain an understanding of 'number-ness' (e.g., three-ness, four-ness) with small quantities using subitizing: visually knowing 'how many' are in a set without actually counting them (Clements, 2004a; Fuson, 1988, 1992a)" (California Department of Education, 2010a, p. 160). Counting builds a foundation for children's future understanding of mathematics. "Literature suggests that the three major basic building blocks for counting are learning of (1) the sequence of number words, (2) one-to-one correspondence, and (3) cardinality (knowing that the last number assigned to the last object counted gives the total number in the set) (Kilpatrick, Swafford, & Findell, 2001; Becker, 1989;

Clements, 2004a; Fuson, 1988, 1992a, 1992b; Hiebert et al., 1997; Sophian, 1988)" (California Department of Education, 2010a, p. 160).

"The preschool years are a critical time for children to master the art of counting small numbers of objects" (California Department of Education, 2010a, p. 160). This basic skill becomes the reference point as children learn to manipulate larger quantities in the future. During the preschool years, they learn to say the number words one to twenty in the correct order.

By the end of the Kindergarten year, the child progresses from counting in order up to 100 and beyond 100. The major advance for the next stage involves understanding teen number composition of ten and additional ones as well as the understanding that in two-digit numbers the first digit represents number of tens and the second digit represents the number of ones. The child is also able to write and read numerals past 20 and up to 100 (Sarama & Clements, 2009).

COG: MATH 3: Number Sense of Math Operations

The developmental processes highlighted in this measure involve children's ability to add and subtract small quantities of objects. "Research indicates that the ability to reason about numbers starts as early as infancy (Wynn, 1992a) Five-months-olds show sensitivity to the effects of addition or subtraction of items on a small collection of objects. Toddlers viewing three balls put into a container and then one being removed know to search for a smaller number of balls, and many search for exactly two balls (Starkey, 1992)" (California Department of Education, 2010b, p. 251).

Growth of numerical competence from age two to age six is facilitated by children's learning of important numerical tools: spoken number words, written number symbols, and cultural solution methods, such as counting and matching. During the preschool years, children are able to solve simple addition and subtraction problems, using objects or fingers to represent numbers by counting out loud to find out the answer. "When asked to combine two sets of objects, they count the two different sets starting from 'one' to determine the answer (the counting-all strategy); therefore, the development of number operations is closely related to the way they learn to count. As children gain experiences, they gradually develop more sophisticated methods" (California Department of Education, 2010a, p. 161). They may "count on" from the second set of objects. Knowing the number of objects in the first set (e.g., four), the child starts with "four" and continues to count "five, six" to find out the total number of objects (4 + 2) rather than starting to count from "one." Children also "become adept at decomposing numbers into smaller chunks for the purpose of adding and subtracting" (California Department of Education, 2010a, p. 161). Their experience with the "concept of decomposition of a number into smaller groups of numbers is the beginning of an important development in mathematical reasoning. Learning the concept that groups or chunks of numbers make up larger numbers supports the understanding of arithmetic operations, for example, children's emerging understanding of different ways the number 10 can be decomposed into groups (e.g., 5 + 5, 4 + 6)" (California Department of Education, 2010a, p. 161).

In the early elementary school years, children are able to represent and solve addition and subtraction word problems with totals up to 20 by using objects, drawings, and equations, and by applying advanced strategies (e.g., count-on), including strategies that reflect understanding of properties of addition and subtraction.

COG: MATH 4: Measurement

This measure highlights the ways in which the child shows an increasing understanding of measurable properties such as size, length, weight, and capacity (volume), and how to quantify those properties. "A typical developmental trajectory involves children first learning to use words that represent quantities or magnitude of a certain attribute. Then, children begin to demonstrate an ability to compare two objects directly and recognize equality or inequality. For example, they may compare two objects to determine which is longer or heavier. After comparing two items, children develop the ability to compare three or more objects and to order them by size (e.g. from shortest to longest) or by other attributes. Finally, children learn to measure, connecting numbers to attributes of objects, such as length, weight, amount, and area (Clements 2004a; Ginsburg, Inoue, & Seo, 1999).

This theoretical sequence establishes the basis for the measurement strand. Children's familiarity with the language required to describe measurement relationships—such as longer, taller, shorter, the same length, holds less, holds the same amount—is an important foundation for the concept of measurement (Greenes, 1999) that should be directly addressed in preschool and, thus, is incorporated as part of the mathematics foundations for children at around 48 months of age. Young preschoolers learn to use words that describe measurement relationships as they compare two objects directly to determine equality or inequality, and as they order three or more objects by size. Older preschool children begin to make progress in reasoning about measuring quantities with less dependence on perceptual cues (Clements 2004a; Clements & Stephan, 2004). Children start to compare the length of objects, indirectly, using transitive reasoning, and to measure the length of objects often by using nonstandard units. They develop the ability to think of the length of a small unit (i.e., a block) as part of the length of the object being measured and to place the smaller unit repeatedly along the length of the larger object" (California Department of Education, 2010a, p. 163–164).

In the early elementary school years, children shift to more accuracy in measuring. They begin to use same size units to measure an object, avoiding gaps or overlaps in the process.

COG: MATH 5: Patterning

This measure highlights the developmental processes that involve the child being able to show an increasing ability to recognize, reproduce, and create patterns of varying complexity. The ability to recognize, identify, and create patterns not only supports the development of algebraic thinking, but it also contributes to broader social development. Through an understanding of patterns, children are able to make predictions about what comes next. For example, predicting what comes next after eating lunch (cleaning up) or after taking a bath (putting on clean clothes) will help a child build confidence in his ability to navigate his environment. In preschool years, young children gradually develop the concept of patterns that includes recognizing, describing, creating, and extending a pattern. To understand a pattern, children should be able to identify similarities and differences among elements of a pattern, note the number of elements in the repeatable group, identify when the first group of elements begins to replicate itself, and make predictions about the order of elements based on given information.

"Klein and Starkey (2004) report that young children experience difficulty at the beginning of the year with a fundamental property of repeating patterns: identifying the core unit of the pattern. However, experiences can have a positive impact on young children's knowledge of duplication and extension of patterns

(Klein & Starkey, 2004; Starkey, Klein, & Wakeley, 2004)" (California Department of Education, 2010a, p. 162). "The developmental trajectory of patterns has been characterized as evolving from three-year-old children's ability to identify repeating pattern to four-year-old children's ability to engage in pattern duplication and pattern extension (Klein & Starkey, 2004). The perception of the initial unit plays a fundamental role in both the duplication and extension of patterns. By the end of preschool years, children can transfer smallest repeating unit of a pattern from one mode, for example, letters, to another, (e.g. shapes). The next stage involves children being able to shift to describing components of a growing pattern by describing their numeric or geometric progressions" (California Department of Education, 2010a, p. 163).

COG: MATH 6: Shapes

This measure highlights the ways in which a child shows an increasing knowledge of shapes and their characteristics. Shape knowledge involves not only recognition and naming, but also an understanding of shape characteristics and properties. By the end of the preschool years, children identify and describe a variety of shapes in different sizes and orientations and use defining attributes such as number of sides or angles to create a representation of the shape.

Van Hiele (1986) highlights the fact that children first identify shapes at the visual level on the basis of their appearance, then represent shapes at the "descriptive" level on the basis of their properties, and finally progress to more formal kinds of geometric thinking that are based on logical reasoning abilities. Thus, preschoolers' early shape categories are centered on prototypes, and the similarity of perceptual surface qualities of a shape are used to determine category inclusion. For example, preschoolers do not accept an inverted triangle as a triangle or non-isosceles triangles as triangles (e.g., Clements et al., 1999). Moreover, they tend to regard squares as a distinct category and not as a special kind of rectangle with four sides that are equal in length (Clements, Swaminathan, Hannibal, & Sarama, 1999). "The literature recommends that young children be given the opportunity to work with many varied examples of a particular shape and many 'nonexamples' of a particular shape (Clements, 2004a)" (California Department of Education, 2010a, p. 164). For example, children need to experience examples of triangles that are not just isosceles triangles. "In addition, children need to experience nonexamples of triangles so that they can develop a robust and explicit sense of the properties of a triangle" (California Department of Education, 2010a, p. 164). By the elementary school years, children's shape categories incorporate deeper knowledge of rules and defined attributes (Burger & Shaughnessy, 1986; Satlow & Newcombe, 1998).

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